

16

Learning Objectives

When you have completed this chapter, you will be able to:

- L01** Define the components of a *time series*.
- L02** Compute a *moving average*.
- L03** Determine a *linear trend equation*.
- L04** Use a trend equation to compute forecasts.
- L05** Compute a nonlinear trend equation.
- L06** Determine and interpret a set of seasonal indexes.
- L07** Deseasonalize data using a seasonal index.
- L08** Calculate seasonally adjusted forecasts.
- L09** Test for autocorrelation.

Time Series and Forecasting



Team Sports Inc. sells sporting goods to high schools and colleges via a nationally distributed catalog. Management at Team Sports estimates it will sell 2,000 Wilson Model A2000 catcher's mitts next year. The deseasonalized sales are projected to be the same for each of the four quarters next year. The seasonal factor for the second quarter is 145. Determine the seasonally adjusted sales for the second quarter of next year. (See Exercise 12 and L08.)

16.1 Introduction

What is a time series?

The emphasis in this chapter is on time series analysis and forecasting. A **time series** is a collection of data recorded over a period of time—weekly, monthly, quarterly, or yearly. Two examples of time series are Microsoft Corporation sales by quarter since 1985 and the annual production of sulfuric acid since 1970.



An analysis of history—a time series—is used by management to make current decisions and plans based on long-term forecasting. We usually assume past patterns will continue into the future. Long-term forecasts extend more than 1 year into the future; 2-, 5-, and 10-year projections are common. Long-range predictions are essen-

tial to allow sufficient time for the procurement, manufacturing, sales, finance, and other departments of a company to develop plans for possible new plants, financing, development of new products, and new methods of assembling.

Forecasting the level of sales, both short-term and long-term, is practically dictated by the very nature of business organizations in the United States. Competition for the consumer's dollar, stress on earning a profit for the stockholders, a desire to procure a larger share of the market, and the ambitions of executives are some of the prime motivating forces in business. Thus, a forecast (a statement of the goals of management) is necessary to have the raw materials, production facilities, and staff to meet the projected demand.

This chapter deals with the use of data to forecast future events. First, we look at the components of a time series. Then, we examine some of the techniques used in analyzing data. Finally, we forecast future events.

16.2 Components of a Time Series

L01 Define the components of a *time series*.

There are four components to a time series: the trend, the cyclical variation, the seasonal variation, and the irregular variation.

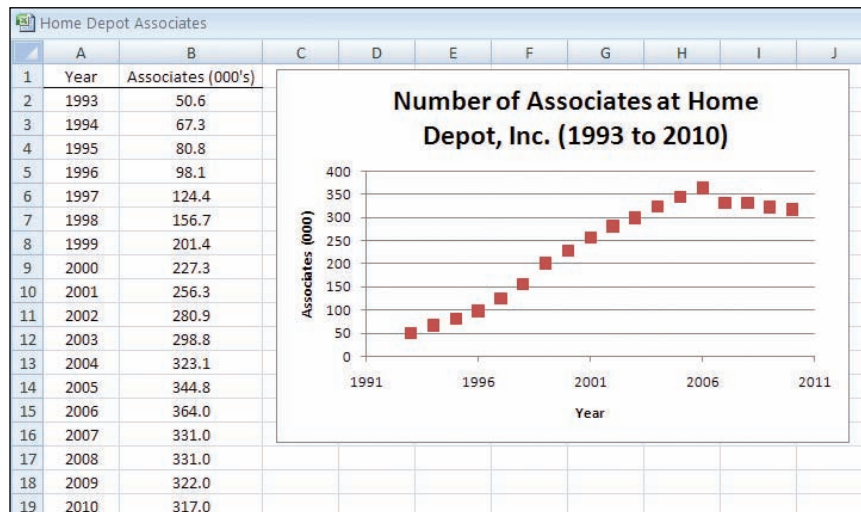
Secular Trend

The long-term trends of sales, employment, stock prices, and other business and economic series follow various patterns. Some move steadily upward, others decline, and still others stay the same over time.

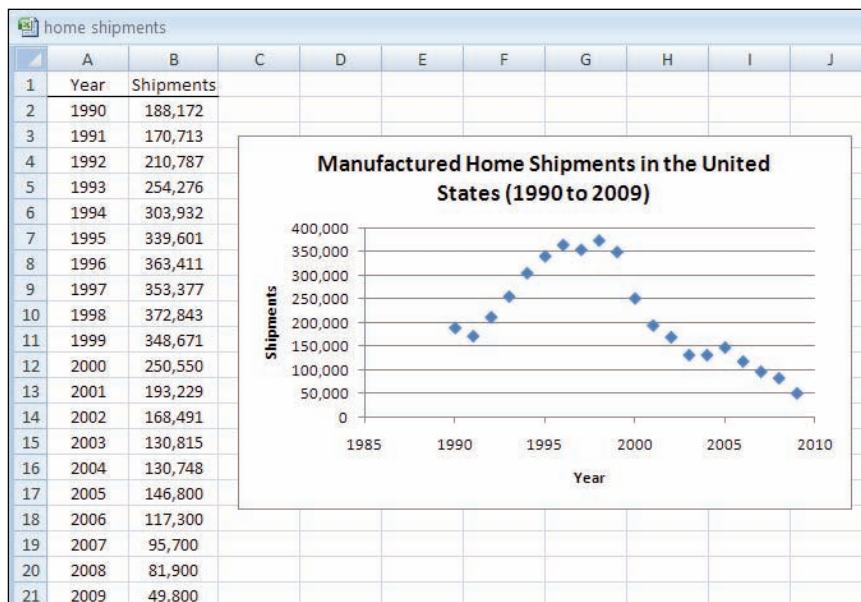
SECULAR TREND The smooth long-term direction of a time series.

The following are several examples of a secular trend.

- Home Depot was founded in 1978 and is the world's largest home improvement retailer. The following chart shows the number of employees at Home Depot Inc. You can see this number has increased rapidly over the last 15 years. In 1993, there were just over 50,000 associates and by 2006 that number increased to 364,000. Since then, the number of associates has declined to 317,000 in 2010.



- The number of manufactured homes shipped in the United States showed a steady increase from 1990 to 1996, then remained about the same until 1999, when the number began to decline. By the year 2002, the number shipped was less than it had been in 1990 and continued to decline to 2009. This information is shown in the following chart.



Cyclical Variation

The second component of a time series is cyclical variation. A typical business cycle consists of a period of prosperity followed by periods of recession, depression, and then recovery. There are sizable fluctuations unfolding over more than one year in time above and below the secular trend. In a recession, for example, employment, production, the Dow Jones Industrial Average, and many other business and economic series are below the long-term trend lines. Conversely, in periods of prosperity they are above their long-term trend lines.

CYCLICAL VARIATION The rise and fall of a time series over periods longer than one year.

Chart 16–1 shows the annual unit sales of batteries sold by National Battery Retailers Inc. from 1991 through 2010. The cyclical nature of business is highlighted. There are periods of recovery, followed by prosperity, then contraction, and finally the cycle bottoms out with depression.

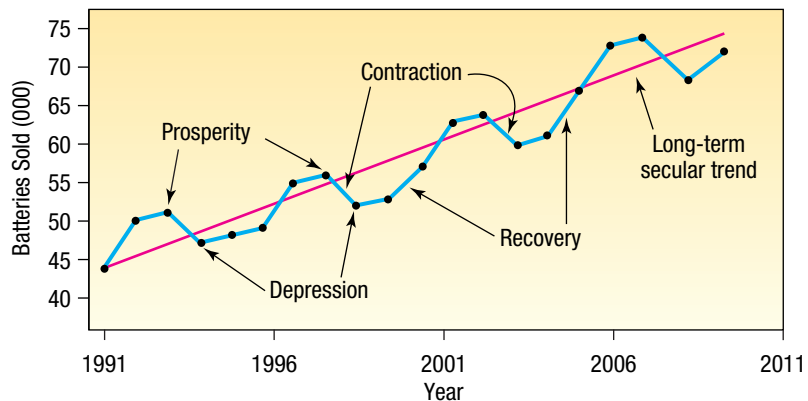


CHART 16–1 Batteries Sold by National Battery Retailers Inc. from 1991 to 2010

Seasonal Variation

The third component of a time series is the **seasonal variation**. Many sales, production, and other series fluctuate with the seasons. The unit of time reported is either quarterly or monthly.

SEASONAL VARIATION Patterns of change in a time series within a year. These patterns tend to repeat themselves each year.

Almost all businesses tend to have recurring seasonal patterns. Men's and boy's clothing, for example, have extremely high sales just prior to Christmas and relatively low sales just after Christmas and during the summer. Toy sales is another example with an extreme seasonal pattern. More than half of the business for the year is usually done in the months of November and December. The lawn care business is seasonal in the northeast and north-central states. Many businesses try to even out the seasonal effects by engaging in an offsetting seasonal business. In the Northeast, you will see the operator of a lawn care business with a snowplow on the front of the truck in an effort to earn income in the off-season. At ski resorts throughout the country, you will often find golf courses nearby. The owners of the lodges try to rent to skiers in the winter and golfers in the summer. This is an effective method of spreading their fixed costs over the entire year rather than a few months.

Chart 16–2 shows the quarterly sales, in millions of dollars, of Hercher Sporting Goods Inc. The Chicago area sporting goods company specializes in selling baseball and softball equipment to high schools, colleges, and youth leagues. It also has several retail outlets in some of the larger shopping malls. There is a distinct seasonal pattern to its business. Most of its sales are in the first and second quarters of the year, when schools and organizations are purchasing equipment for the upcoming season. During the early summer, it keeps busy by selling replacement equipment. It does some business during the holidays (fourth quarter). The late summer (third quarter) is its slow season.



Statistics in Action

Statisticians, economists, and business executives are constantly looking for variables that will forecast the country's economy. The production of crude oil, price of gold on world markets, and the Dow Jones average, as well as many published government indexes are variables that have been used with some success. Variables such as the length of hemlines and the winner of the Super Bowl have also been tried. The variable that seems overall to be the most successful is the price of scrap metal. Why? Scrap metal is the beginning of the manufacturing chain. When its demand increases, this is an indication that manufacturing is also increasing.

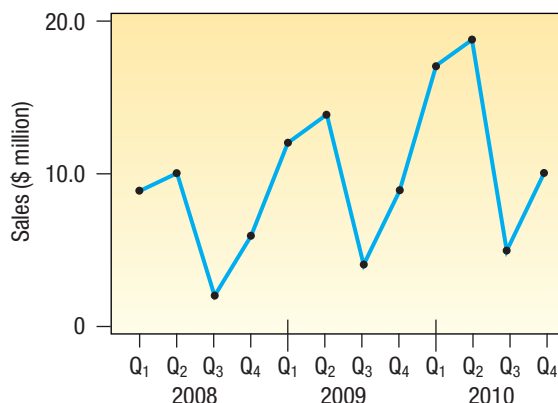


CHART 16-2 Sales of Baseball and Softball Equipment, Hercher Sporting Goods, 2008–2010 by Quarter

Irregular Variation

Many analysts prefer to subdivide the **irregular variation** into *episodic* and *residual* variations. Episodic fluctuations are unpredictable, but they can be identified. The initial impact on the economy of a major strike or a war can be identified, but a strike or war cannot be predicted. After the episodic fluctuations have been removed, the remaining variation is called the residual variation. The residual fluctuations, often called chance fluctuations, are unpredictable, and they cannot be identified. Of course, neither episodic nor residual variation can be projected into the future.

16.3 A Moving Average

Moving-average method smooths out fluctuations

L02 Compute a *moving average*.

A **moving average** is useful in smoothing a time series to see its trend. It is also the basic method used in measuring the seasonal fluctuation, described later in the chapter. In contrast to the least squares method, which expresses the trend in terms of a mathematical equation ($\hat{Y} = a + bt$), the moving-average method merely smooths the fluctuations in the data. This is accomplished by “moving” the arithmetic mean values through the time series.

To apply the moving average to a time series, the data should follow a fairly linear trend and have a definite rhythmic pattern of fluctuations (repeating, say, every three years). The data in the following example have three components—trend, cycle, and irregular, abbreviated *T*, *C*, and *I*. There is no seasonal variation, because the data are recorded annually. What the moving average accomplishes is to average out *C* and *I*. What is left is the trend.

If the duration of the cycles is constant, and if the amplitudes of the cycles are equal, the cyclical and irregular fluctuations are removed entirely using the moving average. The result is a line. For example, in the following time series the cycle repeats itself every seven years, and the amplitude of each cycle is 4; that is, there are exactly four units from the trough (lowest time period) to the peak. The seven-year moving average, therefore, averages out the cyclical and irregular fluctuations perfectly, and the residual is a linear trend.

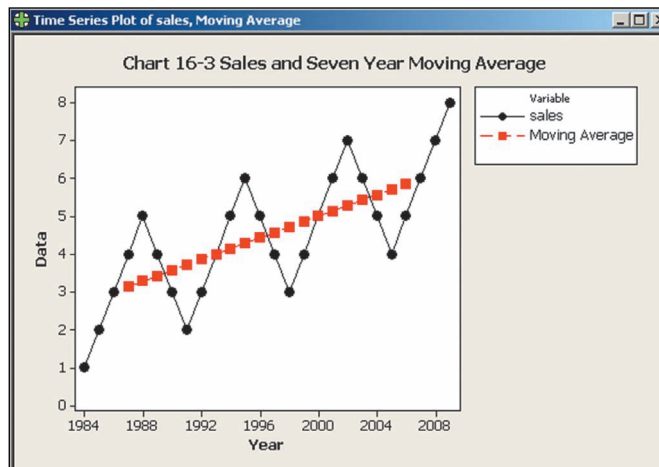
Compute mean of first seven years

The first step in computing the seven-year moving average is to determine the seven-year moving totals. The total sales for the first seven years (1984–90 inclusive) are \$22 million, found by $1 + 2 + 3 + 4 + 5 + 4 + 3$. (See Table 16–1.) The total of \$22 million is divided by 7 to determine the arithmetic mean sales per year. The seven-year total (22) and the seven-year mean (3.143) are positioned opposite the middle year for that group of seven, namely, 1987, as shown in Table 16–1. Then

TABLE 16-1 The Computations for the Seven-Year Moving Average

Year	Sales (\$ millions)	Seven-Year Moving Total	Seven-Year Moving Average
1984	\$1		
1985	2		
1986	3		
1987	4	22	3.143
1988	5	23	3.286
1989	4	24	3.429
1990	3	25	3.571
1991	2	26	3.714
1992	3	27	3.857
1993	4	28	4.000
1994	5	29	4.143
1995	6	30	4.286
1996	5	31	4.429
1997	4	32	4.571
1998	3	33	4.714
1999	4	34	4.857
2000	5	35	5.000
2001	6	36	5.143
2002	7	37	5.286
2003	6	38	5.429
2004	5	39	5.571
2005	4	40	5.714
2006	5	41	5.857
2007	6		
2008	7		
2009	8		

the total sales for the next seven years (1985–91 inclusive) are determined. (A convenient way of doing this is to subtract the sales for 1984 [\$1 million] from the first seven-year total [\$22 million] and add the sales for 1991 [\$2 million], to give the new total of \$23 million.) The mean of this total, \$3.286 million, is positioned opposite the middle year, 1988. The sales data and seven-year moving average are shown graphically in Chart 16–3.

**CHART 16-3** Sales and Seven-Year Moving Average

The number of data values to include in a moving average depends on the character of the data collected. If the data are quarterly, since there are four quarters in a year, then four terms might be typical. If the data are daily, since there are seven days in a week, then seven terms might be appropriate. You might also use trial and error to determine a number that best levels out the chance fluctuations.

A moving average can be easily computed in Excel. In fact, it requires only one command. If the original data are in locations D3 to D20 and you wish a three-period moving average, you can go to position E4 and type $= (D3+D4+D5)/3$ and then copy that same formula down to position E19.

A three-year and a five-year moving average for some production data are shown in Table 16–2 and depicted in Chart 16–4.

TABLE 16–2 A Three-Year Moving Average and a Five-Year Moving Average

Year	Production, Y	Three-Year Moving Total	Three-Year Moving Average	Five-Year Moving Total	Five-Year Moving Average
1991	5				
1992	6	19	6.3		
1993	8	24	8.0	34	6.8
1994	10	23	7.7	32	6.4
1995	5	18	6.0	33	6.6
1996	3	15	5.0	35	7.0
1997	7	20	6.7	37	7.4
1998	10	29	9.7	43	8.6
1999	12	33	11.0	49	9.8
2000	11	32	10.7	55	11.0
2001	9	33	11.0	60	12.0
2002	13	37	12.3	66	13.2
2003	15	46	15.3	70	14.0
2004	18	48	16.0	72	14.4
2005	15	44	14.7	73	14.6
2006	11	40	13.3	75	15.0
2007	14	42	14.0	79	15.8
2008	17	53	17.7		
2009	22				

Sales, production, and other economic and business series usually do not have (1) periods of oscillation that are of equal length or (2) oscillations that have identical amplitudes. Thus, in practice, the application of a moving average does not result precisely in a line. For example, the production series in Table 16–2 repeats about every five years, but the amplitude of the data varies from one oscillation to another. The trend appears to be upward and somewhat linear. Both moving averages—the three-year and the five-year—seem to adequately describe the trend in production since 1991.

Four-year, six-year, and other even-numbered-year moving averages present one minor problem regarding the centering of the moving totals and moving averages. Note in Table 16–3 on the next page that there is no center time period, so the moving totals are positioned *between* two time periods. The total for the first four years (\$42) is positioned between 2002 and 2003. The total for the next four years is \$43. The averages of the first four years and the second four years (\$10.50 and \$10.75, respectively) are averaged, and the resulting figure is centered on 2003. This procedure is repeated until all possible four-year averages are computed.

Determining a moving average for an even-numbered period, such as four years

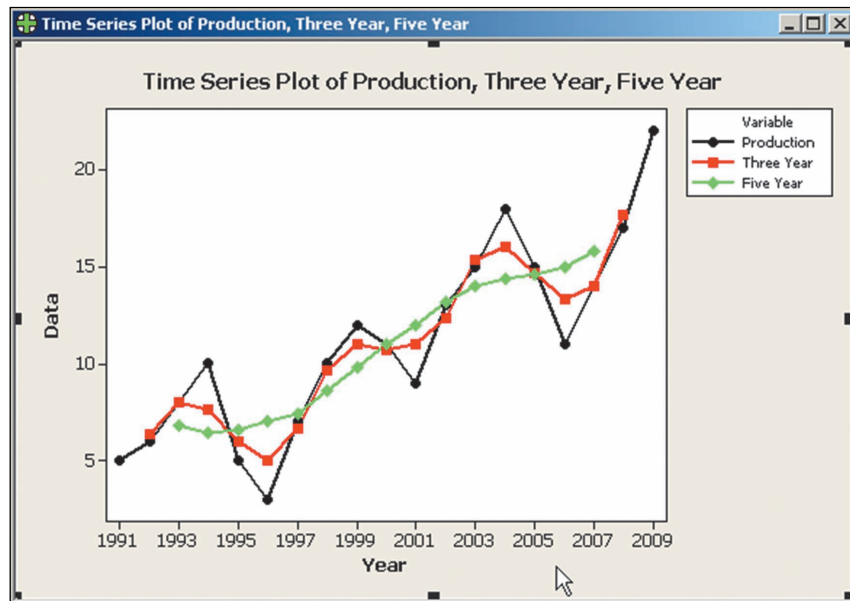


CHART 16-4 A Three-Year and Five-Year Moving Average, 1991 to 2009

TABLE 16-3 A Four-Year Moving Average

Year	Sales, Y	Four-Year Moving Total	Four-Year Moving Average	Centered Four-Year Moving Average
2001	\$ 8			
2002	11			
2003	9	\$42 (8 + 11 + 9 + 14)	\$10.50 (\$42 ÷ 4)	10.625
2004	14	43 (11 + 9 + 14 + 9)	10.75 (\$43 ÷ 4)	10.625
2005	9	42	10.50	10.625
2006	10	43	10.75	10.000
2007	10	37	9.25	9.625
2008	8	40	10.00	
2009	12			

16.4 Weighted Moving Average

A moving average uses the same weight for each observation. For example, a three-year moving total is divided by the value 3 to yield the three-year moving average. To put it another way, each data value has a weight of one-third in this case. Similarly, you can see that for a five-year moving average each data value has a weight of one-fifth.

A natural extension of the weighted mean discussed in Chapter 3 is to compute a weighted moving average. This involves selecting a possibly different weight for each data value and then computing a weighted average of the most recent n values as the smoothed value. In the majority of applications, we use the smoothed value as a forecast of the future. So, the most recent observation receives the most weight, and the weight decreases for older data values. Notice that for both the simple moving average and for the weighted moving average the sum of the weights is equal to 1.

Suppose, for example, we compute a two-year weighted moving average for the data in Table 16–3 giving twice as much weight to the most recent value. In other words, give a weight of $2/3$ to the last year and $1/3$ to the value immediately before that. Then “forecast” sales for 2003 would be found by $(1/3)(\$8) + (2/3)(\$11) = \$10$. The next moving average would be computed as $(1/3)(\$11) + (2/3)(\$9) = \$9.667$. Proceeding in the same fashion, the final or 2010 weighted moving average would be $(1/3)(\$8) + (2/3)(\$12) = \$10.667$. To summarize the technique of using moving averages, its purpose is to help identify the long-term trend in a time series (because it smooths out short-term fluctuations). It is used to reveal any cyclical and seasonal fluctuations.

Example

Cedar Fair operates seven amusement parks and five separately gated water parks. Its combined attendance (in thousands) for the last 17 years is given in the following table. A partner asks you to study the trend in attendance. Compute a three-year moving average and a three-year weighted moving average with weights of 0.2, 0.3, and 0.5 for successive years.



Year	Attendance (000)
1993	5,761
1994	6,148
1995	6,783
1996	7,445
1997	7,405
1998	11,450
1999	11,224
2000	11,703
2001	11,890
2002	12,380
2003	12,181
2004	12,557
2005	12,700
2006	19,300
2007	22,100
2008	22,720
2009	21,136

Solution

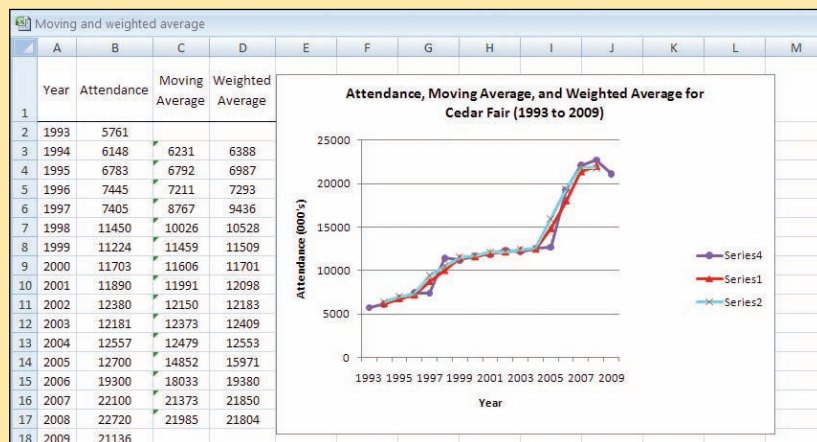
The three-year moving average is:

Year	Attendance (000)	Moving Average	Found by
1993	5,761		
1994	6,148	6,231	$(5,761 + 6,148 + 6,783)/3$
1995	6,783	6,792	$(6,148 + 6,783 + 7,445)/3$
1996	7,445	7,211	$(6,783 + 7,445 + 7,405)/3$
1997	7,405	8,767	$(7,445 + 7,405 + 11,450)/3$
1998	11,450	10,026	$(7,405 + 11,450 + 11,224)/3$
1999	11,224	11,459	$(11,450 + 11,224 + 11,703)/3$
2000	11,703	11,606	$(11,224 + 11,703 + 11,890)/3$
(continued)			

Year	Attendance (000)	Moving Average	Found by
2001	11,890	11,991	$(11,703 + 11,890 + 12,380)/3$
2002	12,380	12,150	$(11,890 + 12,380 + 12,181)/3$
2003	12,181	12,373	$(12,380 + 12,181 + 12,557)/3$
2004	12,557	12,479	$(12,181 + 12,557 + 12,700)/3$
2005	12,700	14,852	$(12,557 + 12,700 + 19,300)/3$
2006	19,300	18,033	$(12,700 + 19,300 + 22,100)/3$
2007	22,100	21,373	$(19,300 + 22,100 + 22,720)/3$
2008	22,720	21,985	$(22,100 + 22,720 + 21,136)/3$
2009	21,136		

The three-year *weighted* moving average is:

Year	Attendance (000)	Weighted Moving Average	Found by
1993	5,761		
1994	6,148	6,388	$.2(5,761) + .3(6,148) + .5(6,783)$
1995	6,783	6,987	$.2(6,148) + .3(6,783) + .5(7,445)$
1996	7,445	7,293	$.2(6,783) + .3(7,445) + .5(7,405)$
1997	7,405	9,436	$.2(7,445) + .3(7,405) + .5(11,450)$
1998	11,450	10,528	$.2(7,405) + .3(11,450) + .5(11,224)$
1999	11,224	11,509	$.2(11,450) + .3(11,224) + .5(11,703)$
2000	11,703	11,701	$.2(11,224) + .3(11,703) + .5(11,890)$
2001	11,890	12,098	$.2(11,703) + .3(11,890) + .5(12,380)$
2002	12,380	12,183	$.2(11,890) + .3(12,380) + .5(12,181)$
2003	12,181	12,409	$.2(12,380) + .3(12,181) + .5(12,557)$
2004	12,557	12,553	$.2(12,181) + .3(12,557) + .5(12,700)$
2005	12,700	15,971	$.2(12,557) + .3(12,700) + .5(19,300)$
2006	19,300	19,380	$.2(12,700) + .3(19,300) + .5(22,100)$
2007	22,100	21,850	$.2(19,300) + .3(22,100) + .5(22,720)$
2008	22,720	21,804	$.2(22,100) + .3(22,720) + .5(21,136)$
2009	21,136		



Study the graph carefully. You will see that the attendance trend is evenly upward with 360,000 added visitors each year. However, there is a “hop” of approximately 3 million per year between 1997 and 1998. That probably reflects the fact Cedar Fair acquired Knott’s Berry Farm in late 1997, leading to a boost in attendance. A similar boost occurred in 2006 with the purchase of King’s Island near Cincinnati. The weighted moving average follows the data more closely than the moving average. This reflects the additional influence given to the most recent period. In other words, the weighted method, where the most recent period is given the largest weight, won’t be quite as smooth. However, it may be more accurate as a forecasting tool.

Self-Review 16–1


Determine a three-year moving average for the sales of Waccamaw Machine Tool Inc. Plot both the original data and the moving average.




Year	Number Produced (thousands)	Year	Number Produced (thousands)
2005	2	2008	5
2006	6	2009	3
2007	4	2010	10

Exercises

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1. Calculate a four-quarter weighted moving average for the number of shares outstanding for the Boxley Box Company for the nine quarters of data. The data are reported in thousands. Apply weights of .1, .2, .3, and .4, respectively, for the quarters. In a few words, describe the trend in the number of subscribers. 

1st Quarter 2008	28,766
2nd Quarter 2008	30,057
3rd Quarter 2008	31,336
4th Quarter 2008	33,240
1st Quarter 2009	34,610
2nd Quarter 2009	35,102
3rd Quarter 2009	35,308
4th Quarter 2009	35,203
1st Quarter 2010	34,386

2. Listed below is the number of movie tickets sold at the Library Cinema-Complex, in thousands, for the period from 1998 to 2010. Compute a five-year weighted moving average using weights of .1, .1, .2, .3, and .3, respectively. Describe the trend in yield. 

1998	8.61	2005	6.61
1999	8.14	2006	5.58
2000	7.67	2007	5.87
2001	6.59	2008	5.94
2002	7.37	2009	5.49
2003	6.88	2010	5.43
2004	6.71		

16.5 Linear Trend

The long-term trend of many business series, such as sales, exports, and production, often approximates a straight line. If so, the equation to describe this growth is:

L03 Determine a linear trend equation.

LINEAR TREND EQUATION

$$\hat{Y} = a + bt$$

[16-1]

where:

\hat{Y} read Y hat, is the projected value of the Y variable for a selected value of t .
 a is the Y -intercept. It is the estimated value of Y when $t = 0$. Another way to put it is: a is the estimated value of Y where the line crosses the Y -axis when t is zero.

Slope of trend line is b

b is the slope of the line, or the average change in \hat{Y} for each increase of one unit in t .

t is any value of time that is selected.

To illustrate the meaning of \hat{Y} , a , b , and t in a time-series problem, a line has been drawn in Chart 16-5 to represent the typical trend of sales. Assume that this company started in business in 2001. This beginning year (2001) has been arbitrarily designated as year 1. Note that sales increased \$2 million on the average every year; that is, based on the straight line drawn through the sales data, sales increased from \$3 million in 2001 to \$5 million in 2002, to \$7 million in 2003, to \$9 million in 2004, and so on. The slope, or b , is therefore 2. Note too that the line intercepts the Y -axis (when $t = 0$) at \$1 million. This point is a . Another way of determining b is to locate the starting place of the straight line in year (1). It is 3 for 2001 in this problem. Then locate the value on the straight line for the last year. It is 19 for 2009. Sales went up \$19 million – \$3 million = \$16 million, in eight years (2001 to 2009). Thus, $16 \div 8 = 2$, which is the slope of the line, or b .

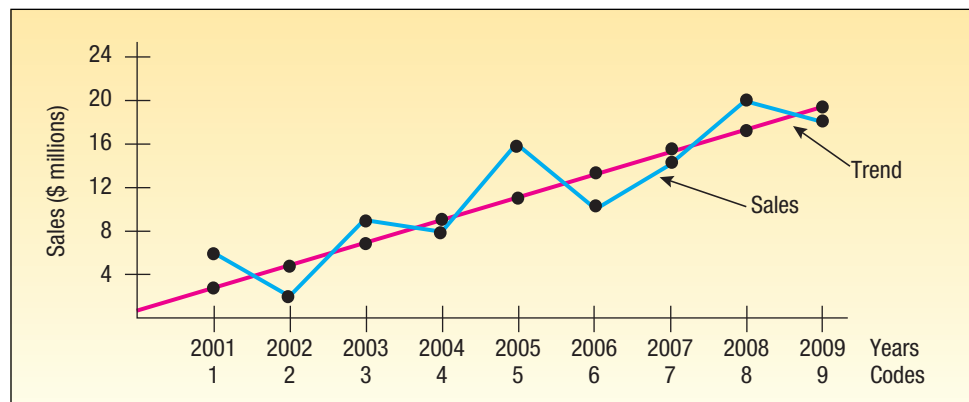


CHART 16-5 A Straight Line Fitted to Sales Data

The equation for the line in Chart 16-5 is:

$$\hat{Y} = 1 + 2t$$

where:

\hat{Y} is sales in millions of dollars.

1 is the intercept with the Y -axis. It is also the sales in millions of dollars for year 0, or 2000.

t refers to the yearly increase in sales.

In Chapter 13, we drew a line through points on a scatter diagram to approximate the regression line. We stressed, however, that this method for determining the regression equation has a serious drawback—namely, the position of the line depends on the judgment of the individual who drew the line. Three people would probably draw three different lines through the scatter plots. Likewise, the line we drew through the sales data in Chart 16–5 might not be the best-fitting line. Because of the subjective judgment involved, this method should be used only when a quick approximation of the straight-line equation is needed, or to check the reasonableness of the least squares line, which is discussed next.

16.6 Least Squares Method

In the discussion of simple linear regression in Chapter 13, we showed how the least squares method is used to find the best linear relationship between two variables. In forecasting methods, time is the independent variable and the value of the time series is the dependent variable. Furthermore, we often code the independent variable time to make the equations easier to interpret. In other words, we let t be 1 for the first year, 2 for the second, and so on. If a time series includes the sales of General Electric for five years starting in 2002 and continuing through 2006, we would code the year 2002 as 1, 2003 as 2, and 2006 as 5.

L04 Use a trend equation to compute forecasts.

Example

The sales of Jensen Foods, a small grocery chain located in southwest Texas, since 2005 are:

Year	Sales (\$ million)
2005	7
2006	10
2007	9
2008	11
2009	13

Determine the regression equation. How much are sales increasing each year? What is the sales forecast for 2012?

Solution

To determine the trend equation, we could use formula (13–4) to find the slope, or b value, and formula (13–5) to locate the intercept, or a value. We would substitute t , the coded values for the year, for X in these equations. Another approach is to use a software package, such as Minitab or Excel. Chart 16–6 is output from

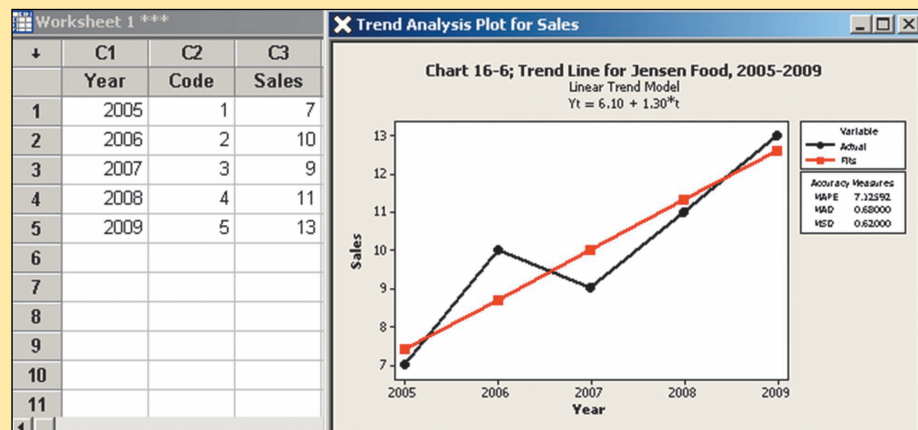


CHART 16–6 Sales and Trend Line, 2005–2009



Statistics in Action

Investors frequently use regression analysis to study the relationship between a particular stock and the general condition of the market. The dependent variable is the monthly percentage change in the value of the stock, and the independent variable is the monthly percentage change in a market index, such as the Standard & Poor's 500 Composite Index. The value of b in the regression equation is the particular stock's *beta coefficient* or just the *beta*. If b is greater than 1, the implication is that the stock is sensitive to market changes. If b is between 0 and 1, the implication is that the stock is not sensitive to market changes.

Minitab. The values for year, coded year, sales, and fitted sales are shown in the lower right portion of the output. The left half is a scatter plot of the data and the fitted regression line.

From the output, the trend equation is $\hat{Y} = 6.1 + 1.3t$. How do we interpret this equation? The sales are in millions of dollars. So the value 1.3 tells us that sales increased at a rate of 1.3 million dollars per year. The value 6.1 is the estimated value of sales in the year 0. That is the estimate for 2004, which is called the base year. For example, to determine the point on the line for 2008, insert the t value of 4 in the equation. Then $\hat{Y} = 6.1 + 1.3(4) = 11.3$.

If sales, production, or other data approximate a linear trend, the equation developed by the least squares technique can be used to estimate future values. It is reasonable that the sales for Jensen Foods follow a linear trend. So we can use the trend equation to forecast future sales.

See Table 16–4. The year 2005 is coded 1, the year 2007 is coded 3, and 2009 is coded 5. Logically, we code 2011 as 7 and 2012 as 8. So we substitute 8 into the trend equation and solve for \hat{Y} .

$$\hat{Y} = 6.1 + 1.3t = 6.1 + 1.3(8) = 16.5$$

Thus, on the basis of past sales, the estimate for 2012 is \$16.5 million.

TABLE 16–4 Calculations for Determining the Points on the Least Squares Line Using the Coded Values

Year	Sales (\$ millions),		t	\hat{Y}	Found by
	Y				
2005	7		1	7.4	$6.1 + 1.3(1)$
2006	10		2	8.7	$6.1 + 1.3(2)$
2007	9		3	10	$6.1 + 1.3(3)$
2008	11		4	11.3	$6.1 + 1.3(4)$
2009	13		5	12.6	$6.1 + 1.3(5)$

In this time series example, there were five years of sales data. Based on those five sales figures, we estimated sales for 2012. Many researchers suggest that we do not project sales, production, and other business and economic series more than $n/2$ time periods into the future where n is the number of data points. If, for example, there are 10 years of data, we would make estimates only up to 5 years into the future ($n/2 = 10/2 = 5$). Others suggest the forecast may be for no longer than 2 years, especially in rapidly changing economic times.

Self-Review 16–2

Annual production of king-size rockers by Wood Products Inc. since 2002 follows.




Year	Production (thousands)	Year	Production (thousands)
2002	4	2006	11
2003	8	2007	9
2004	5	2008	11
2005	8	2009	14

- Plot the production data.
- Determine the least squares equation using a software package.
- Determine the points on the line for 2002 and 2009. Connect the two points to arrive at the line.
- Based on the linear trend equation, what is the estimated production for 2012?


Exercises



3. Listed below is the number of rooms rented at Plantation Resorts of Georgia for the years from 1999 to 2009. 


Year	Rental	Year	Rental	Year	Rental
1999	6,714	2003	9,762	2007	6,162
2000	7,991	2004	10,180	2008	6,897
2001	9,075	2005	8,334	2009	8,285
2002	9,775	2006	8,272		

Determine the least squares equation. According to this information, what is the estimated number of rentals for 2010?

4. Listed below is the net sales in \$ million for Home Depot Inc. and its subsidiaries from 1993 to 2009. 


Year	Net Sales	Year	Net Sales	Year	Net Sales
1993	\$ 9,239	1999	\$38,434	2005	\$81,511
1994	12,477	2000	45,738	2006	90,837
1995	15,470	2001	53,553	2007	77,349
1996	19,535	2002	58,247	2008	71,300
1997	24,156	2003	64,816	2009	66,200
1998	30,219	2004	73,094		

Determine the least squares equation. On the basis of this information, what are the estimated sales for 2010 and 2011?

5. The following table lists the annual amounts of glass cullet produced by Kimble Glass Works Inc. 

Year	Code	Scrap (tons)	Year	Code	Scrap (tons)
2006	1	2	2009	4	5
2007	2	4	2010	5	6
2008	3	3			

Determine the least squares trend equation. Estimate the amount of scrap for the year 2012.

6. The sales by Walker's Milk and Dairy Products in millions of dollars for the period from 2004 to 2010 are reported in the following table. 

Year	Code	Sales (\$ millions)	Year	Code	Sales (\$ millions)
2004	1	17.5	2008	5	24.5
2005	2	19.0	2009	6	26.7
2006	3	21.0	2010	7	27.3
2007	4	22.7			

Determine the least squares regression trend equation. Estimate the sales for 2012.

16.7 Nonlinear Trends

L05 Compute a nonlinear trend equation.

The emphasis in the previous discussion was on a time series whose growth or decline approximated a straight line. A linear trend equation is used to represent the time series when it is believed that the data are increasing (or decreasing) by *equal amounts*, on the average, from one period to another.

Data that increase (or decrease) by *increasing amounts* over a period of time appear *curvilinear* when plotted on an arithmetic scale. To put it another way, data

that increase (or decrease) by *equal percents or proportions* over a period of time appear curvilinear. (See Chart 16–7.)

The trend equation for a time series that does approximate a curvilinear trend, such as the one portrayed in Chart 16–7, is computed by using the logarithms of the data and the least squares method. The general equation for the logarithmic trend equation is:

LOG TREND EQUATION

$$\log \hat{Y} = \log a + \log b(t)$$

[16–2]

The logarithmic trend equation can be determined for the Gulf Shores Importers data in Chart 16–7 using Excel. The first step is to enter the data, then find the log base 10 of each year's imports. Finally, use the regression procedure to find the least squares equation. To put it another way, we take the log of each year's data, then use the logs as the dependent variable and the coded year as the independent variable.

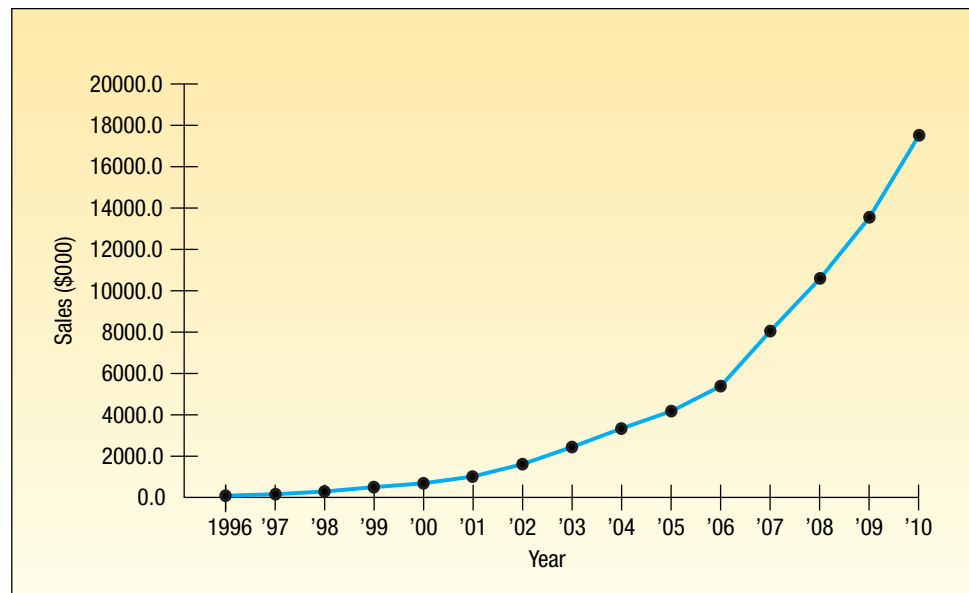


CHART 16–7 Sales for Gulf Shores Importers from 1996–2010

chart 16-7										
	A	B	C	D	E	F	G	H	I	J
1	Year	Sales	Log-Sales	Code		SUMMARY OUTPUT				
2	1996	124.2	2.094122	1						
3	1997	175.6	2.244525	2		Regression Statistics				
4	1998	306.9	2.486997	3		Multiple R	0.994			
5	1999	524.2	2.719497	4		R Square	0.988			
6	2000	714.0	2.853698	5		Adjusted R Square	0.987			
7	2001	1052.0	3.022016	6		Standard Error	0.079			
8	2002	1638.3	3.214393	7		Observations	15			
9	2003	2463.2	3.3915	8						
10	2004	3358.2	3.526107	9		ANOVA				
11	2005	4181.3	3.621311	10			df	SS	MS	F
12	2006	5388.5	3.731468	11		Regression	1	6.585	6.585	1065.228
13	2007	8027.4	3.904575	12		Residual	13	0.080	0.006	
14	2008	10587.2	4.024781	13		Total	14	6.666		
15	2009	13537.4	4.131535	14						
16	2010	17515.6	4.243425	15						
17						Coefficients	Standard Error	t Stat	P-value	
18						Intercept	2.053805	0.0427	48.0741	0.0000
						Code	0.153357	0.0047	32.6378	0.0000

The regression equation is $\hat{Y} = 2.053805 + 0.153357t$. This equation is the log form. We now have a trend equation in terms of percent of change. That is, the value 0.153357 is the percent of change in \hat{Y} for each unit increase in t . This value is similar to the geometric mean described in Section 3.10 in Chapter 3.

The log of b is 0.153357 and its antilog or inverse is 1.423498. If we subtract 1 from this value, as we did in Chapter 3, the value 0.423498 indicates the geometric mean annual rate of increase from 1996 to 2010. We conclude that imports increased at a rate of 42.35 percent annually during the period.

We can also use the logarithmic trend equation to make estimates of future values. Suppose we want to estimate the imports in the year 2014. The first step is to determine the code for the year 2014. It is 19. To explain, the year 2010 has a code of 15 and the year 2014 is four years later, so $15 + 4 = 19$. The log of imports for the year 2014 is

$$\hat{Y} = 2.053805 + 0.153357t = 2.053805 + 0.153357(19) = 4.967588$$

To find the estimated imports for the year 2014, we need the antilog of 4.967588. It is 92,809. This is our estimate of the number of imports for 2014. Recall that the data were in thousands of dollars, so the estimate is \$92,809,000.

Self-Review 16-3 Sales at Tomlin Manufacturing since 2006 are:





Year	Sales (\$ millions)
2006	2.13
2007	18.10
2008	39.80
2009	81.40
2010	112.00

- Determine the logarithmic trend equation for the sales data.
- Sales increased by what percentage annually from 2006–2010?
- What is the projected sales amount for 2011?

Exercises

connect™

Year	Sales (\$ millions)
2003	1.1
2004	1.5
2005	2.0
2006	2.4
2007	3.1

- Sally's Software Inc. is a rapidly growing supplier of computer software to the Sarasota area. Sales for the last five years are given on the left. 
 - Determine the logarithmic trend equation.
 - By what percent did sales increase, on the average, during the period?
 - Estimate sales for the year 2010.
- It appears that the imports of carbon black have been increasing by about 10 percent annually. 

Imports of Carbon Black (thousands of tons)		Imports of Carbon Black (thousands of tons)	
Year		Year	
2000	92.0	2004	135.0
2001	101.0	2005	149.0
2002	112.0	2006	163.0
2003	124.0	2007	180.0

- Determine the logarithmic trend equation.
- By what percent did imports increase, on the average, during the period?
- Estimate imports for the year 2010.

16.8 Seasonal Variation



We mentioned that *seasonal variation* is another of the components of a time series. Business series, such as automobile sales, shipments of soft-drink bottles, and residential construction, have periods of above-average and below-average activity each year. In the area of production, one of the reasons for analyzing seasonal fluctuations is to have a sufficient supply of raw materials on hand to meet the varying seasonal demand. The glass container division of a large glass company, for example, manufactures nonreturnable beer bottles, iodine bottles, aspirin bottles, bottles for rubber cement, and so on. The production scheduling department must know how many bottles to produce and when to produce each kind. A run of too many bottles of one kind may cause a serious storage problem. Production cannot be based entirely on orders on hand, because many orders are telephoned in for immediate shipment. Since the demand for many of the bottles varies according to the season, a forecast a year or two in advance, by month, is essential to good scheduling.

An analysis of seasonal fluctuations over a period of years can also help in evaluating current sales. The typical sales of department stores in the United States, excluding mail-order sales, are expressed as indexes in Table 16–5. Each index represents the average sales for a period of several years. The actual sales for some months were above average (which is represented by an index over 100.0), and the sales for other months were below average. The index of 126.8 for December indicates that, typically, sales for December are 26.8 percent above an average month; the index of 86.0 for July indicates that department store sales for July are typically 14 percent below an average month.

TABLE 16–5 Typical Seasonal Indexes for U.S. Department Store Sales, Excluding Mail-Order Sales

January	87.0	July	86.0
February	83.2	August	99.7
March	100.5	September	101.4
April	106.5	October	105.8
May	101.6	November	111.9
June	89.6	December	126.8

Suppose an enterprising store manager, in an effort to stimulate sales during December, introduced a number of unique promotions, including bands of carolers strolling through the store singing holiday songs, large mechanical exhibits, and clerks dressed in Santa Claus costumes. When the index of sales was computed for that December, it was 150.0. Compared with the typical December sales of 126.8, it was concluded that the promotional program was a huge success.

Determining a Seasonal Index

LO6 Determine and interpret a set of seasonal indexes.

A typical set of monthly indexes consists of 12 indexes that are representative of the data for a 12-month period. Logically, there are four typical seasonal indexes for data reported quarterly. Each index is a percent, with the average for the year equal to 100.0; that is, each monthly index indicates the level of sales, production, or another variable in relation to the annual average of 100.0. A typical index of 96.0 for January indicates that sales (or whatever the variable is) are usually 4 percent below the average for the year. An index of 107.2 for October means that the variable is typically 7.2 percent above the annual average.

Several methods have been developed to measure the typical seasonal fluctuation in a time series. The method most commonly used to compute the typical

seasonal pattern is called the **ratio-to-moving-average method**. It eliminates the trend, cyclical, and irregular components from the original data (Y). In the following discussion, T refers to trend, C to cyclical, S to seasonal, and I to irregular variation. The numbers that result are called the *typical seasonal index*.

We will discuss in detail the steps followed in arriving at typical seasonal indexes using the ratio-to-moving-average method. The data of interest might be monthly or quarterly. To illustrate, we have chosen the quarterly sales of Toys International. First, we will show the steps needed to arrive at a set of typical quarterly indexes. Then we use MegaStat Excel and Minitab software to calculate the seasonal indexes.

Example

Table 16–6 shows the quarterly sales for Toys International for the years 2004 through 2009. The sales are reported in millions of dollars. Determine a quarterly seasonal index using the ratio-to-moving-average method.

TABLE 16–6 Quarterly Sales of Toys International (\$ millions)

Year	Winter	Spring	Summer	Fall
2004	6.7	4.6	10.0	12.7
2005	6.5	4.6	9.8	13.6
2006	6.9	5.0	10.4	14.1
2007	7.0	5.5	10.8	15.0
2008	7.1	5.7	11.1	14.5
2009	8.0	6.2	11.4	14.9

Solution

Chart 16–8 depicts the quarterly sales for Toys International over the six-year period. Notice the seasonal nature of the sales. For each year, the fourth-quarter sales are the largest and the second-quarter sales are the smallest. Also, there is a moderate increase in the sales from one year to the next. To observe this feature, look only at the six fourth-quarter sales values. Over the six-year period, the sales in the fourth quarter increased. If you connect these points in your mind, you can visualize fourth-quarter sales increasing for 2010.

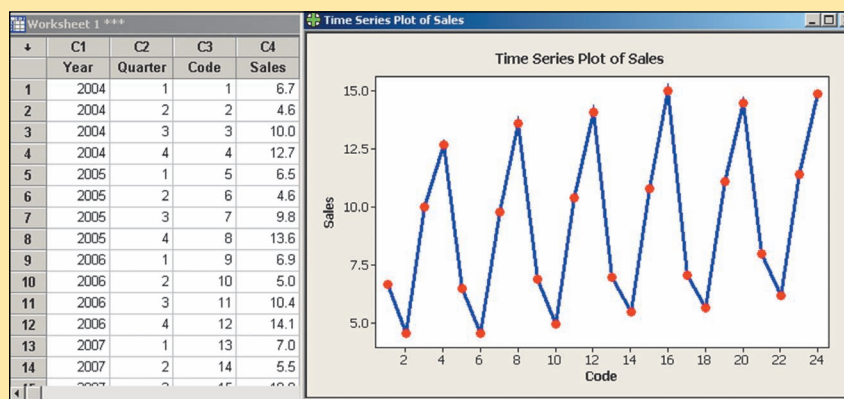


CHART 16–8 Quarterly Sales of Toys International 2004–2009

There are six steps to determining the quarterly seasonal indexes.

Step 1: For the following discussion, refer to Table 16–7. The first step is to determine the four-quarter moving total for 2004. Starting with the winter quarter of 2004, we add \$6.7, \$4.6, \$10.0, and \$12.7. The total is \$34.0 (million). The four-quarter total is “moved along” by adding the spring,

TABLE 16-7 Computations Needed for the Specific Seasonal Indexes

		(1)	(2)	(3)	(4)	(5)
Year	Quarter	Sales (\$ millions)	Four-Quarter Total	Four-Quarter Moving Average	Centered Moving Average	Specific Seasonal
2004	Winter	6.7				
	Spring	4.6				
	Summer	10.0	34.0	8.500	8.475	1.180
	Fall	12.7	33.8	8.450	8.450	1.503
2005	Winter	6.5	33.8	8.450	8.425	0.772
	Spring	4.6	33.6	8.400	8.513	0.540
	Summer	9.8	34.5	8.625	8.675	1.130
	Fall	13.6	34.9	8.725	8.775	1.550
2006	Winter	6.9	35.3	8.825	8.900	0.775
	Spring	5.0	35.9	8.975	9.038	0.553
	Summer	10.4	36.4	9.100	9.113	1.141
	Fall	14.1	36.5	9.125	9.188	1.535
2007	Winter	7.0	37.0	9.250	9.300	0.753
	Spring	5.5	37.4	9.350	9.463	0.581
	Summer	10.8	38.3	9.575	9.588	1.126
	Fall	15.0	38.4	9.600	9.625	1.558
2008	Winter	7.1	38.6	9.650	9.688	0.733
	Spring	5.7	38.9	9.725	9.663	0.590
	Summer	11.1	38.4	9.600	9.713	1.143
	Fall	14.5	39.3	9.825	9.888	1.466
2009	Winter	8.0	39.8	9.950	9.888	0.801
	Spring	6.2	40.1	10.025	10.075	0.615
	Summer	11.4	40.5	10.125		
	Fall	14.9				

summer, and fall sales of 2004 to the winter sales of 2005. The total is \$33.8 (million), found by $4.6 + 10.0 + 12.7 + 6.5$. This procedure is continued for the quarterly sales for each of the six years. Column 2 of Table 16–7 shows all of the moving totals. Note that the moving total 34.0 is positioned between the spring and summer sales of 2004. The next moving total, 33.8, is positioned between sales for summer and fall 2004, and so on. Check the totals frequently to avoid arithmetic errors.

Step 2: Each quarterly moving total in column 2 is divided by 4 to give the four-quarter moving average. (See column 3.) All the moving averages are still positioned between the quarters. For example, the first moving average (8.500) is positioned between spring and summer 2004.

Step 3: The moving averages are then centered. The first centered moving average is found by $(8.500 + 8.450)/2 = 8.475$ and centered opposite summer 2004. The second moving average is found by $(8.450 + 8.450)/2 = 8.450$. The others are found similarly. Note in column 4 that a centered moving average is positioned on a particular quarter.

Step 4: The **specific seasonal index** for each quarter is then computed by dividing the sales in column 1 by the centered moving average in column 4. The specific seasonal index reports the ratio of the original time series value to the moving average. To explain further, if the time series is represented by $TSCI$ and the moving average by TC , then, algebraically, if we compute $TSCI/TC$, the result is the specified seasonal component SI . The specific seasonal index for the summer quarter of 2004 is 1.180, found by $10.0/8.475$.

Step 5: The specific seasonal indexes are organized in Table 16–8. This table will help us locate the specific seasonals for the corresponding quarters. The values 1.180, 1.130, 1.141, 1.126, and 1.143 all represent estimates of the typical seasonal index for the summer quarter. A reasonable method to find a typical seasonal index is to average these values in order to eliminate the irregular component. So we find the typical index for the summer quarter by $(1.180 + 1.130 + 1.141 + 1.126 + 1.143)/5 = 1.144$. We used the arithmetic mean, but the median or a modified mean can also be used.

TABLE 16–8 Calculations Needed for Typical Quarterly Indexes

Year	Winter	Spring	Summer	Fall	
2004			1.180	1.503	
2005	0.772	0.540	1.130	1.550	
2006	0.775	0.553	1.141	1.535	
2007	0.753	0.581	1.126	1.558	
2008	0.733	0.590	1.143	1.466	
2009	0.801	0.615			
Total	3.834	2.879	5.720	7.612	
Mean	0.767	0.576	1.144	1.522	4.009
Adjusted	0.765	0.575	1.141	1.519	4.000
Index	76.5	57.5	114.1	151.9	

Step 6: The four quarterly means (0.767, 0.576, 1.144, and 1.522) should theoretically total 4.00 because the average is set at 1.0. The total of the four quarterly means may not exactly equal 4.00 due to rounding. In this problem, the total of the means is 4.009. A *correction factor* is therefore applied to each of the four means to force them to total 4.00.

**CORRECTION FACTOR
FOR ADJUSTING
QUARTERLY MEANS**

$$\text{Correction factor} = \frac{4.00}{\text{Total of four means}} \quad [16-3]$$

In this example,

$$\text{Correction factor} = \frac{4.00}{4.009} = 0.997755$$

The adjusted winter quarterly index is, therefore, $.767(.997755) = .765$. Each of the means is adjusted downward so that the total of the four quarterly means is 4.00. Usually indexes are reported as percentages, so each value in the last row of Table 16–8 has been multiplied by 100. So the index for the winter quarter is 76.5 and for the fall it is 151.9. How are these values interpreted? Sales for the fall quarter are 51.9 percent above the typical quarter, and for winter they are 23.5 below the typical quarter ($100.0 - 76.5$). These findings should not surprise you. The period prior to Christmas (the fall quarter) is when toy sales are brisk. After Christmas (the winter quarter), sales of the toys decline drastically.

As we noted earlier, there is software that will perform the calculations and output the results. The MegaStat Excel output is shown below. Use of software will greatly reduce the computational time and the chance of an error in arithmetic, but you should understand the steps in the process, as outlined earlier. There can be slight differences in the answers, due to the number of digits carried in the calculations.

Centered Moving Average and Deseasonalization							
t	Year	Quarter	Sales	Centered Moving Average	Ratio to CMA	Seasonal Indexes	Sales Deseasonalized
1	2004	1	6.70			0.765	8.759
2	2004	2	4.60			0.575	8.004
3	2004	3	10.00	8.475	1.180	1.141	8.761
4	2004	4	12.70	8.450	1.503	1.519	8.361
5	2005	1	6.50	8.425	0.772	0.765	8.498
6	2005	2	4.60	8.513	0.540	0.575	8.004
7	2005	3	9.80	8.675	1.130	1.141	8.586
8	2005	4	13.60	8.775	1.550	1.519	8.953
9	2006	1	6.90	8.900	0.775	0.765	9.021
10	2006	2	5.00	9.038	0.553	0.575	8.700
11	2006	3	10.40	9.113	1.141	1.141	9.112
12	2006	4	14.10	9.188	1.535	1.519	9.283
13	2007	1	7.00	9.300	0.753	0.765	9.151
14	2007	2	5.50	9.463	0.581	0.575	9.570
15	2007	3	10.80	9.588	1.126	1.141	9.462
16	2007	4	15.00	9.625	1.558	1.519	9.875
17	2008	1	7.10	9.688	0.733	0.765	9.282
18	2008	2	5.70	9.663	0.590	0.575	9.918
19	2008	3	11.10	9.713	1.143	1.141	9.725
20	2008	4	14.50	9.888	1.466	1.519	9.546
21	2009	1	8.00	9.988	0.801	0.765	10.459
22	2009	2	6.20	10.075	0.615	0.575	10.788
23	2009	3	11.40			1.141	9.988
24	2009	4	14.90			1.519	9.809

Calculation of Seasonal Indexes					
	1	2	3	4	
2004			1.180	1.503	
2005	0.772	0.540	1.130	1.550	
2006	0.775	0.553	1.141	1.535	
2007	0.753	0.581	1.126	1.558	
2008	0.733	0.590	1.143	1.466	
2009	0.801	0.615			
Mean:	0.767	0.576	1.144	1.522	4.009
Adjusted:	0.765	0.575	1.141	1.519	4.000

Now we briefly summarize the reasoning underlying the preceding calculations. The original data in column 1 of Table 16–7 contain trend (*T*), cyclical (*C*), seasonal (*S*), and irregular (*I*) components. The ultimate objective is to remove seasonal (*S*) from the original sales valuation.

Columns 2 and 3 in Table 16–7 are concerned with deriving the centered moving average given in column 4. Basically, we “average out” the seasonal and irregular fluctuations from the original data in column 1. Thus, in column 4 we have only trend and cyclical (*TC*).

Next, we divide the sales data in column 1 (*TCSI*) by the centered fourth-quarter moving average in column 4 (*TC*) to arrive at the specific seasonals in column 5 (*SI*). In terms of letters, $TCSI/TC = SI$. We multiply *SI* by 100.0 to express the typical seasonal in index form.

Finally, we take the mean of all the winter typical indexes, all the spring indexes, and so on. This averaging eliminates most of the irregular fluctuations from the specific seasonals, and the resulting four indexes indicate the typical seasonal sales pattern.

Self-Review 16–4



Teton Village, Wyoming, near Grand Teton Park and Yellowstone Park, contains shops, restaurants, and motels. The village has two peak seasons—winter, for skiing on the 10,000-foot slopes, and summer, for tourists visiting the parks. The number of visitors (in thousands) by quarter for five years follows.

Year	Quarter			
	Winter	Spring	Summer	Fall
2006	117.0	80.7	129.6	76.1
2007	118.6	82.5	121.4	77.0
2008	114.0	84.3	119.9	75.0
2009	120.7	79.6	130.7	69.6
2010	125.2	80.2	127.6	72.0

- Develop the typical seasonal pattern for Teton Village using the ratio-to-moving-average method.
- Explain the typical index for the winter season.

Exercises


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- Victor Anderson, the owner of Anderson Belts Inc., is studying absenteeism among his employees. His workforce is small, consisting of only five employees. For the last three years, he recorded the following number of employee absences, in days, for each quarter.



Year	Quarter			
	I	II	III	IV
2008	4	10	7	3
2009	5	12	9	4
2010	6	16	12	4

Determine a typical seasonal index for each of the four quarters.

10. Appliance Center sells a variety of electronic equipment and home appliances. For the last four years, the following quarterly sales (in \$ millions) were reported. 

Year	Quarter			
	I	II	III	IV
2007	5.3	4.1	6.8	6.7
2008	4.8	3.8	5.6	6.8
2009	4.3	3.8	5.7	6.0
2010	5.6	4.6	6.4	5.9

Determine a typical seasonal index for each of the four quarters.

16.9 Deseasonalizing Data

L07 Deseasonalize data using a seasonal index.

A set of typical indexes is very useful in adjusting a sales series, for example, for seasonal fluctuations. The resulting sales series is called **deseasonalized sales** or **seasonally adjusted sales**. The reason for deseasonalizing the sales series is to remove the seasonal fluctuations so that the trend and cycle can be studied. To illustrate the procedure, the quarterly sales totals of Toys International from Table 16–6 are repeated in column 1 of Table 16–9.

TABLE 16–9 Actual and Deseasonalized Sales for Toys International

Year	Quarter	(1) Sales	(2) Seasonal Index	(3) Deseasonalized Sales
2004	Winter	6.7	0.765	8.76
	Spring	4.6	0.575	8.00
	Summer	10.0	1.141	8.76
	Fall	12.7	1.519	8.36
2005	Winter	6.5	0.765	8.50
	Spring	4.6	0.575	8.00
	Summer	9.8	1.141	8.59
	Fall	13.6	1.519	8.95
2006	Winter	6.9	0.765	9.02
	Spring	5.0	0.575	8.70
	Summer	10.4	1.141	9.11
	Fall	14.1	1.519	9.28
2007	Winter	7.0	0.765	9.15
	Spring	5.5	0.575	9.57
	Summer	10.8	1.141	9.47
	Fall	15.0	1.519	9.87

(continued)

Year	Quarter	(1) Sales	(2) Seasonal Index	(3) Deseasonalized Sales
2008	Winter	7.1	0.765	9.28
	Spring	5.7	0.575	9.91
	Summer	11.1	1.141	9.73
	Fall	14.5	1.519	9.55
2009	Winter	8.0	0.765	10.46
	Spring	6.2	0.575	10.79
	Summer	11.4	1.141	9.99
	Fall	14.9	1.519	9.81

To remove the effect of seasonal variation, the sales amount for each quarter (which contains trend, cyclical, irregular, and seasonal effects) is divided by the seasonal index for that quarter, that is, $TSCI/S$. For example, the actual sales for the first quarter of 2004 were \$6.7 million. The seasonal index for the winter quarter is 76.5 percent, using the MegaStat results on page 626. The index of 76.5 indicates that sales for the first quarter are typically 23.5 percent below the average for a typical quarter. By dividing the actual sales of \$6.7 million by 76.5 and multiplying the result by 100, we find the *deseasonalized sales* value—that is, removed the seasonal effect on sales—for the first quarter of 2004. It is \$8,758,170, found by $(\$6,700,000/76.5)100$. We continue this process for the other quarters in column 3 of Table 16–9, with the results reported in millions of dollars. Because the seasonal component has been removed (divided out) from the quarterly sales, the deseasonalized sales figure contains only the trend (*T*), cyclical (*C*), and irregular (*I*) components. Scanning the deseasonalized sales in column 3 of Table 16–9, we see that the sales of toys showed a moderate increase over the six-year period. Chart 16–9 shows both the actual sales and the deseasonalized sales. It is clear that removing the seasonal factor allows us to focus on the overall long-term trend of sales. We will also be able to determine the regression equation of the trend data and use it to forecast future sales.

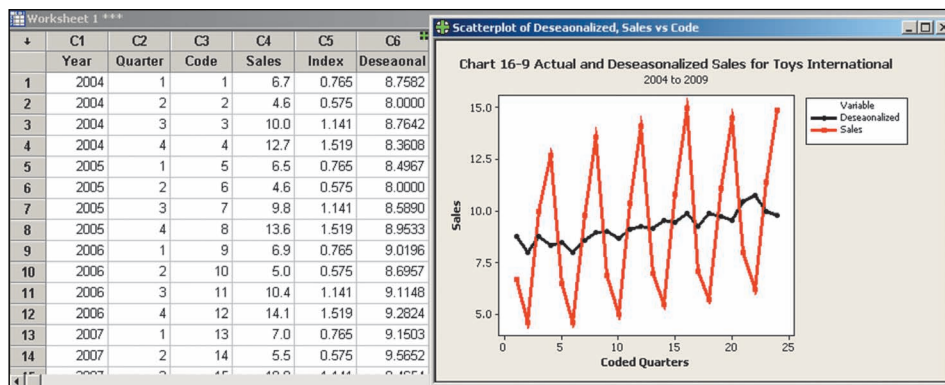


CHART 16–9 Actual and Deseasonalized Sales for Toys International from 2004–2009

Using Deseasonalized Data to Forecast

L08 Calculate seasonally adjusted forecasts.

The procedure for identifying trend and the seasonal adjustments can be combined to yield seasonally adjusted forecasts. To identify the trend, we determine the least squares trend equation on the deseasonalized historical data. Then we project this trend into future periods, and finally we adjust these trend values to account for the seasonal factors. The following example will help to clarify.

Example

Toys International would like to forecast its sales for each quarter of 2010. Use the information in Table 16–9 to determine the forecast.

Solution

The deseasonalized data depicted in Chart 16–9 seems to follow a straight line. Hence, it is reasonable to develop a linear trend equation based on these data. The deseasonalized trend equation is:

$$\hat{Y} = a + bt$$

where:

\hat{Y} is the estimated trend value for Toys International sales for the period t .

a is the intercept of the trend line at time 0.

b is the slope of the line.

t is the coded time period.

The winter quarter of 2004 is the first quarter, so it is coded 1, the spring quarter of 2004 is coded 2, and so on. The last quarter of 2009 is coded 24. A portion of these coded values are shown in the data section of the Minitab output associated with Chart 16–9.

We use Minitab to find the regression equation. The output follows. The output includes a scatter diagram of the coded time periods and the deseasonalized sales as well as the regression line.

The equation for the trend line is:

$$\hat{Y} = 8.109 + .08991t$$

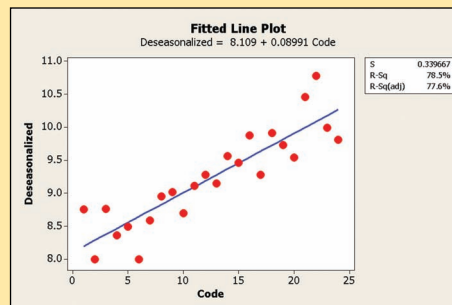
The slope of the trend line is .08991. This shows that over the 24 quarters the deseasonalized sales increased at a rate of 0.08991 (\$ million) per quarter, or \$89,910 per quarter. The value of 8.109 is the intercept of the trend line on the Y-axis (i.e., for $t = 0$).



Statistics in Action

Forecasts are not always correct. The reality is that a forecast may just be a best guess as to what will happen. What are the reasons forecasts are not correct? One expert lists eight common errors:

- (1) Failure to carefully examine the assumptions,
- (2) Limited expertise,
- (3) Lack of imagination,
- (4) Neglect of constraints,
- (5) Excessive optimism,
- (6) Reliance on mechanical extrapolation,
- (7) Premature closure, and
- (8) Overspecification.



The Minitab system also outputs the coefficient of determination. This value, called R^2 , is 78.6 percent. It is shown in the upper right of the Minitab output. We can use this value as an indication of the fit of the data. Because this is *not* sample information, technically we should not use R^2 for judging a regression equation. However, it will serve to quickly evaluate the fit of the deseasonalized sales data. In this instance, because R^2 is rather large, we conclude the deseasonalized sales of Toys International are effectively explained by a linear trend equation.

If we assume that the past 24 periods are a good indicator of future sales, we can use the trend equation to estimate future sales. For example, for the winter quarter of 2010 the value of t is 25. Therefore, the estimated sales of that period is 10.35675, found by

$$\hat{Y} = 8.109 + .08991t = 8.109 + .08991(25) = 10.35675$$

The estimated deseasonalized sales for the winter quarter of 2010 are \$10,356,750. This is the sales forecast, before we consider the effects of seasonality.

We use the same procedure and an Excel spreadsheet to determine a forecast for each of the four quarters of 2010. A partial Excel output follows.

quarterly forecasts						
	A	B	C	D	E	F
1			Quarterly Forecast for Toys International			
2			2010			
3						
4		Quarter	t value	Estimated Sales	Seasonal Index	Quarterly Forecast
5		Winter	25	10.3565	0.765	7.923
6		Spring	26	10.4464	0.575	6.007
7		Summer	27	10.5363	1.141	12.022
8		Fall	28	10.6262	1.519	16.141
9						

Now that we have the forecasts for the four quarters of 2010, we can seasonally adjust them. The index for a winter quarter is 0.765. So we can seasonally adjust the forecast for the winter quarter of 2010 by $10.35675(0.765) = 7.923$. The estimates for each of the four quarters of 2010 are in the right-hand column of the Excel output. Notice how the seasonal adjustments drastically increase the sales estimates for the last two quarters of the year.

Self-Review 16-5



Westberg Electric Company sells electric motors to customers in the Jamestown, New York, area. The monthly trend equation, based on five years of monthly data, is

$$\hat{Y} = 4.4 + 0.5t$$

The seasonal factor for the month of January is 120, and it is 95 for February. Determine the seasonally adjusted forecast for January and February of the sixth year.

Exercises



11. The planning department of Padget and Kure Shoes, the manufacturer of an exclusive brand of women's shoes, developed the following trend equation, in millions of pairs, based on five years of quarterly data.

$$\hat{Y} = 3.30 + 1.75t$$

The following table gives the seasonal factors for each quarter.

	Quarter			
	I	II	III	IV
Index	110.0	120.0	80.0	90.0

- Determine the seasonally adjusted forecast for each of the four quarters of the sixth year.
12. Team Sports Inc. sells sporting goods to high schools and colleges via a nationally distributed catalog. Management at Team Sports estimates it will sell 2,000 Wilson Model A2000 catcher's mitts next year. The deseasonalized sales are projected to be the same for each of the four quarters next year. The seasonal factor for the second quarter is 145. Determine the seasonally adjusted sales for the second quarter of next year.