moon is made of green cheese implies that the earth is round;" and this is the more curious when we realize that it also follows that "The moon is made of green cheese implies that the earth is *not* round."

Why do these true statements seem so curious? It is because we recognize that the shape of the earth and the cheesiness of the moon are utterly irrelevant to each other. As we normally use the word "implies," a statement cannot imply some other statement, false or true, to which it is utterly irrelevant. That is the case when "implies" is used in most of its everyday senses. And yet those "paradoxical" statements in the preceding paragraph are indeed true, and not really problematic at all, because they use the word "implies" in the logical sense of "material implication." The precise meaning of material implication we have made very clear; we understand that to say p materially implies q is only to say that either p is false or q is true.

What needs to be borne in mind is this: *Meaning*—subject matter—is strictly irrelevant to material implication. *Material implication is a truth function*. Only the truth and falsity of the antecedent and the consequent, not their content, are relevant here. There is nothing paradoxical in stating that any disjunction is true that contains one true disjunct. Well, when we say that "The moon is made of green cheese (materially) implies that the earth is round," we know that to be logically equivalent to saying "Either the moon is not made of green cheese or the earth is round"—a disjunction that is most certainly true. And any disjunction we may confront in which "The moon is not made of green cheese" is the first disjunct will certainly be true, no matter what the second disjunct asserts. So, yes, "The moon is made of green cheese (materially) implies that the earth is square" because that is logically equivalent to "The moon is not made of green cheese or the earth is square." A false statement materially implies any statement whatever. A true statement is materially implied by any statement whatever.

Every occurrence of "if–then" should be treated, we have said, as a material implication, and represented with the horseshoe, $\supset$. The justification of this practice, its logical expediency, is the fact that doing so preserves the validity of all valid arguments of the type with which we are concerned in this part of our logical studies. Other symbolizations have been proposed, adequate to other types of implication, but they belong to more advanced parts of logic, beyond the scope of this book.

8.10 The Three "Laws of Thought"

Some early thinkers, after having defined logic as "the science of the laws of thought," went on to assert that there are exactly three *basic* laws of thought, laws so fundamental that obedience to them is both the necessary and the

sufficient condition of correct thinking. These three have traditionally been called:

- The **principle of identity.** This principle asserts that *if any statement is true, then it is true.* Using our notation we may rephrase it by saying that the principle of identity asserts that every statement of the form $p \supset p$ must be true, that every such statement is a tautology.
- The **principle of noncontradiction.** This principle asserts that *no statement can be both true and false.* Using our notation we may rephrase it by saying that the principle of noncontradiction asserts that every statement of the form $p \bullet {\sim}p$ must be false, that every such statement is self-contradictory.
- The **principle of excluded middle.** This principle asserts that *every statement is either true or false.* Using our notation we may rephrase it by saying that the principle of excluded middle asserts that every statement of the form $p \vee {\sim}p$ must be true, that every such statement is a tautology.

It is obvious that these three principles are indeed true—logically true—but the claim that they deserve privileged status as the most fundamental laws of thought is doubtful. The first (identity) and the third (excluded middle) are tautologies, but there are many other tautologous forms whose truth is equally certain. And the second (noncontradiction) is by no means the only self-contradictory form of statement.

We do use these principles in completing truth tables. In the initial columns of each row of a table we place either a **T** or an **F,** being guided by the principle of excluded middle. Nowhere do we put both **T** and **F,** being guided by the principle of noncontradiction. And once having put a **T** under a symbol in a given row, then (being guided by the principle of identity) when we encounter that symbol in other columns of that row, we regard it as still being assigned a **T.** So we could regard the three laws of thought as principles governing the construction of truth tables.

Nevertheless, in regarding the entire system of deductive logic, these three principles are no more important or fruitful than many others. Indeed, there are tautologies that are more fruitful than they for purposes of deduction, and in that sense more important than these three. A more extended treatment of this point lies beyond the scope of this book.[7]

Some thinkers, believing themselves to have devised a new and different logic, have claimed that these three principles are in fact not true, and that obedience to them has been needlessly confining. But these criticisms have been based on misunderstandings.

The principle of identity has been attacked on the ground that things change, and are always changing. Thus, for example, statements that were true of the United States when it consisted of the thirteen original states are no

longer true of the United States today, which has fifty states. But this does not undermine the principle of identity. The sentence, "There are only thirteen states in the United States," is incomplete, an elliptical formulation of the statement that "There were only thirteen states in the United States *in 1790*"—and that statement is as true today as it was in 1790. When we confine our attention to complete, nonelliptical formulations of propositions, we see that their truth (or falsity) does not change over time. The principle of identity is true, and it does not interfere with our recognition of continuing change.

The principle of noncontradiction has been attacked by Hegelians and Marxists on the grounds that genuine contradiction is everywhere pervasive, that the world is replete with the inevitable conflict of contradictory forces. That there are conflicting forces in the real word is true, of course—but to call these conflicting forces "contradictory" is a loose and misleading use of that term. Labor unions and the private owners of industrial plants may indeed find themselves in conflict—but neither the owner nor the union is the "negation" or the "denial" or the "contradictory" of the other. The principle of noncontradiction, understood in the straightforward sense in which it is intended by logicians, is unobjectionable and perfectly true.

The principle of excluded middle has been the object of much criticism, on the grounds that it leads to a "two-valued orientation," which implies that things in the world must be either "white or black," and which thereby hinders the realization of compromise and less than absolute gradations. This objection also arises from misunderstanding. Of course the statement "This is black" cannot be jointly true with the statement "This is white"—where "this" refers to exactly the same thing. But although these two statements cannot both be true, they can both be false. "This" may be neither black nor white; the two statements are *contraries*, not contradictory. The contradictory of the statement "This is white" is the statement "It is not the case that this is white" and (if "white" is used in precisely the same sense in both of these statements) one of them must be true and the other false. The principle of excluded middle is inescapable.

All three of these "laws of thought" are unobjectionable—so long as they are applied to statements containing unambiguous, nonelliptical, and precise terms. They may not deserve the honorific status assigned to them by some philosophers,* but they are indubitably true.

*Plato appealed explicitly to the principle of noncontradiction in Book IV of his *Republic* (at nos. 436 and 439); Aristotle discussed all three of these principles in Books IV and XI of his *Metaphysics*. Of the principle of noncontradiction, Aristotle wrote: "That the same attribute cannot at the same time belong and not belong to the same subject and in the same respect" is a principle "which everyone must have who understands anything that is," and which "everyone must already have when he comes to a special study." It is, he concluded, "the most certain of all principles."

SUMMARY

This chapter has presented the fundamental concepts of modern symbolic logic.

In Section 8.1, we explained the general approach of modern symbolic logic, and its need for an artificial symbolic language.

In Section 8.2, we introduced and defined the symbols for negation (the curl: ~); and for the truth-functional connectives of conjunction (the dot: •) and disjunction (the wedge: ∨). We also explained logical punctuation.

In Section 8.3, we discussed the different senses of implication, and defined the truth-functional connective material implication (the horseshoe: ⊃).

In Section 8.4, we explained the formal structure of arguments, defined argument forms, and explained other concepts essential in analyzing deductive arguments.

In Section 8.5, we gave a precise account of valid and invalid argument forms.

In Section 8.6, we explained the truth-table method of testing the validity of argument forms.

In Section 8.7, we identified and described a few very common argument forms, some valid and some invalid.

In Section 8.8, we explained the formal structure of statements and defined essential terms for dealing with statement forms. We introduced tautologous, contradictory, and contingent statement forms, and defined a fourth truth-functional connective, material equivalence (three bars: ≡).

In Section 8.9, we introduced and defined a powerful new relation, logical equivalence, using the symbol $\overset{T}{\equiv}$. We explained why statements that are logically equivalent may be substituted for one another, while statements that are merely materially equivalent cannot replace one another. We introduced several logical equivalences of special importance: De Morgan's theorems, the principle of double negation, and the definition of material implication.

In Section 8.10, we discussed certain logical equivalences that have been thought by many to be fundamental in all reasoning: the principle of identity, the principle of noncontradiction, and the principle of the excluded middle.

End Notes

[1]Somewhat more complicated definitions have been proposed by David H. Sanford in "What Is a Truth Functional Component?" *Logique et Analyse* 14 (1970): 483–486.

[2]Ted Turner, quoted in *The New Yorker*, 30 April 2001.

[3]"The Firm" *The New Yorker*, 8 March 1999.

[4]Peter J. Bertocci, "Chavez' Plight Must Come from Arrogance," *The New York Times*, 19 January 2001.

[5]Rabbi Ammiel Hirsch, "Grand Canyon," *The New York Times*, 10 October 2005.

[6]Orlando Patterson, "The Speech Misheard Round the World," *The New York Times*, 22 January 2005. Mr. Patterson's wording of the syllogism is very slightly different but has exactly the same logical force.

[7]For further discussion of these matters, the interested reader can consult I. M. Copi and J. A. Gould, eds., *Readings on Logic*, 2d ed. (New York: Macmillan, 1972), part 2; and I. M. Copi and J. A. Gould, eds., *Contemporary Philosophical Logic* (New York: St. Martin's Press, 1978), part 8.