

3. A researcher wonders about the sources of support for restrictive immigration policies and uses a nationally representative U.S. sample to test the relationships between a variety of variables (including gender, occupation, and education) and support for the idea that immigration should be reduced. If the researcher finds significant relationships in the sample, he will conclude that those variables are related in the population (all adult Americans).

The **chi square ( $\chi^2$ ) test** has probably been the most frequently used test of hypothesis in the social sciences—a popularity that is due largely to the fact that the assumptions and requirements in step 1 of the five-step model are easy to satisfy. Specifically, the test can be conducted with variables measured at the nominal level (the lowest level of measurement), and because it is a **nonparametric**, or “distribution free,” test, chi square requires no assumption at all about the shape of the population or sampling distribution.

Why is it an advantage to have easy-to-satisfy assumptions and requirements? The decision to reject the null hypothesis (step 5) is not specific; it means only that one statement in the model (step 1) *or* the null hypothesis (step 2) is wrong. Usually, of course, we single out the null hypothesis for rejection. The more certain we are of the model, the greater our confidence that the null hypothesis is the faulty assumption. A “weak” or easily satisfied model means that our decision to reject the null hypothesis can be made with even greater certainty.

Chi square has also been popular for its flexibility. Not only can it be used with variables at any level of measurement, but it also can be used with variables that have many categories or scores. For example, in Chapter 8, we tested the significance of the difference in the proportions of black and white citizens who were “highly participatory” in voluntary associations. What if the researcher wished to expand the test to include Hispanic and Asian Americans? The two-sample test would no longer be applicable, but chi square handles the more complex variable easily. Also, unlike the ANOVA test presented in Chapter 10, the chi square test can be conducted with variables at any level of measurement.

## The Bivariate Table

Chi square is computed from **bivariate tables**—so called because they display the scores of cases on two different variables at the same time. Bivariate tables are used to ascertain if there is a significant relationship between the variables and for other purposes that we will investigate in later chapters. In fact, these tables are very commonly used in research and a detailed examination of them is in order.

First of all, bivariate tables have (of course) two dimensions. We refer to the horizontal dimension (across) in terms of **rows** and the vertical dimension (up and down) in terms of **columns**. Each column or row represents a score on a variable, and the intersections of the rows and columns (**cells**) represent the combined scores on both variables.

Let us use an example to clarify. Suppose a researcher is interested in senior citizens and wants to see if their participation in voluntary groups, community-service organizations, and so forth, is affected by their marital status. To simplify the analysis, the researcher has confined the sample to people who are presently married or not married (including people who are single and divorced) and has measured involvement as a simple dichotomy on which people are classified as either high or low.

By convention, the independent variable (the variable that is taken to be the cause) is placed in the columns and the dependent variable in the rows. In the example at hand, marital status is the causal variable (the question was “Is membership *affected by* marital status?”) and each column will represent a score on this variable. On the other hand, each row will represent a score on level of involvement (high or low). Table 10.1 displays the outline of the bivariate table for a sample of 100 senior citizens.

Note some further details of the table. First, subtotals have been added to each column and row. These are called the row or column **marginals**, and in this case, they tell us that 50 members of the sample are married and 50 are not married (the column marginals), while 50 are high in participation and 50 are rated low (the row marginals). Second, the total number of cases in the sample ( $N = 100$ ) is reported at the intersection of the row and column marginals. Finally, take careful note of the labeling of the table. Each row and column is identified, and the table has a descriptive title that includes the names of the variables, with the dependent variable listed first. Clear, complete labels and concise titles must be included in *all* tables, graphs, and charts.

As you have noticed, Table 10.1 lacks one piece of crucial information: the numbers in the body of the table. To finish the table, we need to classify the marital status and level of participation of each member of the sample, keep count of how often each combination of scores occurs, and record these numbers in the appropriate cell of the table. Because each variable has two scores, four combinations of scores are possible—each corresponding to a cell in the table. For example, married people with high levels of participation would be counted in the upper left-hand cell, nonmarried people with low levels of participation would be counted in the lower right-hand cell, and so forth. When we are finished counting, each cell will display the number of times each *combination* of scores occurred.

Finally, note how we could expand the bivariate table to accommodate variables with more scores. If we wished to include people with other marital statuses (widowed, separated, and so forth), we would simply add columns. More elaborate dependent variables could also be easily accommodated. If we had measured participation rates with three categories (e.g., high, moderate, and low) rather than two, we would simply add a row to the table.

**TABLE 10.1 Rates of Participation in Voluntary Associations by Marital Status for 100 Senior Citizens**

| Participation Rates | Marital Status |             |     |
|---------------------|----------------|-------------|-----|
|                     | Married        | Not Married |     |
| High                |                |             | 50  |
| Low                 |                |             | 50  |
|                     | 50             | 50          | 100 |

## The Logic of Chi Square

The chi square test has several different uses, but we will cover only the *chi square test for independence*. We have encountered the term *independence* in connection with the requirements for the two-sample case (Chapter 8) and for the ANOVA test (Chapter 9). In the context of chi square, the concept of **independence** refers to the relationship between the variables, not between the samples. Two variables are independent if the classification of a case into a particular category of one variable has no effect on the probability that the case will fall into any particular category of the second variable. For example, marital status and participation in voluntary associations would be independent of each other if the classification of a case as married or not married has no effect on the classification of the case as high or low on participation. In other words, the variables are independent if level of participation and marital status are completely unrelated to each other.

Consider Table 10.1 again. If the variables are independent, the cell frequencies will be determined solely by random chance and we would find that just as an honest coin will show heads about 50% of the time, about half the married respondents will rank high on participation and half will rank low. The same pattern would hold for the 50 nonmarried respondents. In fact, each of the four cells would have 25 cases in it, as illustrated in Table 10.2. This pattern of cell frequencies indicates that marital status has no effect on the probability that a person would be either high or low in participation. The probability of being classified as high or low would be 0.50 for both marital statuses, and the variables would therefore be independent.

The null hypothesis for chi square is that the variables are independent. Under the assumption that the null hypothesis is true, the cell frequencies we would expect to find if only random chance were operating are computed. These frequencies—called **expected frequencies** (symbolized  $f_e$ )—are then compared, cell by cell, with the frequencies actually observed in the table (**observed frequencies**, symbolized  $f_o$ ). If the null hypothesis is true and the variables are independent, then there should be little difference between the expected and observed frequencies. However, if the null hypothesis is false, there should be large differences between the two. The greater the differences between expected ( $f_e$ ) and observed ( $f_o$ ) frequencies, the less likely that the variables are independent and the more likely that we will be able to reject the null hypothesis.

**TABLE 10.2** The Cell Frequencies That Would Be Expected If Rates of Participation and Marital Status Were Independent

| Participation Rates | Marital Status |             |     |
|---------------------|----------------|-------------|-----|
|                     | Married        | Not Married |     |
| High                | 25             | 25          | 50  |
| Low                 | 25             | 25          | 50  |
|                     | 50             | 50          | 100 |

## The Computation of Chi Square

As with all tests of hypothesis, with chi square, we compute a test statistic— $\chi^2$  (**obtained**)—from the sample data and then place that value on the sampling distribution of all possible sample outcomes. Specifically,  $\chi^2$ (obtained) will be compared with the value of  $\chi^2$ (**critical**) that will be determined by consulting a chi square table (Appendix C) for a particular alpha level and degrees of freedom. Prior to conducting the formal test of hypothesis, let us take a moment to consider the calculation of chi square, as defined by Formula 10.1:

$$\text{FORMULA 10.1} \quad \chi^2 (\text{obtained}) = \sum \frac{(f_o - f_e)^2}{f_e}$$

where:  $f_o$  = the cell frequencies observed in the bivariate table  
 $f_e$  = the cell frequencies that would be expected  
 if the variables were independent

We must work cell by cell to solve this formula. To compute chi square, subtract the expected frequency from the observed frequency for each cell, square the result, divide by the expected frequency for that cell, and then sum the resultant values for all cells.

This formula requires an expected frequency for each cell in the table. In Table 10.2, the marginals are the same value for all rows and columns and the expected frequencies are obvious by intuition:  $f_e = 25$  for all four cells. In the more usual case, the expected frequencies will not be obvious, marginals will be unequal, and we must use Formula 10.2 to find the expected frequency for each cell:

$$\text{FORMULA 10.2} \quad f_e = \frac{\text{Row marginal} \times \text{Column marginal}}{N}$$

That is, the expected frequency for any cell is equal to the total number of cases in the row (the row marginal) multiplied by the total number of cases in the column (the column marginal) divided by the total number of cases in the table ( $N$ ).

### A Computational Example

An example using Table 10.3 should clarify these procedures. A random sample of 100 social work majors has been classified in terms of whether the Council on Social Work Education has accredited their undergraduate programs (the column, or independent, variable) and whether they were hired in social work positions within three months of graduation (the row, or dependent, variable).

Beginning with the upper left-hand cell (graduates of accredited programs who are working as social workers), the expected frequency for this cell—using Formula 10.2—is  $(40 \times 55)/100$ , or 22. For the other cell in this row (graduates of nonaccredited programs who are working as social workers), the expected frequency is  $(40 \times 45)/100$ , or 18. For the two cells in the bottom row, the expected frequencies are  $(60 \times 55)/100$ , or 33, and  $(60 \times 45)/100$ , or 27, respectively. The expected frequencies for all four cells are displayed in Table 10.4.

**TABLE 10.3 Employment of 100 Social Work Majors by Accreditation Status of Undergraduate Program**

| Employment Status              | Accreditation Status |                | Totals    |
|--------------------------------|----------------------|----------------|-----------|
|                                | Accredited           | Not Accredited |           |
| Working as a social worker     | 30                   | 10             | 40        |
| Not working as a social worker | <u>25</u>            | <u>35</u>      | <u>60</u> |
| Totals                         | 55                   | 45             | 100       |

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**TABLE 10.4 Expected Frequencies for Table 10.3**

| Employment Status              | Accreditation Status |                | Totals    |
|--------------------------------|----------------------|----------------|-----------|
|                                | Accredited           | Not Accredited |           |
| Working as a social worker     | 22                   | 18             | 40        |
| Not working as a social worker | <u>33</u>            | <u>27</u>      | <u>60</u> |
| Totals                         | 55                   | 45             | 100       |

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**TABLE 10.5 Computational Table for Table 10.3**

| (1)       | (2)       | (3)         | (4)                               | (5)                 |
|-----------|-----------|-------------|-----------------------------------|---------------------|
| $f_o$     | $f_e$     | $f_o - f_e$ | $(f_o - f_e)^2$                   | $(f_o - f_e)^2/f_e$ |
| 30        | 22        | 8           | 64                                | 2.91                |
| 10        | 18        | -8          | 64                                | 3.56                |
| 25        | 33        | -8          | 64                                | 1.94                |
| <u>35</u> | <u>27</u> | <u>8</u>    | 64                                | <u>2.37</u>         |
| $N = 100$ | $N = 100$ | 0           | $\chi^2(\text{obtained}) = 10.78$ |                     |

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Note that the row and column marginals as well as the total number of cases in Table 10.4 are exactly the same as those in Table 10.3. The row and column marginals for the expected frequencies must *always* equal those of the observed frequencies—a relationship that provides a convenient way of checking your arithmetic to this point.

The value for chi square for these data can now be found by solving Formula 10.1. It will be helpful to use a computing table, such as Table 10.5, to organize the several steps required to compute chi square. The table lists the observed frequencies ( $f_o$ ) in column 1 in order from the upper left-hand cell to the lower right-hand cell, moving left to right across the table and top to bottom. The second column lists the expected frequencies ( $f_e$ ) in exactly the same order. Double-check to make sure you have listed the cell frequencies in the same order for both of these columns.

The next step is to subtract the expected frequency from the observed frequency for each cell and then list these values in the third column. To complete the fourth column, square the value in column 3; then, in column 5, divide the column 4 value by the expected frequency for that cell. Finally, add up column 5. The sum of this column is  $\chi^2(\text{obtained})$ :  $\chi^2(\text{obtained}) = 10.78$ .

## ONE STEP AT A TIME Computing Chi Square

Begin by preparing a computing table similar to Table 10.5. List the observed frequencies ( $f_o$ ) in column 1. The total for column 1 is the number of cases ( $N$ ).

### Step Operation

*To Find the Expected Frequencies ( $f_e$ ) by Using Formula 10.2*

1. Start with the upper left-hand cell and multiply the row marginal by the column marginal for that cell.
2. Divide the quantity you found in step 1 by  $N$ . The result is the expected frequency ( $f_e$ ) for that cell. Record this value in the second column of the computing table. Make sure you place the value of  $f_e$  in the same row as the observed frequency for that cell.
3. Repeat steps 1 and 2 for each cell in the table. Double-check to make sure you are using the correct row and column marginals. Record each  $f_e$  in the second column of the computational table.
4. Find the total of the expected frequencies column. This total *must* equal the total of the observed frequencies column (which is the same as  $N$ ). If the two totals do not match (within rounding error), you have made a mistake and need to check your computations.

*To Find Chi Square by Using Formula 10.1*

1. For each cell, subtract the expected frequency ( $f_e$ ) from the observed frequency ( $f_o$ ) and then list these values in the third column of the computational table ( $f_o - f_e$ ). Find the total for this column. If this total is not zero, you have made a mistake and need to check your computations.
2. Square each of the values in the third column of the table and then record the result in the fourth column, labeled  $(f_o - f_e)^2$ .
3. Divide each value in column 4 by the expected frequency for that cell and then record the result in the fifth column, labeled  $(f_o - f_e)^2/f_e$ .
4. Find the total for the fifth column. This value is  $\chi^2(\text{obtained})$ .

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Note that the totals for columns 1 and 2 ( $f_o$  and  $f_e$ ) are exactly the same. This will always be the case, and if the totals do not match, you have made a computational error—probably in the calculation of the expected frequencies. Also note that the sum of column 3 will always be zero—another convenient way to check your math to this point.

This sample value for chi square must still be tested for its significance. (*For practice in computing chi square, see any problem at the end of this chapter.*)

## The Chi Square Test for Independence

We are now ready to conduct the chi square test for independence. Recall that if the variables are independent, the score of a case on one variable will have no relationship with its score on the other variable. As always, the five-step model for significance testing will provide the framework for organizing our decision making. The data presented in Table 10.3 will serve as our example.



**Step 1. Making Assumptions and Meeting Test Requirements.** Note that we make no assumptions at all about the shape of the sampling distribution.

Model: Independent random samples  
Level of measurement is nominal

**Step 2. Stating the Null Hypothesis.** The null hypothesis states that the two variables are independent. If the null hypothesis is true, the differences between the observed and expected frequencies will be small. As usual, the research hypothesis directly contradicts the null hypothesis. Thus, if we reject  $H_0$ , the research hypothesis will be supported:

$H_0$ : The two variables are independent.  
( $H_1$ : The two variables are dependent.)

**Step 3. Selecting the Sampling Distribution and Establishing the Critical Region.** The sampling distribution of sample chi squares, unlike the  $Z$  and  $t$  distributions, is positively skewed, with higher values of sample chi squares in the upper tail of the distribution (to the right). Thus, with the chi square test, the critical region is established in the upper tail of the sampling distribution.

Values for  $\chi^2(\text{critical})$  are given in Appendix C. This table is similar to the  $t$  table, with alpha levels arrayed across the top and degrees of freedom down the side. However, a major difference is that degrees of freedom ( $df$ ) for chi square are found by this formula:

FORMULA 10.3

$$df = (r - 1)(c - 1)$$

A table with two rows and two columns (a  $2 \times 2$  table) has one degree of freedom regardless of the number of cases in the sample.<sup>1</sup> A table with two rows and three columns would have  $(2 - 1)(3 - 1)$ , or two degrees of freedom. Our sample problem involves a  $2 \times 2$  table with  $df = 1$ , so if we set alpha at 0.05, the critical chi square score would be 3.841. Summarizing these decisions, we have:

$$\begin{aligned}\text{Sampling distribution} &= \chi^2 \text{ distribution} \\ \text{Alpha} &= 0.05 \\ \text{Degrees of freedom} &= 1 \\ \chi^2(\text{critical}) &= 3.841\end{aligned}$$

---

<sup>1</sup>Degrees of freedom are the number of values in a distribution that are free to vary for any particular statistic. A  $2 \times 2$  table has one degree of freedom because for a given set of marginals, once one cell frequency is determined, all other cell frequencies are fixed and no longer free to vary. For example, in Table 10.3, if any cell frequency is known, all others are determined. If the upper left-hand cell is 30, the other cell in that row must be 10 because there are 40 cases in the row and  $40 - 30 = 10$ . Once the cell frequencies in the top row are known, those in the bottom row can be found by subtraction from the column marginal. Incidentally, this relationship can be used to quickly compute expected frequencies. In a  $2 \times 2$  table, only one expected frequency needs to be computed. All others can be found by subtraction.

**Step 4. Computing the Test Statistic.** The mechanics of these computations were introduced earlier. As you recall, we had:

$$\chi^2(\text{obtained}) = \sum \frac{(f_o - f_e)^2}{f_e}$$

$$\chi^2(\text{obtained}) = 10.78$$

**Step 5. Making a Decision and Interpreting the Results of the Test.** Comparing the test statistic with the critical region,

$$\chi^2(\text{obtained}) = 10.78$$

$$\chi^2(\text{critical}) = 3.841$$

we see that the test statistic falls into the critical region; therefore, we reject the null hypothesis of independence. The pattern of cell frequencies observed in Table 10.3 is unlikely to have occurred by chance alone. The variables are dependent. Specifically, based on these sample data, the probability of securing employment in the field of social work is dependent on the accreditation status of the program. (*For practice in conducting and interpreting the chi square test for independence, see problems 10.2 to 10.16.*)

Let us stress exactly what the chi square test does and does not tell us. A significant chi square means that the variables are (probably) dependent on each other in the population; accreditation status makes a difference in whether a person is working as a social worker. What chi square does *not* tell us is the exact nature of the relationship. In our example, it does not tell us which type of graduate is more likely to be working as a social worker. To make this determination, we must perform some additional calculations. We can figure out how the independent variable (accreditation status) is affecting the dependent variable (employment as a social worker) by computing **column percentages** or by calculating percentages within each column of the bivariate table. This procedure is analogous to calculating percentages for frequency distributions (see Chapter 2).

To calculate column percentages, divide each cell frequency by the total number of cases in the column (the column marginal) and then multiply the result by 100. For Table 10.3, starting in the upper left-hand cell, we see that there are 30 cases in this cell and 55 cases in the column. Thus, 30 of the 55 graduates of accredited programs are working as social workers. The column percentage for this cell is therefore  $(30/55) \times 100 = 54.55\%$ . For the lower left-hand cell, the column percentage is  $(25/55) \times 100 = 45.45\%$ . For the two cells in the right-hand column (graduates of nonaccredited programs), the column percentages are  $(10/45) \times 100 = 22.22$  and  $(35/45) \times 100 = 77.78$ . Table 10.6 displays all column percentages for Table 10.3.

Column percentages allow us to examine the bivariate relationship in more detail and show us exactly how the independent variable affects the dependent variable. In this case, they reveal that graduates of accredited programs are more likely to find work as social workers. We explore column percentages more extensively when we discuss bivariate association in Chapters 11 and 12.

Column percentages make the relationship between the variables more obvious, and we can easily see from Table 10.6 that it is the students from accredited programs who are more likely to be working as social workers. Nearly 55% of these students are working as social workers vs. about 22% of the students from



**TABLE 10.6 Column Percentages for Table 10.3**

| Employment Status              | Accreditation Status |                | Totals  |
|--------------------------------|----------------------|----------------|---------|
|                                | Accredited           | Not Accredited |         |
| Working as a social worker     | 54.55%               | 22.22%         | 40.00%  |
| Not working as a social worker | 45.45%               | 77.78%         | 60.00%  |
| Totals                         | 100.00%              | 100.00%        | 100.00% |
|                                | (55)                 | (45)           |         |

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nonaccredited programs. We already know that this relationship is significant (unlikely to be caused by random chance). With the aid of column percentages, we know how the two variables are related. According to these results, graduates from accredited programs have a decided advantage in securing social work jobs.

### ONE STEP AT A TIME Computing Column Percentages

#### Step Operation

1. Start with the upper left-hand cell. Divide the cell frequency (the number of cases in the cell) by the total number of cases in that column (or the column marginal). Multiply the result by 100 to convert to a percentage.
2. Move down one cell and then repeat step 1. Continue moving down the column, cell by cell, until you have converted all cell frequencies to percentages.
3. Move to the next column. Start with the cell in the top row and then repeat step 1 (making sure you use the correct column total in the denominator of the fraction).
4. Continue moving down the second column until you have converted all cell frequencies to percentages.
5. Continue these operations, moving from column to column one at a time, until you have converted all cell frequencies to percentages.

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### Applying Statistics 10.1 The Chi Square Test

Do men and women vary in their opinions about cohabitation? A random sample of 47 males and females have been asked if they approve or disapprove of “living together.” The results are:

| Support for Cohabitation | Gender |         | Totals |
|--------------------------|--------|---------|--------|
|                          | Males  | Females |        |
| Approve                  | 15     | 5       | 20     |
| Disapprove               | 10     | 17      | 27     |
| Totals                   | 25     | 22      | 47     |

The frequencies we would expect to find if the null hypothesis ( $H_0$ : the variables are independent) were true are:

| Support for Cohabitation | Group |         | Totals |
|--------------------------|-------|---------|--------|
|                          | Males | Females |        |
| Approve                  | 10.64 | 9.36    | 20.00  |
| Disapprove               | 14.36 | 12.64   | 27.00  |
| Totals                   | 25.00 | 22.00   | 47.00  |

(continued next page)

### Applying Statistics 10.1 (continued)

Expected frequencies are found on a cell-by-cell basis by means of the formula

$$f_e = \frac{\text{Row marginal} \times \text{Column marginal}}{N}$$

and the calculation of chi square will be organized into a computational table:

| (1)      | (2)         | (3)         | (4)                       | (5)                   |
|----------|-------------|-------------|---------------------------|-----------------------|
| $f_o$    | $f_e$       | $f_o - f_e$ | $(f_o - f_e)^2$           | $(f_o - f_e)^2 / f_e$ |
| 15       | 10.64       | 4.36        | 19.01                     | 1.79                  |
| 5        | 9.36        | -4.36       | 19.01                     | 2.03                  |
| 10       | 14.36       | -4.36       | 19.01                     | 1.32                  |
| 17       | 12.64       | 4.36        | 19.01                     | 1.50                  |
| $N = 47$ | $N = 47.00$ | 0.00        | $\chi^2(\text{obtained})$ | $= 6.64$              |

$$\chi^2(\text{obtained}) = 6.64$$

#### Step 1. Making Assumptions and Meeting Test Requirements.

Model: Independent random samples  
Level of measurement is nominal

#### Step 2. Stating the Null Hypothesis.

$H_0$ : The two variables are independent.

( $H_1$ : The two variables are dependent.)

#### Step 3. Selecting the Sampling Distribution and Establishing the Critical Region.

Sampling distribution =  $\chi^2$  distribution  
Alpha = 0.05

Degrees of freedom = 1

$$\chi^2(\text{critical}) = 3.841$$

#### Step 4. Computing the Test Statistic.

$$\chi^2(\text{obtained}) = \sum \frac{(f_o - f_e)^2}{f_e}$$

$$\chi^2(\text{obtained}) = 6.64$$

**Step 5. Making a Decision and Interpreting the Results of the Test.** With an obtained  $\chi^2$  of 6.64, we would reject the null hypothesis of independence. For this sample, there is a statistically significant relationship between gender and support for cohabitation.

To complete the analysis, it would be useful to know exactly how the two variables are related. We can determine this by computing and analyzing column percentages:

| Support for Cohabitation | Males   | Females | Totals  |
|--------------------------|---------|---------|---------|
| Approve                  | 60.00%  | 22.73%  | 42.55%  |
| Disapprove               | 40.00%  | 77.27%  | 57.45%  |
| Totals                   | 100.00% | 100.00% | 100.00% |

The column percentages show that 60% of males in this sample approve of cohabitation vs. only about 23% of females. We have already concluded that the relationship is significant, and now we know the pattern of the relationship: Males are more supportive.

Let us highlight two points in summary:

1. Chi square is a test of statistical significance. It tests the null hypothesis that the variables are independent in the population. If we reject the null hypothesis, we are concluding, with a known probability of error (determined by the alpha level), that the variables are dependent on each other in the population. In the terms of our example, this means that accreditation status makes a difference in the likelihood of finding work as a social worker. However, by itself, chi square does not tell us the nature or pattern of the relationship.