convenient origin as an [illegible]

Example 20.1 Given the data below, test the hypothesis that the means of the three populations are equal. Let $\alpha = 0.05$.

Sample 1	Sample 2	Sample 3
40	70	45
50	65	38
60	66	60
65	50	42

(i) We state our null and alternative hypotheses as

$H_0 : \mu_1 = \mu_2 = \mu_3$, *i.e.* all the three means are equal, and

H_1 : Not all three means are equal.

(ii) The significance level is set at $\alpha = 0.05$.

(iii) The test-statistic to use is

$$F = \frac{s_b^2}{s_w^2},$$

which, if H_0 is true, has an F-distribution with $v_1 = k-1$ and $v_2 = n-k$ degrees of freedom.

(iv) The computations are carried out as below:

	Sample 1 X_{i1} (X_{i1}^2)	Sample 2 X_{i2} (X_{i2}^2)	Sample 3 X_{i3} (X_{i3}^2)	Total	$\sum_j X_{ij}^2$
	40 (1600)	70 (4900)	45 (2025)	---	8525
	50 (2500)	65 (4225)	38 (1444)	---	8169
	60 (3600)	66 (4356)	60 (3600)	---	11556
	65 (4225)	50 (2500)	42 (1764)	---	8489
$T_{.j}$	215	251	185	651	36739
$T_{.j}^2$	46225	63001	34225	143451	↑
$\sum_i X_{ij}^2$	11925	15981	8833	36739	← check

$$\text{Correction Factor } (C.F.) = \frac{T_{..}^2}{n} = \frac{(651)^2}{12} = 35316.75$$

$$\text{Total} \qquad SS = \sum_i \sum_j X_{ij}^2 - C.F.$$

$$= 36739 - 35316.75 = 1422.25$$

$$\text{Correction Factor } (C.F.) = \frac{T_{..}^2}{n} = \frac{(651)^2}{12} = 35316.75$$

$$\text{Total } SS = \sum_i \sum_j X_{ij}^2 - C.F.$$

$$= 36739 - 35316.75 = 1422.25$$

$$\text{Between } SS = \frac{\sum_j T_{.j}^2}{r} - C.F.$$

$$= \frac{143451}{4} - 35316.75 = 546.00, \text{ and}$$

$$\text{Within } SS = \text{Total } SS - \text{Between } SS = 1422.25 - 546.00 = 876.25$$

The Analysis of Variance table is:

Source of Variation	*d.f.*	Sum of Squares	Mean Square	Computed *F*
Between Samples	2	546.00	273.00	$\frac{273.00}{97.36} = 2.80$
Within Samples	9	876.25	97.36	....
Total Variation	11	1422.25	---	...

(v) The critical region is $F \geq F_{0\cdot05}(2, 9) = 4.26$

(vi) **Conclusion.** Since the calculated value of $F = 2.80$ does not fall in the critical region, so we accept our null hypothesis and conclude that all the three means are equal.