11

Managing Economies of Scale in a Supply Chain: Cycle Inventory

PowerPoint presentation to accompany Chopra and Meindl Supply Chain Management, 5e

Learning Objectives

1. Balance the appropriate costs to choose the optimal lot size and cycle inventory in a supply chain.
2. Understand the impact of quantity discounts on lot size and cycle inventory.
3. Devise appropriate discounting schemes for a supply chain.
4. Understand the impact of trade promotions on lot size and cycle inventory.
5. Identify managerial levers that reduce lot size and cycle inventory in a supply chain without increasing cost.

Role of Cycle Inventory in a Supply Chain

- *Lot* or *batch size* is the quantity that a stage of a supply chain either produces or purchases at a time
- *Cycle inventory* is the average inventory in a supply chain due to either production or purchases in lot sizes that are larger than those demanded by the customer

Q: Quantity in a lot or batch size

D: Demand per unit time

Inventory Profile

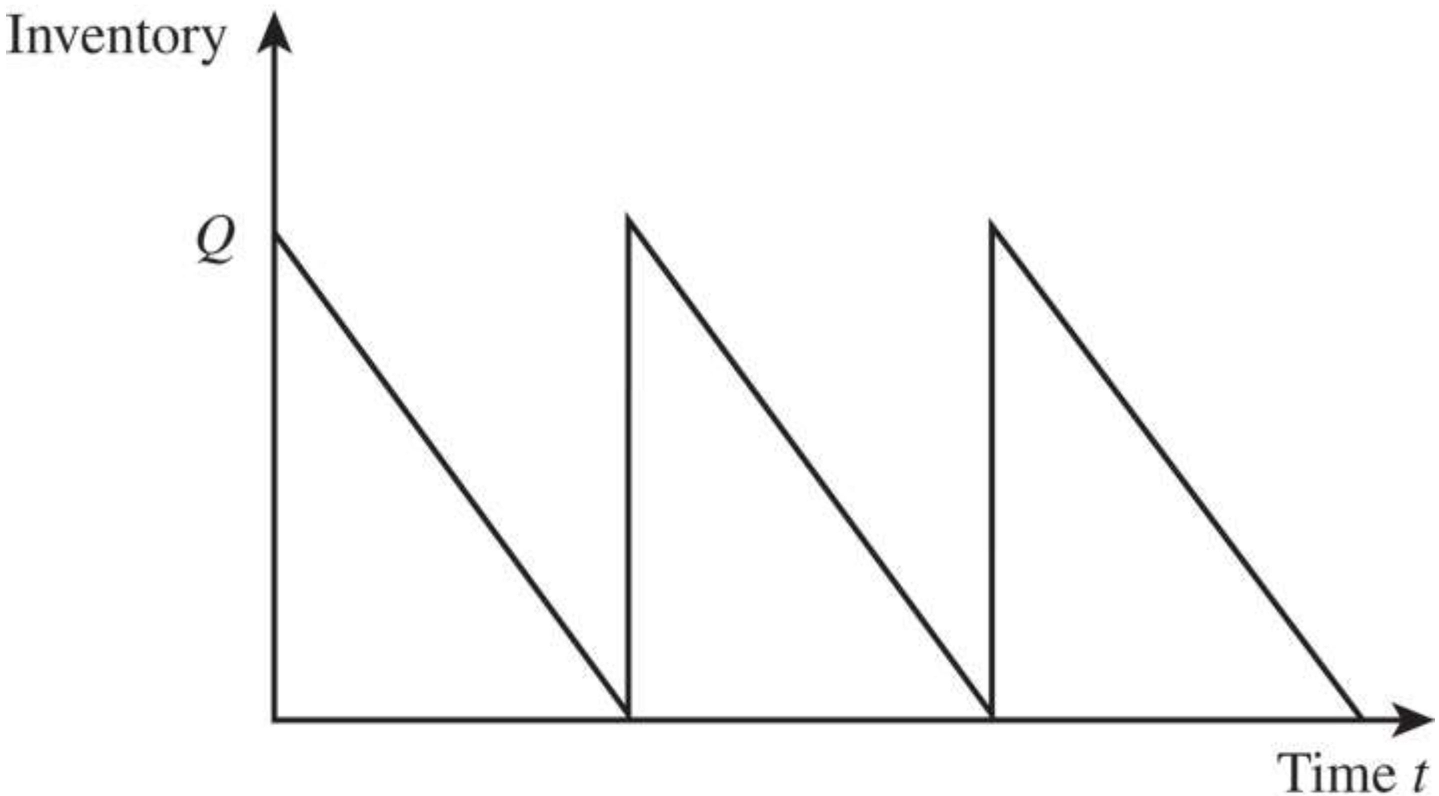


Figure 11-1

Role of Cycle Inventory in a Supply Chain

$$\text{Cycle inventory} = \frac{\text{lot size}}{2} = \frac{Q}{2}$$

$$\text{Average flow time} = \frac{\text{average inventory}}{\text{average flow rate}}$$

$$\text{Average flow time resulting from cycle inventory} = \frac{\text{cycle inventory}}{\text{demand}} = \frac{Q}{2D}$$

Role of Cycle Inventory in a Supply Chain

- Lower cycle inventory has
  - Shorter average flow time
  - Lower working capital requirements
  - Lower inventory holding costs
- Cycle inventory is held to
  - Take advantage of economies of scale
  - Reduce costs in the supply chain

Role of Cycle Inventory in a Supply Chain

- *Average price paid per unit purchased* is a key cost in the lot-sizing decision

  Material cost = C

- *Fixed ordering cost* includes all costs that do not vary with the size of the order but are incurred each time an order is placed

  Fixed ordering cost = S

- *Holding cost* is the cost of carrying one unit in inventory for a specified period of time

  Holding cost = $H = hC$

Role of Cycle Inventory in a Supply Chain

- Primary role of cycle inventory is to allow different stages to purchase product in lot sizes that minimize the sum of material, ordering, and holding costs
- Ideally, cycle inventory decisions should consider costs across the entire supply chain
- In practice, each stage generally makes its own supply chain decisions
- Increases total cycle inventory and total costs in the supply chain

Role of Cycle Inventory in a Supply Chain

- Economies of scale exploited in three typical situations
  1. A fixed cost is incurred each time an order is placed or produced
  2. The supplier offers price discounts based on the quantity purchased per lot
  3. The supplier offers short-term price discounts or holds trade promotions

Estimating Cycle Inventory Related Costs in Practice

- Inventory Holding Cost
  - Cost of capital

$$WACC = \frac{E}{D+E}\left(R_f + b' \; MRP\right) + \frac{D}{D+E} R_b (1 - t)$$

where

E	= amount of equity
D	= amount of debt
R_f	= risk-free rate of return
β	= the firm’s beta
MRP	= market risk premium
R_b	= rate at which the firm can borrow money
t	= tax rate

Estimating Cycle Inventory Related Costs in Practice

- Inventory Holding Cost
  - Cost of capital

Adjusted for pre-tax setting

$$\text{Pretax } WACC = \text{after-tax } WACC / (1 - t)$$

Estimating Cycle Inventory Related Costs in Practice

- Inventory Holding Cost
  - Obsolescence cost
  - Handling cost
  - Occupancy cost
  - Miscellaneous costs
    - Theft, security, damage, tax, insurance

Estimating Cycle Inventory Related Costs in Practice

- Ordering Cost
  - Buyer time
  - Transportation costs
  - Receiving costs
  - Other costs

Economies of Scale to Exploit Fixed Costs

- Lot sizing for a single product (EOQ)

  D = Annual demand of the product

  S = Fixed cost incurred per order

  C = Cost per unit

  H = Holding cost per year as a fraction of product cost

- Basic assumptions
  - Demand is steady at D units per unit time
  - No shortages are allowed
  - Replenishment lead time is fixed

Economies of Scale to Exploit Fixed Costs

- Minimize
  - Annual material cost
  - Annual ordering cost
  - Annual holding cost

Lot Sizing for a Single Product

$$\text{Annual material cost} = CD$$

$$\text{Number of orders per year} = \frac{D}{Q}$$

$$\text{Annual ordering cost} = \left(\frac{D}{Q}\right)S$$

$$\text{Annual holding cost} = \left(\frac{Q}{2}\right)H = \left(\frac{Q}{2}\right)hC$$

$$\text{Total annual cost, } TC = CD + \left(\frac{D}{Q}\right)S + \left(\frac{Q}{2}\right)hC$$

Lot Sizing for a Single Product

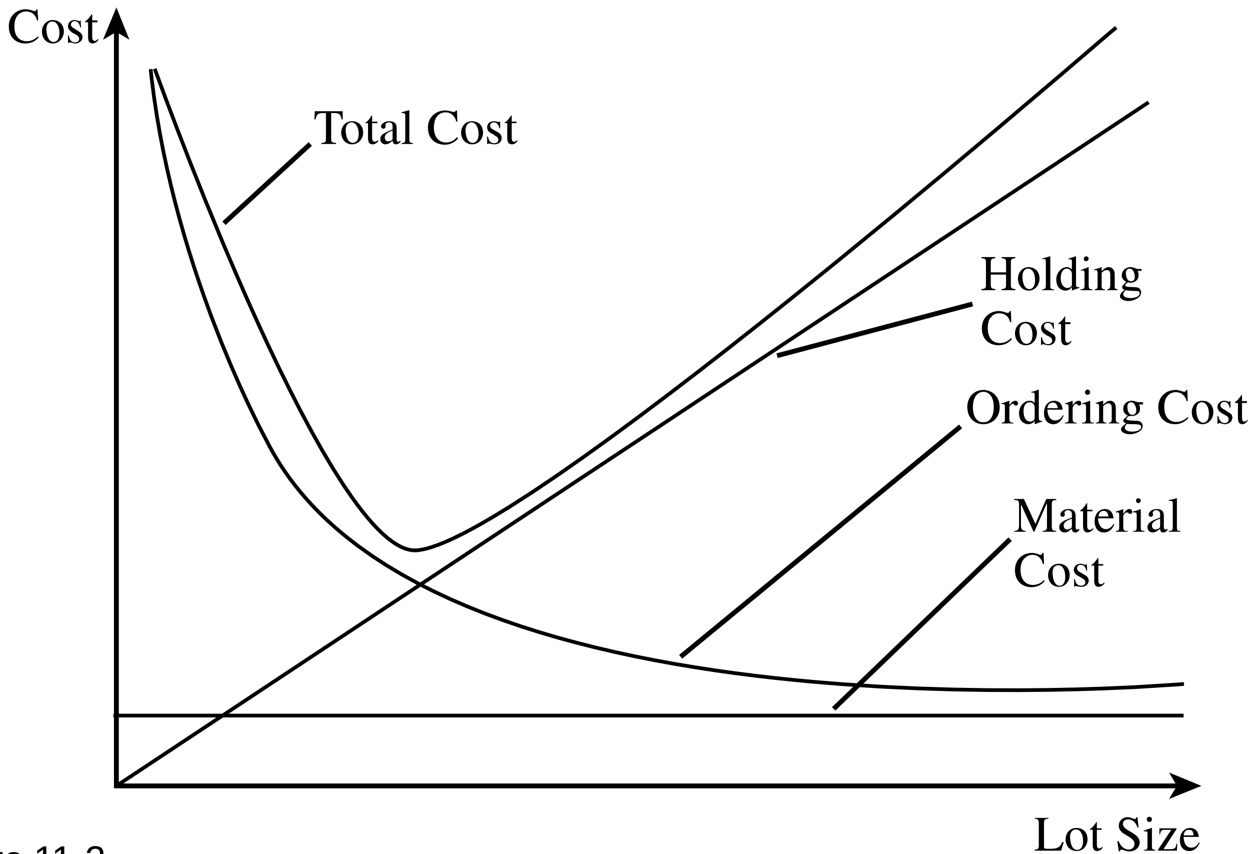


Figure 11-2

Lot Sizing for a Single Product

- The *economic order quantity* (*EOQ*)

$$\text{Optimal lot size, } Q^* = \sqrt{\frac{2DS}{hC}}$$

- The optimal ordering frequency

$$n^* = \frac{D}{Q^*} = \sqrt{\frac{DhC}{2S}}$$

EOQ Example

Annual demand, D = 1,000 x 12 = 12,000 units

Order cost per lot, S = $4,000

Unit cost per computer, C = $500

Holding cost per year as a fraction of unit cost, h = 0.2

$$\text{Optimal order size} = Q^* = \sqrt{\frac{2 \times 12{,}000 \times 4{,}000}{0.2 \times 500}} = 980$$

EOQ Example

$$\text{Cycle inventory} = \frac{Q^*}{2} = \frac{980}{2} = 490$$

$$\text{Number of orders per year} = \frac{D}{Q^*} = 12.24$$

$$\text{Annual ordering and holding cost} = \frac{D}{Q^*}S + \left(\frac{Q^*}{2}\right)hC = 97{,}980$$

$$\text{Average flow time} = \frac{Q^*}{2D} = \frac{490}{12{,}000} = 0.041 = 0.49 \text{ month}$$

EOQ Example

- Lot size reduced to Q = 200 units

$$\text{Annual inventory-related costs} = \frac{D}{Q^*}S + \left(\frac{Q^*}{2}\right)hC = 250{,}000$$

Lot Size and Ordering Cost

- If the lot size $Q^* = 200$, how much should the ordering cost be reduced?

Desired lot size, $Q^* = 200$

Annual demand, $D = 1{,}000 \times 12 = 12{,}000$ units

Unit cost per computer, $C = \$500$

Holding cost per year as a fraction of inventory value, $h = 0.2$

$$S = \frac{hC(Q^*)^2}{2D} = \frac{0.2 \times 500 \times 200^2}{2 \times 12{,}000} = 166.7$$

Production Lot Sizing

- The entire lot does not arrive at the same time
- Production occurs at a specified rate P
- Inventory builds up at a rate of $P - D$

$$Q^P = \sqrt{\frac{2DS}{(1 - D/P)hC}}$$

Annual setup cost

$$\left(\frac{D}{Q^P}\right)S$$

Annual holding cost

$$(1 - D/P)\left(\frac{Q^P}{2}\right)hC$$

Aggregating Multiple Products in a Single Order

- Savings in transportation costs
  - Reduces fixed cost for each product
  - Lot size for each product can be reduced
  - Cycle inventory is reduced
- Single delivery from multiple suppliers or single truck delivering to multiple retailers
- Receiving and loading costs reduced

Lot Sizing with Multiple Products or Customers

- Ordering, transportation, and receiving costs grow with the variety of products or pickup points
- Lot sizes and ordering policy that minimize total cost

D_i: Annual demand for product i

S: Order cost incurred each time an order is placed, independent of the variety of products in the order

s_i: Additional order cost incurred if product i is included in the order

Lot Sizing with Multiple Products or Customers

- Three approaches
  1. Each product manager orders his or her model independently
  2. The product managers jointly order every product in each lot
  3. Product managers order jointly but not every order contains every product; that is, each lot contains a selected subset of the products

Multiple Products Ordered and Delivered Independently

Demand

D_L = 12,000/yr, D_M = 1,200/yr, D_H = 120/yr

Common order cost

S = $4,000

Product-specific order cost

s_L = $1,000, s_M = $1,000, s_H = $1,000

Holding cost

h = 0.2

Unit cost

C_L = $500, C_M = $500, C_H = $500

Multiple Products Ordered and Delivered Independently

	Litepro	Medpro	Heavypro
Demand per year	12,000	1,200	120
Fixed cost/order	$5,000	$5,000	$5,000
Optimal order size	1,095	346	110
Cycle inventory	548	173	55
Annual holding cost	$54,772	$17,321	$5,477
Order frequency	11.0/year	3.5/year	1.1/year
Annual ordering cost	$54,772	$17,321	$5,477
Average flow time	2.4 weeks	7.5 weeks	23.7 weeks
Annual cost	$109,544	$34,642	$10,954

Table 11-1

- Total annual cost = $155,140

Lots Ordered and Delivered Jointly

$$S^* = S + s_L + s_M + s_H$$

$$\text{Annual order cost} = S^* n$$

$$\text{Annual holding cost} = \frac{D_L hC_L}{2n} + \frac{D_M hC_M}{2n} + \frac{D_H hC_H}{2n}$$

$$\text{Total annual cost} = \frac{D_L hC_L}{2n} + \frac{D_M hC_M}{2n} + \frac{D_H hC_H}{2n} + S^* n$$

$$n^* = \sqrt{\frac{D_L hC_L + D_M hC_M + D_H hC_H}{2S^*}}$$

$$n^* = \sqrt{\frac{\sum_{i=1}^{k} D_i hC_i}{2S^*}}$$

Products Ordered and Delivered Jointly

$$S^* = S + s_A + s_B + s_C = \$7{,}000 \text{ per order}$$

$$n^* = \sqrt{\frac{12{,}000 \times 100 + 1{,}200 \times 100 + 120 \times 100}{2 \times 7{,}000}} = 9.75$$

Annual order cost = 9.75 x 7,000 = $68,250

Annual ordering
and holding cost = $61,512 + $6,151 + $615 + $68,250
= $136,528

Products Ordered and Delivered Jointly

	Litepro	Medpro	Heavypro
Demand per year (D)	12,000	1,200	120
Order frequency ($n*$)	9.75/year	9.75/year	9.75/year
Optimal order size ($D/n*$)	1,230	123	12.3
Cycle inventory	615	61.5	6.15
Annual holding cost	$61,512	$6,151	$615
Average flow time	2.67 weeks	2.67 weeks	2.67 weeks

Table 11-2

Aggregation with Capacity Constraint

- W.W. Grainger example

  Demand per product, D_i = 10,000

  Holding cost, h = 0.2

  Unit cost per product, C_i = \$50

  Common order cost, S = \$500

  Supplier-specific order cost, s_i = \$100

Aggregation with Capacity Constraint

$$S^* = S + s_1 + s_2 + s_3 + s_4 = \$900 \text{ per order}$$

$$n^* = \sqrt{\frac{\sum_{i=1}^{4} D_1 h C_1}{2S^*}} = \sqrt{\frac{4 \times 10{,}000 \times 0.2 \times 50}{2 \times 900}} = 14.91$$

$$\text{Annual order cost} = 14.91 \times \frac{900}{4} = \$3{,}354$$

$$\text{Annual holding cost per supplier} = \frac{hC_iQ}{2} = 0.2 \times 50 \times \frac{671}{2} = \$3{,}355$$

Aggregation with Capacity Constraint

Total required capacity per truck = 4 x 671 = 2,684 units

Truck capacity = 2,500 units

Order quantity from each supplier = 2,500/4 = 625

Order frequency increased to 10,000/625 = 16

Annual order cost per supplier increases to $3,600

Annual holding cost per supplier decreases to $3,125.

Lots Ordered and Delivered Jointly for a Selected Subset

Step 1: Identify the most frequently ordered product assuming each product is ordered independently

$$\bar{n}_i = \sqrt{\frac{hC_iD_i}{2(S+s_i)}}$$

Step 2: For all products $i \neq i^*$, evaluate the ordering frequency

$$\bar{\bar{n}}_i = \sqrt{\frac{hC_iD_i}{2s_i}}$$

Lots Ordered and Delivered Jointly for a Selected Subset

Step 3: For all $i \neq i^*$, evaluate the frequency of product i relative to the most frequently ordered product i^* to be m_i

$$m_i = \left\lceil \bar{n} / \bar{n}_i \right\rceil$$

Step 4: Recalculate the ordering frequency of the most frequently ordered product i^* to be n

$$n = \sqrt{\frac{\sum_{i=1}^{l} hC_i m_i D}{2\left(S + \sum_{i=1}^{l} s_i / m_i\right)}}$$

Lots Ordered and Delivered Jointly for a Selected Subset

Step 5: Evaluate an order frequency of $n_i = n/m_i$ and the total cost of such an ordering policy

$$TC = nS + \sum_{i=1}^{l} n_i s_i + \sum_{i-1}^{l} \left(\frac{D_i}{2n_i} \right) hC_1$$

Tailored aggregation – higher-demand products ordered more frequently and lower-demand products ordered less frequently

Ordered and Delivered Jointly – Frequency Varies by Order

- Applying Step 1

$$\bar{n}_L = \sqrt{\frac{hC_L D_L}{2(S + s_L)}} = 11.0$$

$$\bar{n}_M = \sqrt{\frac{hC_M D_M}{2(S + s_M)}} = 3.5$$

$$\bar{n}_L = \sqrt{\frac{hC_H D_H}{2(S + s_H)}} = 1.1$$

Thus

$$\bar{n} = 11.0$$

Ordered and Delivered Jointly – Frequency Varies by Order

- Applying Step 2

$$\bar{\bar{n}}_M = \sqrt{\frac{hC_M D_M}{2s_M}} = 7.7 \text{ and } \bar{\bar{n}}_H = \sqrt{\frac{hC_H D_H}{2s_H}} = 2.4$$

- Applying Step 3

$$m_M = \left[\frac{\bar{n}}{\bar{\bar{n}}_M}\right] = \left[\frac{11.0}{7.7}\right] = 2 \text{ and } m_H = \left[\frac{\bar{n}}{\bar{\bar{n}}_H}\right] = \left[\frac{11.0}{2.4}\right] = 5$$

Ordered and Delivered Jointly – Frequency Varies by Order

	Litepro	Medpro	Heavypro
Demand per year (D)	12,000	1,200	120
Order frequency ($n*$)	11.47/year	5.74/year	2.29/year
Optimal order size ($D/n*$)	1,046	209	52
Cycle inventory	523	104.5	26
Annual holding cost	$52,307	$10,461	$2,615
Average flow time	2.27 weeks	4.53 weeks	11.35 weeks

Table 11-3

Ordered and Delivered Jointly – Frequency Varies by Order

- Applying Step 4

$$n = 11.47$$

- Applying Step 5

$$n_L = 11.47/\text{yr} \qquad n_M = 11.47/2 = 5.74/\text{yr}$$

$$n_H = 11.47/5 = 2.29/\text{yr}$$

Annual order cost

$$nS + n_L s_L + n_M s_M + n_H s_H = \$65{,}383.5$$

Total annual cost

$$\$130{,}767$$

Economies of Scale to Exploit Quantity Discounts

- *Lot size-based discount* – discounts based on quantity ordered in a single lot
- *Volume based discount* – discount is based on total quantity purchased over a given period
- Two common schemes
  - All-unit quantity discounts
  - Marginal unit quantity discount or multi-block tariffs

Quantity Discounts

- Two basic questions
  1. What is the optimal purchasing decision for a buyer seeking to maximize profits? How does this decision affect the supply chain in terms of lot sizes, cycle inventories, and flow times?
  2. Under what conditions should a supplier offer quantity discounts? What are appropriate pricing schedules that a supplier seeking to maximize profits should offer?

All-Unit Quantity Discounts

- Pricing schedule has specified quantity break points $q_0, q_1, \ldots, q_r$, where $q_0 = 0$
- If an order is placed that is at least as large as q_i but smaller than q_{i+1}, then each unit has an average unit cost of C_i
- Unit cost generally decreases as the quantity increases, i.e., $C_0 > C_1 > \ldots > C_r$
- Objective is to decide on a lot size that will minimize the sum of material, order, and holding costs

All-Unit Quantity Discounts

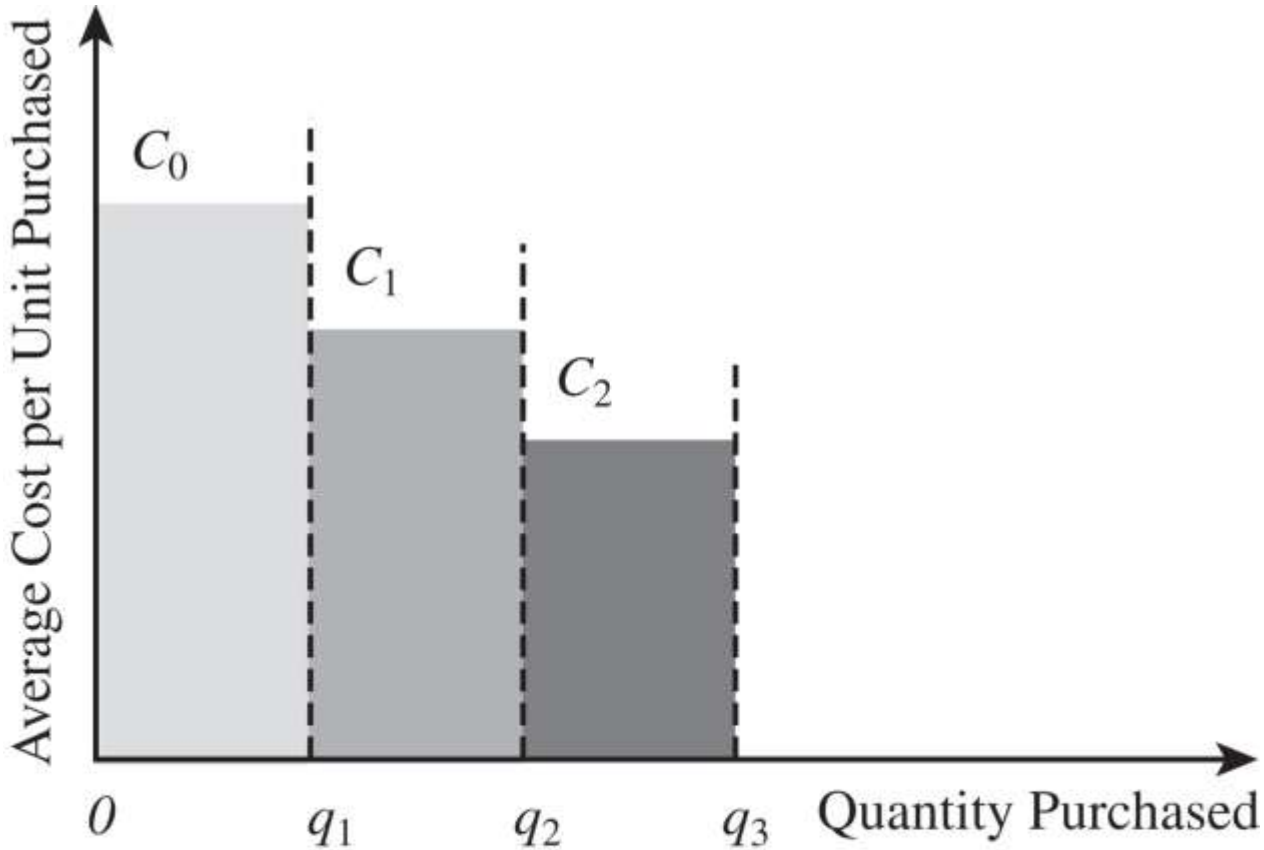


Figure 11-3

All-Unit Quantity Discounts

Step 1: Evaluate the optimal lot size for each price $C_i, 0 \le i \le r$ as follows

$$Q_i = \sqrt{\frac{2DS}{hC_i}}$$

All-Unit Quantity Discounts

Step 2: We next select the order quantity Q^*_i for each price C_i

1. $q_i \le Q_i < q_{i+1}$
2. $Q_i < q_i$
3. $Q_i \ge q_{i+1}$

- Case 3 can be ignored as it is considered for Q_{i+1}
- For Case 1 if $q_i \le Q_i < q_{i+1}$, then set $Q^*_i = Q_i$
- If $Q_i < q_i$, then a discount is not possible
- Set $Q^*_i = q_i$ to qualify for the discounted price of C_i

All-Unit Quantity Discounts

Step 3: Calculate the total annual cost of ordering Q^*_i units

$$\text{Total annual cost, } TC_i = \left(\frac{D}{Q_i^*}\right)S + \left(\frac{Q_i^*}{2}\right)hC_i + DC_i$$

All-Unit Quantity Discounts

Step 4: Select Q^*_i with the lowest total cost TC_i

- Cutoff price

$$C^* = \frac{1}{D}\left(DC_r + \frac{DS}{q_r} + \frac{h}{2}q_r C_r - \sqrt{2hDSC_r}\right)$$

All-Unit Quantity Discount Example

Order Quantity	Unit Price
0–4,999	$3.00
5,000–9,999	$2.96
10,000 or more	$2.92

$q_0 = 0$, $q_1 = 5{,}000$, $q_2 = 10{,}000$

$C_0 = \$3.00$, $C_1 = \$2.96$, $C_2 = \$2.92$

$D = 120{,}000$/year, $S = \$100$/lot, $h = 0.2$

All-Unit Quantity Discount Example

Step 1

$$Q_0 = \sqrt{\frac{2DS}{hC_0}} = 6{,}324;\ Q_1 = \sqrt{\frac{2DS}{hC_1}} = 6{,}367;\ Q_2 = \sqrt{\frac{2DS}{hC_2}} = 6{,}410$$

Step 2

Ignore $i = 0$ because $Q_0 = 6{,}324 > q_1 = 5{,}000$
For $i = 1, 2$

$$Q_1^* = Q_1 = 6{,}367;\ \ Q_2^* = q_2 = 10{,}000$$

All-Unit Quantity Discount Example

Step 3

$$TC_1 = \left(\frac{D}{Q_1^*}\right)S + \left(\frac{Q_1^*}{2}\right)hC_1 + DC_1 = \$358{,}969; \quad TC_2 = \$354{,}520$$

Lowest total cost is for i = 2

Order $Q_2^* = 10{,}000$ bottles per lot at $2.92 per bottle

Marginal Unit Quantity Discounts

- *Multi-block tariffs* – the *marginal cost* of a unit that decreases at a breakpoint

For each value of i, $0 \le i \le r$, let V_i be the cost of ordering q_i units

$$V_i = C_0(q_1 - q_0) + C_1(q_2 - q_1) + \ldots + C_{i-1}(q_i - q_{i-1})$$

Marginal Unit Quantity Discounts

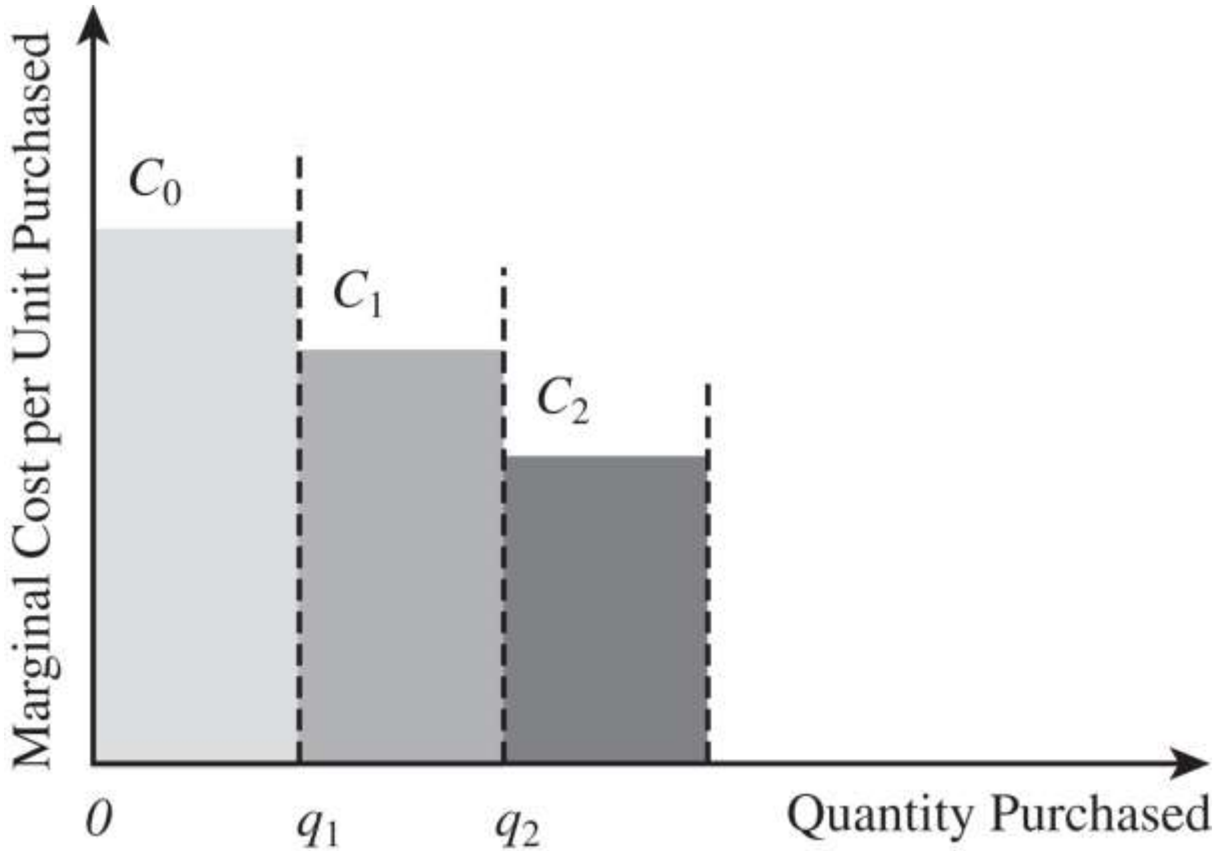


Figure 11-4

Marginal Unit Quantity Discounts

Material cost of each order Q is $V_i + (Q - q_i)C_i$

$$\text{Annual order cost} = \left(\frac{D}{Q}\right)S$$

$$\text{Annual holding cost} = \left[V_i + (Q - q_i)C_i\right]h/2$$

$$\text{Annual materials cost} = \frac{D}{Q}\left[V_i + (Q - q_i)C_i\right]$$

$$\text{Total annual cost} = \left(\frac{D}{Q}\right)S + \left[V_i + (Q - q_i)C_i\right]h/2 + \frac{D}{Q}\left[V_i + (Q - q_i)C_i\right]$$

Marginal Unit Quantity Discounts

Step 1: Evaluate the optimal lot size for each price C_i

Optimal lot size for C_i is $Q_i = \sqrt{\frac{2D(S + V_i - q_i C_i)}{hC_i}}$

Marginal Unit Quantity Discounts

Step 2: Select the order quantity Q_i* for each price C_i

1. If q_i £ Q_i £ q_{i+1} then set $Q_i^* = Q_i$
2. If $Q_i < q_i$ then set $Q_i^* = q_i$
3. If $Q_i > q_{i+1}$ then set $Q_i^* = q_{i+1}$

Marginal Unit Quantity Discounts

Step 3: Calculate the total annual cost of ordering Q_i*

$$TC_i = \left(\frac{D}{Q_i^*}\right)S + \left[V_i + (Q_i^* - q_i)C_i\right]h/2 + \frac{D}{Q_i^*}\left[V_i + (Q_i^* - q_i)C_i\right]$$

Step 4: Select the order size Q_i* with the lowest total cost TC_i

Marginal Unit Quantity Discount Example

- Original data now a marginal discount

Order Quantity	Unit Price
0–4,999	\$3.00
5,000–9,999	\$2.96
10,000 or more	\$2.92

$q_0 = 0,\ q_1 = 5{,}000,\ q_2 = 10{,}000$

$C_0 = \$3.00,\ C_1 = \$2.96,\ C_2 = \$2.92$

$D = 120{,}000\text{/year},\ S = \$100\text{/lot},\ h = 0.2$

Marginal Unit Quantity Discount Example

$$V_0 = 0; \quad V_1 = 3(5{,}000 - 0) = \$15{,}000$$

$$V_2 = 3(5{,}000 - 0) + 2.96(10{,}000 - 5{,}000) = \$29{,}800$$

Step 1

$$Q_0 = \sqrt{\frac{2D(S + V_0 - q_0 C_0)}{hC_0}} = 6{,}324$$

$$Q_1 = \sqrt{\frac{2D(S + V_1 - q_1 C_1)}{hC_1}} = 11{,}028$$

$$Q_2 = \sqrt{\frac{2D(S + V_2 - q_2 C_2)}{hC_2}} = 16{,}961$$

Marginal Unit Quantity Discount Example

Step 2

$$Q_0^* = q_1 = 5{,}000 \text{ because } Q_0 = 6{,}324 > 5{,}000$$

$$Q_1^* = q_2 = 10{,}000; \; Q_2 = Q_2 = 16{,}961$$

Step 3

$$TC_0 = \left(\frac{D}{Q_0^*}\right)S + \left[V_0 + (Q_0^* - q_0)C_0\right]h/2 + \frac{D}{Q_0^*}\left[V_0 + (Q_0^* - q_0)C_0\right] = \$363{,}900$$

$$TC_1 = \left(\frac{D}{Q_1^*}\right)S + \left[V_1 + (Q_1^* - q_1)C_1\right]h/2 + \frac{D}{Q_1^*}\left[V_1 + (Q_1^* - q_1)C_1\right] = \$361{,}780$$

$$TC_2 = \left(\frac{D}{Q_2^*}\right)S + \left[V_2 + (Q_2^* - q_2)C_2\right]h/2 + \frac{D}{Q_2^*}\left[V_2 + (Q_2^* - q_2)C_2\right] = \$360{,}365$$

Why Quantity Discounts?

- Quantity discounts can increase the supply chain surplus for the following two main reasons
  1. Improved coordination to increase total supply chain profits
  2. Extraction of surplus through price discrimination

Quantity Discounts for Commodity Products

D = 120,000 bottles/year, S_R = \$100, h_R = 0.2, C_R = \$3

S_M = \$250, h_M = 0.2, C_M = \$2

$$Q_R = \sqrt{\frac{2DS_R}{h_R C_R}} = \sqrt{\frac{2 \times 120{,}000 \times 100}{0.2 \times 3}} = 6{,}324$$

$$\text{Annual cost for DO} = \left(\frac{D}{Q_R}\right) S_R + \left(\frac{Q_R}{2}\right) h_R C_R = \$3{,}795$$

$$\text{Annual cost for manufacturer} = \left(\frac{D}{Q_R}\right) S_M + \left(\frac{Q_R}{2}\right) h_M C_M = \$6{,}009$$

Annual supply chain cost (manufacturer + DO) = \$6,009 + \$3,795 = \$9,804

Locally Optimal Lot Sizes

Annual cost for DO and manufacturer $= \left(\frac{D}{Q}\right)S_R + \left(\frac{Q}{2}\right)h_R C_R + \left(\frac{D}{Q}\right)S_M + \left(\frac{Q}{2}\right)h_M C_M$

$$Q^* = \sqrt{\frac{2D(S_R + S_M)}{h_R C_R + h_M C_M}} = 9{,}165$$

Annual cost for DO $= \left(\frac{D}{Q^*}\right)S_R + \left(\frac{Q^*}{2}\right)h_R C_R = \$4{,}059$

Annual cost for manufacturer $= \left(\frac{D}{Q^*}\right)S_M + \left(\frac{Q^*}{2}\right)h_M C_M = \$5{,}106$

Annual supply chain cost (manufacturer + DO) = \$5,106 + \$4,059 = \$9,165

Designing a Suitable Lot Size-Based Quantity Discount

- Design a suitable quantity discount that gets DO to order in lots of 9,165 units when its aims to minimize only its own total costs
- Manufacturer needs to offer an incentive of at least $264 per year to DO in terms of decreased material cost if DO orders in lots of 9,165 units
- Appropriate quantity discount is $3 if DO orders in lots smaller than 9,165 units and $2.9978 for orders of 9,165 or more

Quantity Discounts When Firm Has Market Power

Demand curve = 360,000 – 60,000p

Production cost = C_M = \$2 per bottle

$$Prof_R = (p - C_R)(360{,}000 - 60{,}000p)$$

$$Prof_M = (C_R - C_M)(360{,}000 - 60{,}000p)$$

p to maximize $Prof_R$ $\quad p = 3 + \dfrac{C_R}{2}$

$$Prof_M = (C_R - C_M)\left(360{,}000 - 60{,}000\left(3 + \frac{C_R}{2}\right)\right)$$

$$= (C_R - 2)(180{,}000 - 30{,}000C_R)$$

Quantity Discounts When Firm Has Market Power

C_R = \$4 per bottle, p = \$5 per bottle

Total market demand = 360,000 – 60,000p = 60,000

$Prof_R$ = (5 – 4)(360,000 – 60,000 × 5) = \$60,000

$Prof_M$ = (4 – 2)(360,000 – 60,000 × 5) = \$120,000

$$Prof_{SC} = (p - C_M)(360{,}000 - 60{,}000p)$$

Coordinated retail price $p = 3 + \frac{C_M}{2} = 3 + \frac{2}{2} = \$4$

$$Prof_{SC} = (\$4 - \$2) \times 120{,}000 = \$240{,}000$$

Two-Part Tariff

- Manufacturer charges its entire profit as an up-front franchise fee *ff*
- Sells to the retailer at cost
- Retail pricing decision is based on maximizing its profits
- Effectively maximizes the coordinated supply chain profit

Volume-Based Quantity Discounts

- Design a volume-based discount scheme that gets the retailer to purchase and sell the quantity sold when the two stages coordinate their actions

Lessons from Discounting Schemes

- Quantity discounts play a role in supply chain coordination and improved supply chain profits
- Discount schemes that are optimal are volume based and not lot size based unless the manufacturer has large fixed costs associated with each lot
- Even in the presence of large fixed costs for the manufacturer, a two-part tariff or volume-based discount, with the manufacturer passing on some of the fixed cost to the retailer, optimally coordinates the supply chain and maximizes profits

Lessons from Discounting Schemes

- Lot size–based discounts tend to raise the cycle inventory in the supply chain
- Volume-based discounts are compatible with small lots that reduce cycle inventory
- Retailers will tend to increase the size of the lot toward the end of the evaluation period, the *hockey stick phenomenon*
- With multiple retailers with different demand curves optimal discount continues to be volume based with the average price charged to the retailers decreasing as the rate of purchase increases

Price Discrimination to Maximize Supplier Profits

- Firm charges differential prices to maximize profits
- Setting a fixed price for all units does not maximize profits for the manufacturer
- Manufacturer can obtain maximum profits by pricing each unit differently based on customers' marginal evaluation at each quantity
- Quantity discounts are one mechanism for price discrimination because customers pay different prices based on the quantity purchased

Short-Term Discounting: Trade Promotions

- Trade promotions are price discounts for a limited period of time
- Key goals
  1. Induce retailers to use price discounts, displays, or advertising to spur sales
  2. Shift inventory from the manufacturer to the retailer and the customer
  3. Defend a brand against competition

Short-Term Discounting: Trade Promotions

- Impact on the behavior of the retailer and supply chain performance
- Retailer has two primary options
  1. Pass through some or all of the promotion to customers to spur sales
  2. Pass through very little of the promotion to customers but purchase in greater quantity during the promotion period to exploit the temporary reduction in price (*forward buy*)

Forward Buying Inventory Profile

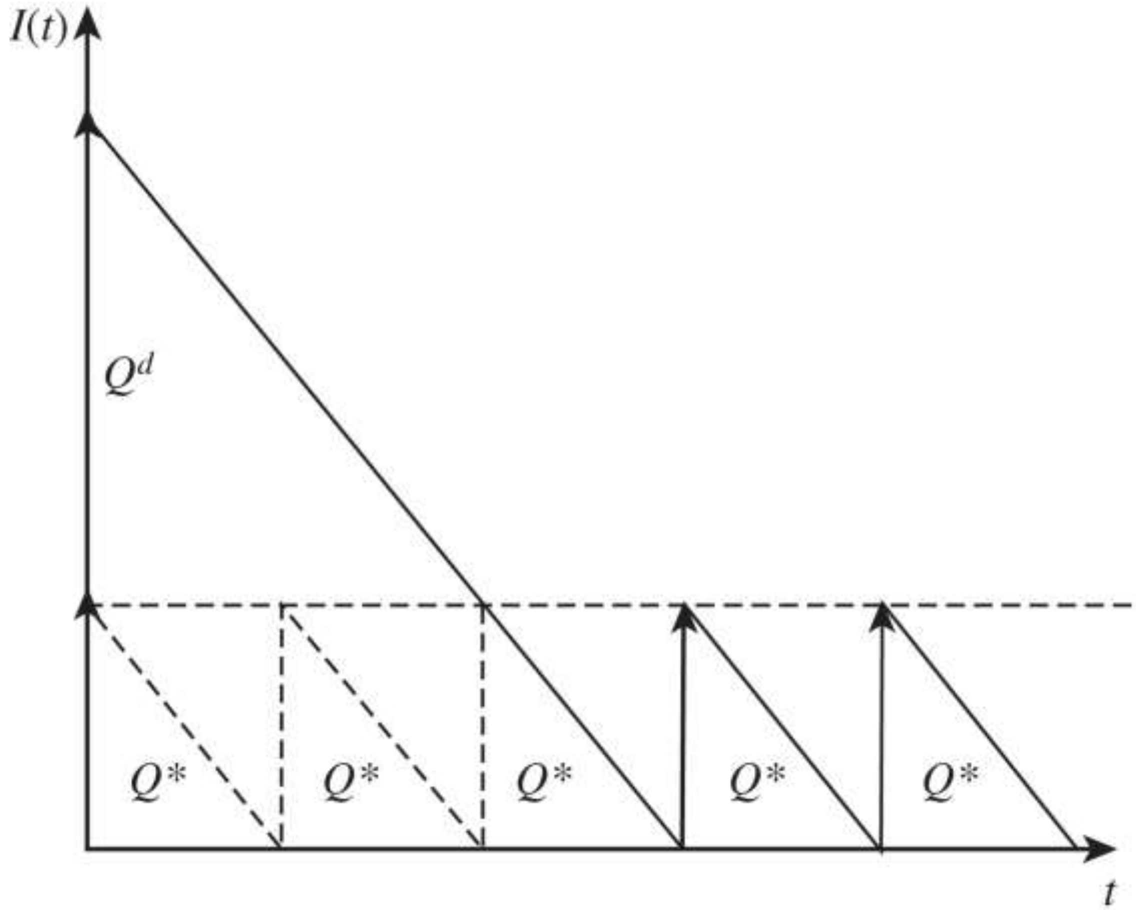


Figure 11-5

Forward Buy

- Costs to be considered – material cost, holding cost, and order cost
- Three assumptions
  1. The discount is offered once, with no future discounts
  2. The retailer takes no action to influence customer demand
  3. Analyze a period over which the demand is an integer multiple of Q^*

Forward Buy

- Optimal order quantity

$$Q^d = \frac{dD}{(C-d)h} + \frac{CQ^*}{C-d}$$

- Retailers are often aware of the timing of the next promotion

$$\text{Forward buy} = Q^d - Q^*$$

Impact of Trade Promotions on Lot Sizes

$Q^* = 6{,}324$ bottles, $C = \$3$ per bottle

$d = \$0.15$, $D = 120{,}000$, $h = 0.2$

Cycle inventory at DO = $Q^*/2 = 6{,}324/2 = 3{,}162$ bottles

Average flow time = $Q^*/2D = 6{,}324/(2D) = 0.3162$ months

$$Q^d = \frac{dD}{(C-d)h} + \frac{CQ^*}{C-d}$$

$$= \frac{0.15 \times 120{,}000}{(3.00 - 0.15) \times 0.20} + \frac{3 \times 6{,}324}{3.00 - 0.15} = 38{,}236$$

Impact of Trade Promotions on Lot Sizes

- With trade promotions

Cycle inventory at DO = $Q^d/2$ = 38,236/2 = 19,118 bottles

Average flow time = $Q^d/2D$ = 38,236/(20,000)

= 1.9118 months

Forward buy = $Q^d - Q^*$ = 38,236 – 6,324 = 31,912 bottles

How Much of a Discount Should the Retailer Pass Through?

- Profits for the retailer

$$Prof_R = (300{,}000 - 60{,}000p)p - (300{,}000 - 60{,}000p)C_R$$

- Optimal price

$$p = (300{,}000 + 60{,}000C_R)/120{,}000$$

- Demand with no promotion

$$D_R = 30{,}000 - 60{,}000p = 60{,}000$$

- Optimal price with discount

$$p = (300{,}000 + 60{,}000 \text{ x } 2.85)/120{,}000 = \$3.925$$

- Demand with promotion

$$D_R = 300{,}000 - 60{,}000p = 64{,}500$$

Trade Promotions

- Trade promotions generally increase cycle inventory in a supply chain and hurt performance
- Counter measures
  - EDLP (every day low pricing)
  - Discount applies to items sold to customers (sell-through) not the quantity purchased by the retailer (sell-in)
  - Scan based promotions

Managing Multiechelon Cycle Inventory

- Multi-echelon supply chains have multiple stages with possibly many players at each stage
- Lack of coordination in lot sizing decisions across the supply chain results in high costs and more cycle inventory than required
- The goal is to decrease total costs by coordinating orders across the supply chain

Managing Multiechelon Cycle Inventory

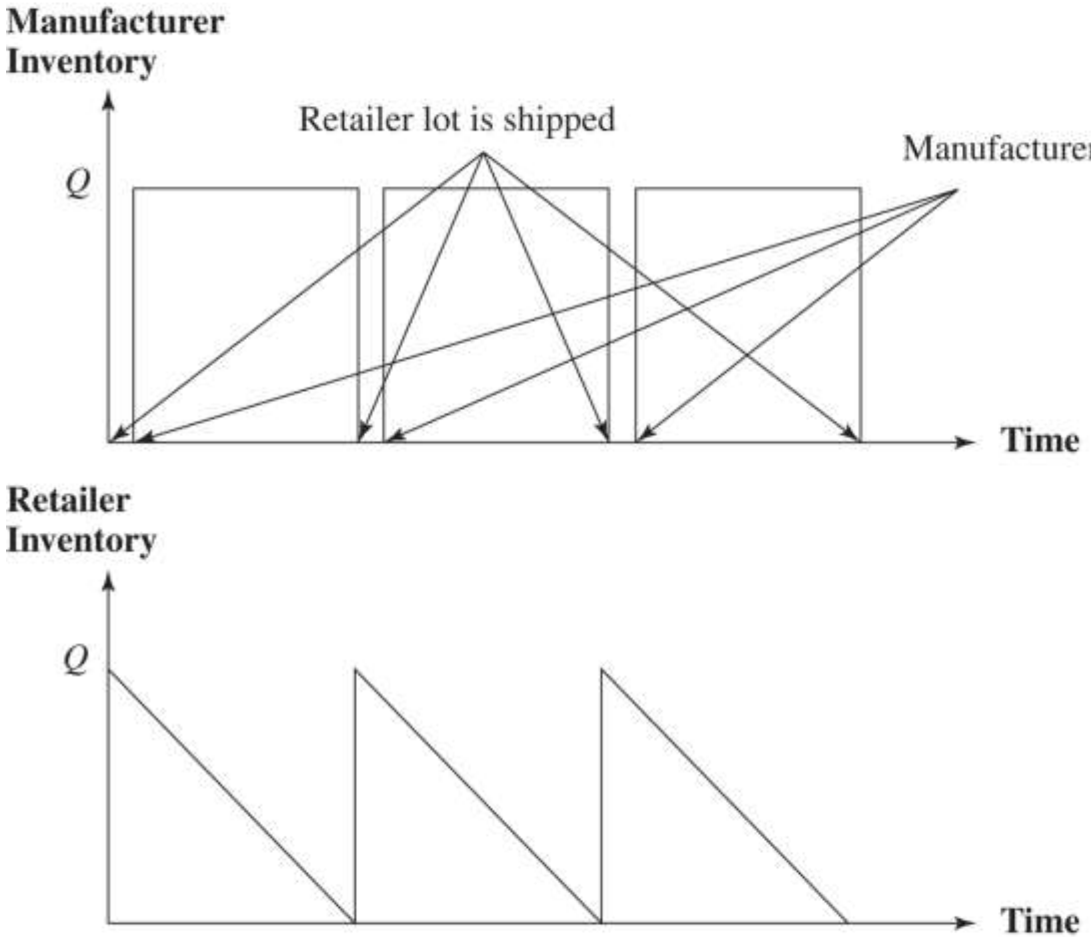


Figure 11-6

Integer Replenishment Policy

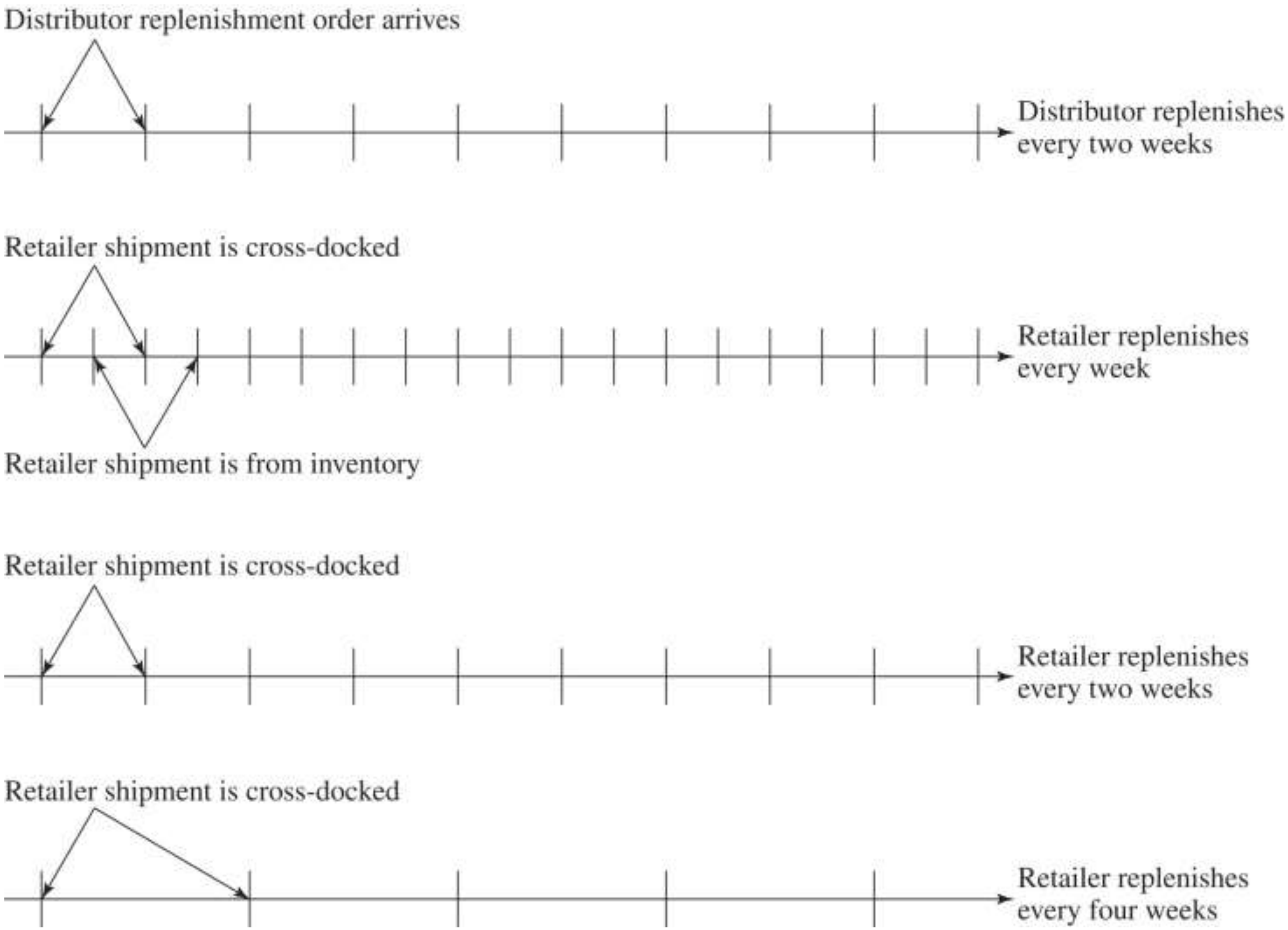
- Divide all parties within a stage into groups such that all parties within a group order from the same supplier and have the same reorder interval
- Set reorder intervals across stages such that the receipt of a replenishment order at any stage is synchronized with the shipment of a replenishment order to at least one of its customers
- For customers with a longer reorder interval than the supplier, make the customer’s reorder interval an integer multiple of the supplier’s interval and synchronize replenishment at the two stages to facilitate cross-docking

Integer Replenishment Policy

- For customers with a shorter reorder interval than the supplier, make the supplier's reorder interval an integer multiple of the customer's interval and synchronize replenishment at the two stages to facilitate cross-docking
- The relative frequency of reordering depends on the setup cost, holding cost, and demand at different parties
- These polices make the most sense for supply chains in which cycle inventories are large and demand is relatively predictable

Integer Replenishment Policy

Figure 11-7



Integer Replenishment Policy

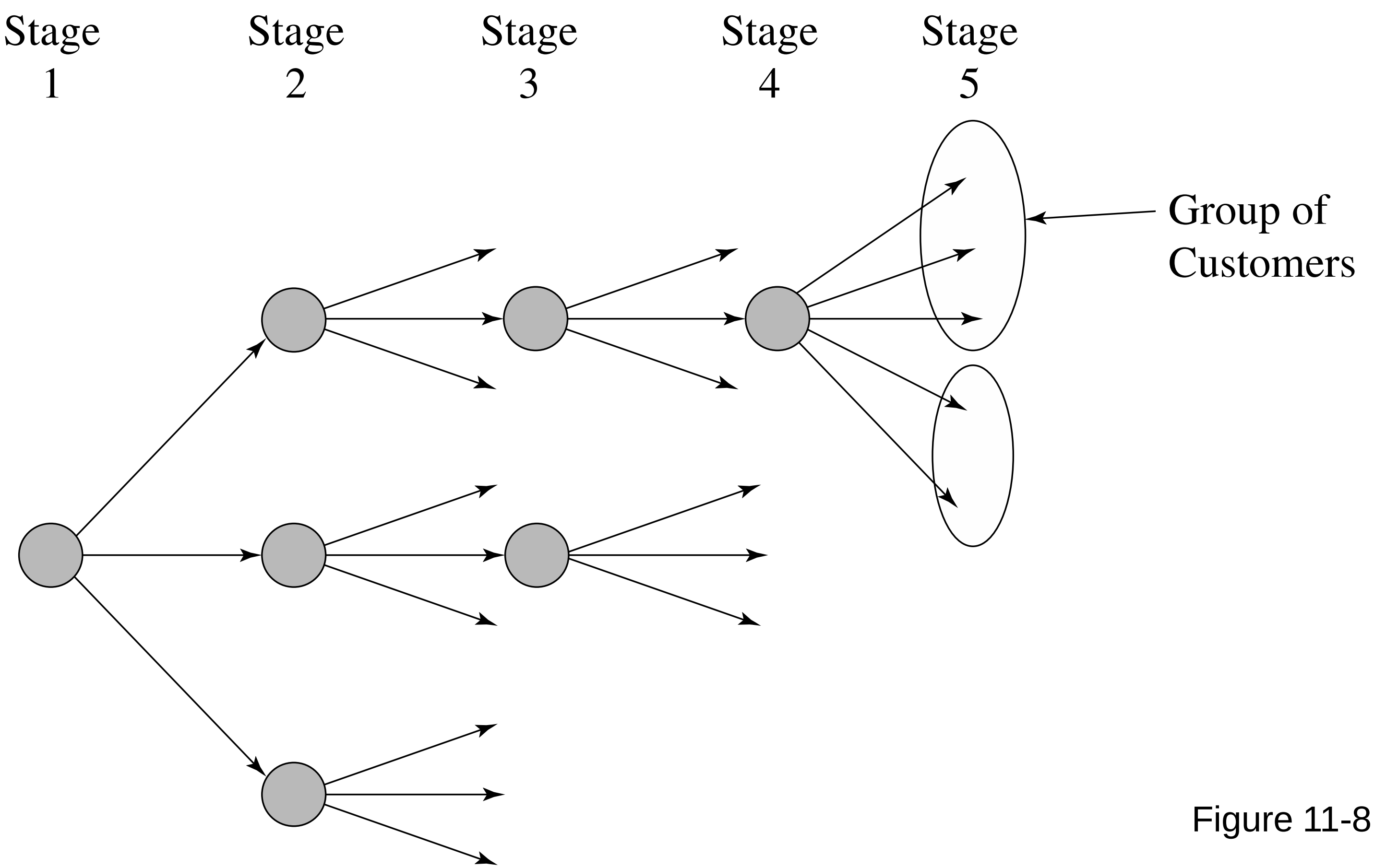


Figure 11-8

Summary of Learning Objectives

1. Balance the appropriate costs to choose the optimal lot size and cycle inventory in a supply chain
2. Understand the impact of quantity discounts on lot size and cycle inventory
3. Devise appropriate discounting schemes for a supply chain
4. Understand the impact of trade promotions on lot size and cycle inventory

Summary of Learning Objectives

5. Identify managerial levers that reduce lot size and cycle inventory in a supply chain without increasing cost
   - Reduce fixed ordering and transportation costs incurred per order
   - Implement volume-based discounting schemes rather than individual lot size–based discounting schemes
   - Eliminate or reduce trade promotions and encourage EDLP – base trade promotions on sell-through rather than sell-in to the retailer