

## IMMEDIATE INFERENCE

In logic, we distinguish two varieties of deductive inference: immediate and mediated inferences. An immediate inference occurs in an argument consisting of two propositions: one premise and a conclusion. For example, from "all men are mortal," one can immediately deduce that some men are mortal. The immediate inference involves two and only two terms: men and mortal; whereas mediated inferences (syllogisms) have three and only three terms. Immediate inferences are the subject of this chapter; treatment of syllogisms is left for the next. With both varieties of inference, it is important to distinguish valid from invalid inference.

### Valid Inference

No doubt, the student has noted the numerous references to valid inference or valid argument form. The former will receive attention forthwith; the latter is reserved for the next chapter, since its explication falls within the scope of mediated inference. But first, a reminder about the use of the word form, for it has more than one meaning. The primary meaning of this word is in reference to the four standard propositions: A, E, I, and O. When speaking about the form of an argument, the student can take it to mean the "bare bones" of an argument, so to speak; or, its outline or skeleton. More definitive language about the form of an argument must be postponed until Chapter Four.

Now for a very important definition borrowed from Gordon H. Clark:

- *An inference is to be counted as valid whenever the form of the conclusion is true every time the forms of the premises are.*
- *Or, If the form of the conclusion is not true every time the forms of the premises are true, then the inference is invalid.*

Chart 2.1 below shows how many times a form can be true. Five sets of circles correspond to five ways in which two terms (subject and predicate terms) can be related in the four forms. The circles are numbered as cases 1 through 5 for easy reference.

There are five possibilities, 2 Terms Related in 5 Ways.

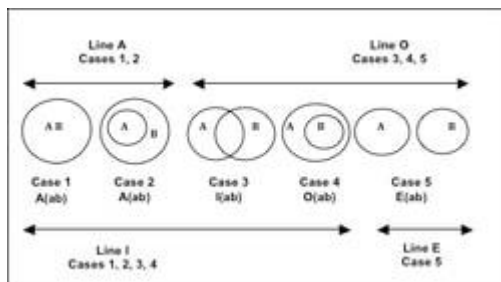


Chart 2.1: Euler Circles

- Case 1: One sense of A(ab), where All a is b & All b is a.
- Case 2: Another sense of A(ab), where All a is b, but not All b is a.
- Case 3: Corresponds to I(ab), Some a is b; & Some b is a.

- Case 4: Corresponds to  $O(ab)$ , Some a is not b.
- Case 5: Corresponds to  $E(ab)$ , No a is b, & No b is a.

To repeat. An inference is valid if the form of the conclusion is true every time the forms of the premises are. In other words, a valid inference from premises to conclusion depends on the arrangement of the subject and predicate being true in the conclusion, every time the arrangement of the same subject and predicate is true in the premises.

- Line A designates All; Line O designates No; Line I designates Some; and Line E designates Some—is not—.
- An inspection of the five sets or cases of circles reveals that  $A(ab)$ , or All a is b, is true in two of the five sets of circles: Cases 1 and 2. Line A covers the two cases.
- $I(ab)$ , Some a is b, is true in four sets: Cases 1, 2, 3, and 4; line I includes the four cases.
- $O(ab)$ , Some a is not b, is true three times, in the 3rd, 4th, and 5th Cases, as shown by line O.
- $E(ab)$ , or No a is b, is true only once, in the 5th case, as shown by line E.

Recall that under discussion is the nature of Immediate Inference. Chart 3.1 and the discussion following may challenge the student; however, it is essential that one have a thorough understanding of what is meant by immediate inference.

An application of the diagram should convince the student of its usefulness. For example,  $A(ab)$  logically implies  $I(ab)$ , since  $I(ab)$  is true every time  $A(ab)$  is true. Similarly,  $E(ab)$  implies  $O(ab)$  is a valid inference, since  $O(ab)$  is true every time  $E(ab)$  is true as an inspection of Chart 3.1 circles and lines O and E show. On the other hand one cannot validly infer from  $O(ab)$ , the form  $E(ab)$ , since  $E(ab)$  is not true every time  $O(ab)$  is true;  $O(ab)$  is true three times,  $E(ab)$  only once. Similarly, that  $I(ab)$  implies  $A(ab)$  is not a valid inference, since  $A(ab)$  is not true every time  $I(ab)$  is true. (Examine the diagrams!)

### **Square of Opposition**

The valid inferences of the previous paragraph belong to a set of sixteen which are captured in a good memory device, the square of opposition, shown below. The student should become familiar with the various kinds of opposition shown between the four forms. It should be kept in mind that the square of opposition does not justify the immediate inferences but merely displays them in the form of a chart.

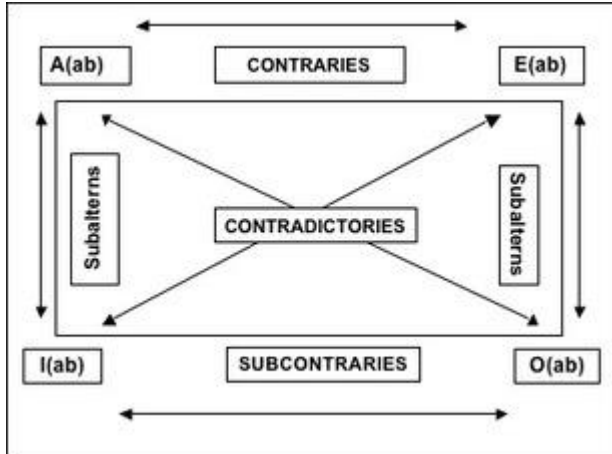


Chart 2.2 Square of Opposition

The four relationships are contraries, subcontraries, subalternation, and contradiction. Definitions follow in the order listed with examples.

### Contraries

By contraries we mean that the two forms A(ab) and E(ab) cannot both be true together; however, both may be false. Examine Chart 2.1. Note that lines A and E do not overlap which means they cannot both be true in any instance. Since the lines A and E do not exhaust all five cases, they could both be false together. If, for example, some Christians are Calvinists (Case 3 or the third set of circles), then the corresponding A (All Christians are Calvinists) and E (No Christians are Calvinists) are both false.

### Subcontraries

The forms, I(ab) and O(ab), are subcontraries, meaning that they cannot both be false together, but they could both be true. Referring again to Chart 2.1, the lines I and O exhaust all 5 cases, and overlap each other to show that they can both be true together -- as in, Some Christians are Calvinists, and Some Christians are not Calvinists.

### Subalternations

Subalterns are two forms that are both true together or both false together. There are two pairs of subalterns: (1) A(ab) & I(ab); and (2) E(ab) & O(ab). The Chart shows that lines A and I are both true under cases 1 and 2, and both false in Case 5. In Case 5, if it is true that No men are angels, then the corresponding A and I are both false. A similar analysis applies to the second pair of subalterns. If, All men are sinners, then the corresponding E and O forms are both false.

(It should be noted here that logic alone does not assert the existence or the nonexistence of anything. The existence or nonexistence of men, sinners, or angels in these propositions is a matter for history or biology, as Clark suggests, or some other discipline. Clark, G. H. Logic, p. 89)

In short, the truth of the A or E includes and necessitates the truth of the I or the O, respectively. From the truth of I or O, we have no right to infer the truth or falsity of the A or E, respectively. However, from the falsity of the I, the falsity of the A is valid inference, and from the falsity of O, the falsity of E is valid

inference.

### Contradiction

The strongest form of opposition is contradiction. Two forms are contradictories, if they cannot both be true together and cannot both be false together. Lines A and O, and I and E can be seen to meet without overlapping and, at the same time, each pair exhausts all cases. A(ab) & O(ab), and E(ab) & I(ab) are contradictories.

As previously mentioned, the square of opposition incorporates a number of useful relationships that hold among the four forms. With it, we can determine, for example, whether the following inference is valid or not: "Since it is the case that all men are mortal; it is false that some men are not mortal." The premise is an A proposition; the conclusion is an O proposition; the A and O forms are contradictories. Another way of stating this valid inference is to say that from the truth of an A proposition, one can infer the falsity of its contradictory, the O proposition. Or, if the A is false, then the O is true. Similar valid inferences occur between the contradictories E and I. These relationships can be charted.

Chart 2.3: Immediate Inferences

	A is	E is	I is	O is
If A be true	true	false	true	false
If E be true	false	true	false	true
If I be true	***	false	true	***
If O be true	false	***	***	true

\*\*\*Truth-Value is undetermined (Always in pairs)

### Square of Opposition Inferences

Before we list the immediate inferences depicted by the Square of Opposition, two observations are in order.

1. First, note that the not attached to a form below means the form is false; otherwise, true. (In every case, assume that a form is true, unless it is designated false by the prefix "not".)
2. Second, it is permitted to speak of immediate inferences as logical implications in accord with the logic of necessary inference. (Grammatically, experts note the difference between infer versus imply. Thus to imply may mean to state indirectly, and to infer may mean to deduce a statement or a conclusion. In our use, logical implication is but another way of expressing necessary implication or inference.)

For example, one could ask: Is I(ab) a necessary consequence of A(ab)? That is to say: Does A(ab) logically imply I(ab)? Below, we list which of the four forms is logically implied by each.

- 1a. Does A(ab) logically imply I(ab)?
- Answer: Yes, by Subalternation.
- Example: if it is true that "All men are mortal," then "Some men are mortal" is true.

- For additional Square of Opposition Immediate Inferences, [Click here](#).

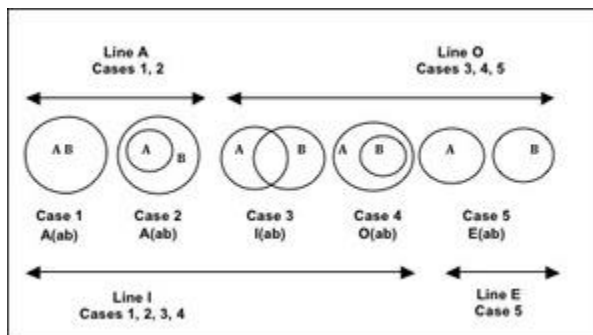
### Invalid Inferences

We have shown the value of Chart 2.1 in testing the validity of the immediate inferences depicted in the Square of Opposition. Obviously, use of the same methods proves invalidity as well, for if a logical implication is not valid, then it must be invalid -- the only other possibility.

Consider the following list of expressions. The list is the number of ways two forms can be combined to form logical implications. The purpose here is to again make use of our charts, in particular Chart 3.1. Using “<” for “logically implies” which of the inferences does the chart show invalidity? For example, the first implication in Table 3.1 First Figure is  $A(ab) < E(ab)$ . It can be read as a conditional posed as a question:

- Is “if  $A(ab)$ , then  $E(ab)$ ” valid or invalid?
- Answer: Invalid because  $E(ab)$  is not true every time  $A(ab)$  is true;  $E(ab)$  is true once in the fifth Case;  $A(ab)$  is true twice in Cases 1 and 2.

Chart 2.1: Clark Diagram: 5 Possibilities, 2 Terms Related in 5 Ways



Another example: the last inference in Table 2.1,  $O(ab) < I(ab)$ , is invalid, since  $I(ab)$  is true four times (Cases 1, 2, 3, and 4), while  $O(ab)$  is true three times (Cases 3, 4, and 5), and for a valid inference the form of the conclusion must be true every time the form of the premise is true. Practice using the chart is essential; therefore, 1.2 through 1.7 should receive similar treatment as shown with the first and last.

Table 2.1: First Figure

- |                      |                      |
|----------------------|----------------------|
| 1.1. $A(ab) < E(ab)$ | 1.5. $I(ab) < A(ab)$ |
| 1.2. $A(ab) < I(ab)$ | 1.6. $I(ab) < O(ab)$ |
| 1.3. $E(ab) < A(ab)$ | 1.7. $O(ab) < E(ab)$ |
| 1.4. $E(ab) < O(ab)$ | 1.8. $O(ab) < I(ab)$ |

The list of implications above (1.1-1.8) are said to be in the First Figure. Reordering the terms of the conclusion produces another set of implications below said to be in the Second Figure. For example:  $A(ab)$  is in the First Figure;  $A(ba)$  is in the Second Figure. It may be expressed the other way around:  $A(ba)$  is in the First Figure;  $A(ab)$  is in the Second Figure. There are only two figures for immediate

inferences. Perhaps sufficient information has been provided for the student to do an exercise: show which of the implications in the Second Figure that follow are invalid. (Notice that the terms of the conclusions of 2.1-2.8 have been interchanged.)

*Table 2.2: Second Figure*

2.1. $A(ab) < E(ba)$	2.5. $I(ab) < A(ba)$
2.2. $A(ab) < I(ba)$	2.6. $I(ab) < O(ba)$
2.3. $E(ab) < A(ba)$	2.7. $O(ab) < E(ba)$
2.4. $E(ab) < O(ba)$	2.8. $O(ab) < I(ba)$

### **Other Immediate Inferences**

Three other immediate inferences are available for the four forms. These are known as conversion, obversion, and contraposition. Definitions with commentary of these inferences are accessible through the link below:

- **To view these inferences, Click [here](#).**

The remaining immediate inferences are three: reflexive, symmetrical, and transitive. These inferences apply to relationships, like "is greater than," or "is less than," when speaking of numbers or quantities. One or more may apply to other types of relationships; for example, family relationships, "the son of" or "the sister of," and so forth. Some relationships exhibit one or more; some none of these.

### **Reflexive**

The reflexive relationship is one that holds between one of its objects and the object itself. In logic, implication is reflexive because every proposition implies itself. Note, however, that while  $A(ab)$  logically implies  $I(ab)$ , it is not the case that  $I(ab)$  logically implies  $A(ab)$ . Each form implies itself. Thus,  $1 = 1$  in arithmetic is reflexive; "is greater than" and "is less than" are not reflexive.

### **Symmetrical**

Symmetrical relationships are those which hold for  $a$  and  $b$ , and also for  $b$  and  $a$ . If  $a$  is the cousin of  $b$ , does it follow that  $b$  is the cousin of  $a$ ? Obviously! But, do you see that symmetry is not present if  $a$  is the sister of  $b$ ? (Assume  $b$  is male.) What can be said of "is the twin of?" Is it symmetrical? Logical Implication is not symmetrical, with the exception of the Law of Identity – every proposition implies itself.

### **Transitive**

Transitive relationships are a bit more complicated to explain but easier to illustrate. The relationships "is less than," "is greater than," "is subsequent to," "is parallel to," link together three terms in a unique fashion. If  $a$  is greater than  $b$ , and  $b$  is greater than  $c$ , then it follows necessarily that  $a$  is greater than  $c$ . The relationship "is the brother of" is not transitive. Logical implication is transitive: if  $a$  logically implies  $b$ , and  $b$  logically implies  $c$ , then  $a$  logically implies  $c$ . The basic principle should be obvious: a relation is transitive when it holds for two things,  $a$  and  $b$ , and it also holds for  $b$  and  $c$ , then it also holds between  $a$  and  $c$ .

### **Summary**

The wealth of immediate inferences available from a set of four standard form propositions may come

as a surprise to a beginner. As has been shown, knowledge of the definitions of immediate inferences when applied correctly will distinguish valid from invalid inferences. Here's another example.

What valid inferences (necessary consequences) follow from "There is therefore now no condemnation to them which are in Christ Jesus, who walk not after the flesh, but after the Spirit," reworded so that the sense of the A form is clear? All persons-who-are-in-Christ Jesus-who-walk-not-after-the-flesh-but-after-the-Spirit are persons-for-whom-there-is-now-no-condemnation. The related I form is true by subalternation; the E form is false by contraries; the contradictory O form is false.

Perhaps enough information has been presented in this chapter to show the beginner the impressive power of the logic of immediate inference. We turn in the next chapter to the power of mediated logic, the syllogism.