

①

Example - Given TR & TC functions

$$TR = 22Q - 2Q^2$$

$$TC = \frac{1}{3}Q^3 - 10Q^2 + 50Q + 45$$

(a) Find the quantity Q at which TR maximum.

(b) Find Q , at which π is maximum what will be π .

(c) Prove that $MR = MC$ at the Q where π is maximum.

(2)

Solutions -

(a) Already done in Previous Lecture.

(b) we know that

$$\pi = TR - TC$$

$$\pi = 22Q - 2Q^2 - \left(\frac{1}{3}Q^3 - 10Q^2 + 50Q + 45\right)$$

$$\pi = 22Q - 2Q^2 - \frac{1}{3}Q^3 + 10Q^2 - 50Q - 45$$

$$\pi = -\frac{1}{3}Q^3 + 8Q^2 - 28Q - 45$$

NC/ $\frac{d\pi}{dQ} = -Q^2 + 16Q - 28$

set $\frac{d\pi}{dQ} = 0$

$$-Q^2 + 16Q - 28 = 0$$

$$Q^2 - 16Q + 28 = 0$$

By factoring we get

$$Q = 2, \quad Q = 14$$

SC/ $\frac{d^2\pi}{dQ^2} = -2Q + 16$

at $Q=2$.

$$\frac{d^2\pi}{dQ^2} = -2(2) + 16$$

$$= -4 + 16$$

$$= 12 > 0$$

at $Q=14$

$$\frac{d^2\pi}{dQ^2} = -2(14) + 16$$

$$= -28 + 16$$

$$= -12 < 0$$

At $Q=14$, π will be Max

For finding Max. π , put $Q=14$ in Profit function.

$$\pi = \frac{-1}{3}(14)^3 + 8(14)^2 - 28(14) - 45$$

$$\pi = \frac{-2744}{3} + 8 \times 196 - 28 \times 14 - 45$$

$$= -914.67 + 1568 - 392 - 45$$

$$= -1351.67 + 1568$$

$$\pi = 216.33$$

$$e) \quad MR = \frac{dTR}{dQ} = \frac{d}{dQ} (22Q - 2Q^2)$$

$$MR = 22 - 4Q$$

$$MC = \frac{dTC}{dQ} = \frac{d}{dQ} \left(\frac{1}{3}Q^3 - 10Q^2 + 50Q + 45 \right)$$

$$MC = Q^2 - 20Q + 50$$

Maximum profit will be where

$$MR = MC$$

Put $Q=14$ in both MC & MR functions.

$$MR = 22 - 4(14)$$

$$= 22 - 56$$

$$MR = -34$$

$$MC = (14)^2 - 20(14) + 50$$

$$MC = 196 - 280 + 50$$

$$MC = -34$$

Thus $MR = MC \Rightarrow -34 = -34$