1 BASIC CONCEPTS OF LOGIC

1. WHAT IS LOGIC?

Logic may be defined as the science of reasoning. However, this is not to suggest that logic is an empirical (i.e., experimental or observational) science like physics, biology, or psychology. Rather, logic is a non-empirical science like mathematics. Also, in saying that logic is the science of reasoning, we do not mean that it is concerned with the *actual* mental (or physical) process employed by a thinking entity when it is reasoning. The investigation of the actual reasoning *process* falls more appropriately within the province of psychology, neurophysiology, or cybernetics.

Even if these empirical disciplines were considerably more advanced than they presently are, the most they could disclose is the exact process that goes on in a being's head when he or she (or it) is reasoning. They could not, however, tell us whether the being is reasoning correctly or incorrectly.

Distinguishing correct reasoning from incorrect reasoning is the task of logic.

2. INFERENCES AND ARGUMENTS

Reasoning is a special mental activity called *inferring*, what can also be called making (or performing) *inferences*. The following is a useful and simple definition of the word 'infer'.

To ***infer*** is to draw conclusions from premises.

In place of word 'premises', you can also put: 'data', 'information', 'facts'.

Examples of Inferences:

(1) You see smoke and *infer* that there is a fire.

(2) You count 19 persons in a group that originally had 20, and you *infer* that someone is missing.

Note carefully the difference between 'infer' and 'imply', which are sometimes confused. We *infer* the fire on the basis of the smoke, but we do not *imply* the fire. On the other hand, the smoke *implies* the fire, but it does not *infer* the fire. The word 'infer' is not equivalent to the word 'imply', nor is it equivalent to 'insinuate'.

The reasoning process may be thought of as beginning with *input* (premises, data, etc.) and producing *output* (conclusions). In each specific case of drawing (inferring) a conclusion C from premises P_1, P_2, P_3, ..., the details of the actual mental process (how the "gears" work) is not the proper concern of logic, but of psychology or neurophysiology. The proper concern of logic is whether the inference of C on the basis of P_1, P_2, P_3, ... is warranted (correct).

Inferences are made on the basis of various sorts of things – data, facts, information, states of affairs. In order to simplify the investigation of reasoning, logic treats all of these things in terms of a single sort of thing – *statements*. Logic correspondingly treats inferences in terms of collections of statements, which are called *arguments*. The word 'argument' has a number of meanings in ordinary English. The definition of 'argument' that is relevant to logic is given as follows.

> An ***argument*** is a collection of **statements**, one of which is designated as the **conclusion**, and the remainder of which are designated as the **premises**.

Note that this is *not* a definition of a *good* argument. Also note that, in the context of ordinary discourse, an argument has an additional trait, described as follows.

> Usually, the premises of an argument are intended to **support** (**justify**) the conclusion of the argument.

Before giving some concrete examples of arguments, it might be best to clarify a term in the definition. The word 'statement' is intended to mean *declarative sentence*. In addition to declarative sentences, there are also interrogative, imperative, and exclamatory sentences. The sentences that make up an argument are all declarative sentences; that is, they are all statements. The following may be taken as the official definition of 'statement'.

> A ***statement*** is a **declarative sentence**, which is to say a sentence that is capable of being **true** or **false**.

The following are examples of statements.

it is raining
I am hungry
2+2 = 4
God exists

On the other hand the following are examples of sentences that are *not* statements.

are you hungry?
shut the door, please
#$%@!!! (replace '#$%@!!!' by your favorite expletive)

Observe that whereas a *statement* is capable of being true or false, a *question*, or a *command*, or an *exclamation* is not capable of being true or false.

Note that in saying that a statement is capable of being true or false, we are not saying that we *know for sure* which of the two (true, false) it is. Thus, for a sentence to be a statement, it is not necessary that humankind knows for sure whether it is true, or whether it is false. An example is the statement 'God exists'.

Now let us get back to inferences and arguments. Earlier, we discussed two examples of inferences. Let us see how these can be represented as arguments. In the case of the smoke-fire inference, the corresponding argument is given as follows.

(a1)	there is smoke	(premise)
	therefore, there is fire	(conclusion)

Here the argument consists of two statements, 'there is smoke' and 'there is fire'. The term 'therefore' is not strictly speaking part of the argument; it rather serves to designate the conclusion ('there is fire'), setting it off from the premise ('there is smoke'). In this argument, there is just one premise.

In the case of the missing-person inference, the corresponding argument is given as follows.

(a2)	there were 20 persons originally	(premise)
	there are 19 persons currently	(premise)
	therefore, someone is missing	(conclusion)

Here the argument consists of three statements – 'there were 20 persons originally', 'there are 19 persons currently', and 'someone is missing'. Once again, 'therefore' sets off the conclusion from the premises.

In principle, any collection of statements can be treated as an argument simply by designating which statement in particular is the conclusion. However, not every collection of statements is *intended to be* an argument. We accordingly need criteria by which to distinguish arguments from other collections of statements.

There are no hard and fast rules for telling when a collection of statements is intended to be an argument, but there are a few rules of thumb. Often an argument can be identified as such because its conclusion is marked. We have already seen one conclusion-marker – the word 'therefore'. Besides 'therefore', there are other words that are commonly used to mark conclusions of arguments, including 'consequently', 'hence', 'thus', 'so', and 'ergo'. Usually, such words indicate that what follows is the conclusion of an argument.

Other times an argument can be identified as such because its premises are marked. Words that are used for this purpose include: 'for', 'because', and 'since'. For example, using the word 'for', the smoke-fire argument (a1) earlier can be rephrased as follows.

(a1′) there is fire
for there is smoke

Note that in (a1′) the conclusion comes *before* the premise.

Other times neither the conclusion nor the premises of an argument are marked, so it is harder to tell that the collection of statements is intended to be an argument. A general rule of thumb applies in this case, as well as in previous cases.

> In an argument, the premises are intended to support (justify) the conclusion.

To state things somewhat differently, when a person (speaking or writing) advances an argument, he(she) expresses a statement he(she) believes to be true (the conclusion), and he(she) cites other statements as a reason for believing that statement (the premises).

3. DEDUCTIVE LOGIC VERSUS INDUCTIVE LOGIC

Let us go back to the two arguments from the previous section.

(a1) there is smoke;
therefore, there is fire.

(a2) there were 20 people originally;
there are 19 persons currently;
therefore, someone is missing.

There is an important difference between these two inferences, which corresponds to a division of logic into two branches.

On the one hand, we know that the existence of smoke does not *guarantee* (ensure) the existence of fire; it only makes the existence of fire *likely* or *probable*. Thus, although inferring fire on the basis of smoke is reasonable, it is nevertheless *fallible*. Insofar as it is *possible* for there to be smoke without there being fire, we *may* be wrong in asserting that there is a fire.

The investigation of inferences of this sort is traditionally called *inductive logic*. Inductive logic investigates the process of drawing *probable* (likely, plausible) *though fallible* conclusions from premises. Another way of stating this: inductive logic investigates arguments in which the truth of the premises *makes likely* the truth of the conclusion.

Inductive logic is a very difficult and intricate subject, partly because the practitioners (experts) of this discipline are not in complete agreement concerning what constitutes correct inductive reasoning.

Inductive logic is *not* the subject of this book. If you want to learn about inductive logic, it is probably best to take a course on probability and statistics. Inductive reasoning is often called statistical (or probabilistic) reasoning, and forms the basis of experimental science.

Inductive reasoning is important to science, but so is deductive reasoning, which *is* the subject of this book.

Consider argument (a2) above. In this argument, *if* the premises are in fact true, *then* the conclusion is *certainly* also true; or, to state things in the subjunctive mood, *if* the premises *were* true, *then* the conclusion *would certainly* also be true.

Still another way of stating things: the truth of the premises *necessitates* the truth of the conclusion.

The investigation of these sorts of arguments is called *deductive logic*.

The following should be noted. suppose that you have an argument and suppose that the truth of the premises *necessitates* (guarantees) the truth of the conclusion. Then it follows (logically!) that the truth of the premises *makes likely* the truth of the conclusion. In other words, if an argument is judged to be *deductively correct*, then it is also judged to be *inductively correct* as well. The converse is not true: not every inductively correct argument is also deductively correct; the smoke-fire argument is an example of an inductively correct argument that is not deductively correct. For whereas the existence of smoke *makes likely* the existence of fire it does not *guarantee* the existence of fire.

In deductive logic, the task is to distinguish deductively correct arguments from deductively incorrect arguments. Nevertheless, we should keep in mind that, although an argument may be judged to be *deductively incorrect*, it may still be *reasonable*, that is, it may still be *inductively correct*.

Some arguments are not inductively correct, and therefore are not deductively correct either; they are just plain unreasonable. Suppose you flunk intro logic, and suppose that *on the basis of this* you conclude that it will be a breeze to get into law school. Under these circumstances, it seems that your reasoning is faulty.

4. STATEMENTS VERSUS PROPOSITIONS

Henceforth, by 'logic' I mean deductive logic.

Logic investigates inferences in terms of the arguments that represent them. Recall that an argument is a collection of statements (declarative sentences), one of which is designated as the conclusion, and the remainder of which are designated as the premises. Also recall that usually in an argument the premises are offered to support or justify the conclusions.

Statements, and sentences in general, are linguistic objects, like words. They consist of strings (sequences) of sounds (spoken language) or strings of symbols (written language). Statements must be carefully distinguished from the *propositions* they express (assert) when they are uttered. Intuitively, statements stand in the same relation to propositions as nouns stand to the objects they denote. Just as the word 'water' denotes a substance that is liquid under normal circumstances, the *sentence* (statement) 'water is wet' denotes the *proposition* that water is wet; equivalently, the sentence denotes the state of affairs the wetness of water.

The difference between the five letter word 'water' in English and the liquid substance it denotes should be obvious enough, and no one is apt to confuse the word and the substance. Whereas 'water' consists of letters, water consists of molecules. The distinction between a statement and the proposition it expresses is very much like the distinction between the word 'water' and the substance water.

There is another difference between statements and propositions. Whereas statements are always part of a particular language (e.g., English), propositions are not peculiar to any particular language in which they might be expressed. Thus, for example, the following are different statements in different languages, yet they all express the same proposition – namely, the whiteness of snow.

snow is white
der Schnee ist weiss
la neige est blanche

In this case, quite clearly different sentences may be used to express the same proposition. The opposite can also happen: the same sentence may be used in different contexts, or under different circumstances, to express different propositions, to denote different states of affairs. For example, the statement 'I am hungry' expresses a different proposition for each person who utters it. When I utter it, the proposition expressed pertains to my stomach; when you utter it, the proposition pertains to your stomach; when the president utters it, the proposition pertains to his(her) stomach.

5. FORM VERSUS CONTENT

Although propositions (or the meanings of statements) are always lurking behind the scenes, logic is primarily concerned with statements. The reason is that statements are in some sense easier to point at, easier to work with; for example, we can write a statement on the blackboard and examine it. By contrast, since they are essentially abstract in nature, propositions cannot be brought into the classroom, or anywhere. Propositions are unwieldy and uncooperative. What is worse, no one quite knows exactly what they are!

There is another important reason for concentrating on statements rather than propositions. Logic analyzes and classifies arguments according to their *form*, as opposed to their *content* (this distinction will be explained later). Whereas the form of a statement is fairly easily understood, the form of a proposition is not so easily understood. Whereas it is easy to say what a statement consists of, it is not so easy to say what a proposition consists of.

A statement consists of words arranged in a particular order. Thus, the form of a statement may be analyzed in terms of the arrangement of its constituent words. To be more precise, a statement consists of *terms*, which include simple terms and compound terms. A simple term is just a single word together with a specific grammatical role (being a noun, or being a verb, etc.). A compound term is a string of words that act as a grammatical unit within statements. Examples of compound terms include noun phrases, such as 'the president of the U.S.', and predicate phrases, such as 'is a Democrat'.

For the purposes of logic, terms divide into two important categories – *descriptive terms* and *logical terms*. One must carefully note, however, that this distinction is *not* absolute. Rather, the distinction between descriptive and logical terms depends upon the level (depth) of logical analysis we are pursuing.

Let us pursue an analogy for a moment. Recall first of all that the core meaning of the word 'analyze' is *to break down a complex whole into its constituent parts*. In physics, matter can be broken down (analyzed) at different levels; it can be analyzed into molecules, into atoms, into elementary particles (electrons, protons, etc.); still deeper levels of analysis are available (e.g., quarks). The basic idea in breaking down matter is that in order to go deeper and deeper one needs ever increasing amounts of energy, and one needs ever increasing sophistication.

The same may be said about logic and the analysis of language. There are many levels at which we can analyze language, and the deeper levels require more logical sophistication than the shallower levels (they also require more energy on the part of the logician!)

In the present text, we consider three different levels of logical analysis. Each of these levels is given a name – Syllogistic Logic, Sentential Logic, and Predicate Logic. Whereas syllogistic logic and sentential logic represent relatively superficial (shallow) levels of logical analysis, predicate logic represents a relatively deep level of analysis. Deeper levels of analysis are available.

Each level of analysis – syllogistic logic, sentential logic, and predicate logic – has associated with it a special class of logical terms. In the case of syllogistic logic, the logical terms include only the following: 'all', 'some', 'no', 'not', and 'is/are'. In the case of sentential logic, the logical terms include only sentential connectives (e.g., 'and', 'or', 'if...then', 'only if'). In the case of predicate logic, the logical terms include the logical terms of both syllogistic logic and sentential logic.

As noted earlier, logic analyzes and classifies arguments according to their *form*. The (logical) form of an argument is a function of the forms of the individual statements that constitute the argument. The logical form of a statement, in turn, is a function of the arrangement of its terms, where the logical terms are regarded as more important than the descriptive terms. Whereas the logical terms have to do with the form of a statement, the descriptive terms have to do with its content.

Note, however, that since the distinction between logical terms and descriptive terms is relative to the particular level of analysis we are pursuing, the notion of logical form is likewise relative in this way. In particular, for each of the different logics listed above, there is a corresponding notion of logical form.

The distinction between form and content is difficult to understand in the abstract. It is best to consider some actual examples. In a later section, we examine this distinction in the context of syllogistic logic.

As soon as we can get a clear idea about form and content, then we can discuss how to classify arguments into those that are deductively correct and those that are not deductively correct.

6. PRELIMINARY DEFINITIONS

In the present section we examine some of the basic ideas in logic which will be made considerably clearer in subsequent chapters.

As we saw in the previous section there is a distinction in logic between *form* and *content*. There is likewise a distinction in logic between arguments that are *good in form* and arguments that are *good in content*. This distinction is best understood by way of an example or two. Consider the following arguments.

(a1) all cats are dogs
all dogs are reptiles
therefore, all cats are reptiles

(a2) all cats are vertebrates
all mammals are vertebrates
therefore, all cats are mammals

Neither of these arguments is good, but they are bad for different reasons. Consider first their content. Whereas all the statements in (a1) are false, all the statements in (a2) are true. Since the premises of (a1) are not all true this is not a good argument *as far as content goes*, whereas (a2) is a good argument *as far as content goes*.

Now consider their forms. This will be explained more fully in a later section. The question is this: do the premises *support* the conclusion? Does the conclusion *follow from* the premises?

In the case of (a1), the premises do in fact support the conclusion, the conclusion does in fact follow from the premises. Although the premises are not true, if they *were* true then the conclusion *would* also be true, of necessity.

In the case of (a2), the premises are all true, and so is the conclusion, but nevertheless the truth of the conclusion is *not* conclusively supported by the premises; in (a2), the conclusion does not follow from the premises. To see that the conclusion does not follow from the premises, we need merely substitute the term 'reptiles' for 'mammals'. Then the premises are both true but the conclusion is false.

All of this is meant to be at an intuitive level. The details will be presented later. For the moment, however we give some rough definitions to help us get started in understanding the ways of classifying various arguments.

In examining an argument there are basically two questions one should ask.

Question 1:	Are all of the premises true?

Question 2:	Does the conclusion ***follow from*** the premises?

The classification of a given argument is based on the answers to these two questions. In particular, we have the following definitions.

An argument is ***factually correct***
if and only if
all of its *premises* are true.

An argument is ***valid***
if and only if
its conclusion *follows from* its premises.

An argument is ***sound***
if and only if
it is *both* factually correct *and* valid.

Basically, a factually correct argument has *good content*, and a valid argument has *good form*, and a sound argument has *both* good content *and* good form.

Note that a factually correct argument *may* have a false conclusion; the definition only refers to the premises.

Whether an argument is valid is sometimes difficult to decide. Sometimes it is hard to know whether or not the conclusion follows from the premises. Part of the problem has to do with knowing what 'follows from' means. In studying logic we are attempting to understand the meaning of 'follows from'; more importantly perhaps, we are attempting to learn how to distinguish between valid and invalid arguments.

Although logic can teach us something about validity and invalidity, it can teach us very little about factual correctness. The question of the truth or falsity of individual statements is primarily the subject matter of the sciences, broadly construed.

As a rough-and-ready definition of validity, the following is offered.

An argument is ***valid***
if and only if
it is **impossible** for
the conclusion to be **false**
while the premises are **all true**.

An alternative definition might be helpful in understanding validity.

To say that an argument is ***valid***
is to say that
if the premises **were** true,
then the conclusion **would necessarily** also be true.

These will become clearer as you read further, and as you study particular examples.

7. FORM AND CONTENT IN SYLLOGISTIC LOGIC

In order to understand more fully the notion of logical form, we will briefly examine syllogistic logic, which was invented by Aristotle (384-322 B.C.).

The arguments studied in syllogistic logic are called syllogisms (more precisely, categorical syllogisms). Syllogisms have a couple of distinguishing characteristics, which make them peculiar as arguments. First of all, every syllogism has exactly two premises, whereas in general an argument can have any number of premises. Secondly, the statements that constitute a syllogism (two premises, one conclusion) come in very few models, so to speak; more precisely, all such statements have forms similar to the following statements.

(1)	all **Lutherans** are **Protestants**	all **dogs** are **collies**
(2)	some **Lutherans** are **Republicans**	some **dogs** are **cats**
(3)	no **Lutherans** are **Methodists**	no **dogs** are **pets**
(4)	some **Lutherans** are not **Democrats**	some **dogs** are not **mammals**

In these examples, the words written in bold-face letters are descriptive terms, and the remaining words are logical terms, relative to syllogistic logic.

In syllogistic logic, the descriptive terms all refer to *classes*, for example, the class of cats, or the class of mammals. On the other hand, in syllogistic logic, the logical terms are all used to express relations among classes. For example, the statements on line (1) state that a certain class (Lutherans/dogs) is entirely contained in another class (Protestants/collies).

Note the following about the four pairs of statements above. In each case, the pair contains both a true statement (on the left) and a false statement (on the right). Also, in each case, the statements are about different things. Thus, we can say that the two statements differ in *content*. Note, however, that in each pair above, the two statements have the same *form*. Thus, although ‘all Lutherans are Protestants’ *differs in content* from ‘all dogs are collies’, these two statements *have the same form*.

The sentences (1)-(4) are what we call *concrete sentences*; they are all actual sentences of a particular actual language (English). Concrete sentences are to be distinguished from *sentence forms*. Basically, a sentence form may be obtained from a concrete sentence by replacing all the descriptive terms by letters, which serve as place holders. For example, sentences (1)-(4) yield the following sentence forms.

(f1) all X are Y
(f2) some X are Y
(f3) no X are Y
(f4) some X are not Y

The process can also be reversed: concrete sentences may be obtained from sentence forms by *uniformly* substituting descriptive terms for the letters. Any concrete sentence obtained from a sentence form in this way is called a *substitution instance* of that form. For example, 'all cows are mammals' and 'all cats are felines' are both substitution instances of sentence form (f1).

Just as there is a distinction between concrete statements and statement forms, there is also a distinction between concrete arguments and argument forms. A *concrete argument* is an argument consisting entirely of concrete statements; an *argument form* is an argument consisting entirely of statement forms. The following are examples of concrete arguments.

(a1) all Lutherans are Protestants
some Lutherans are Republicans
/ some Protestants are Republicans

(a2) all Lutherans are Protestants
some Protestants are Republicans
/ some Lutherans are Republicans

Note: henceforth, we use the slash symbol (/) to abbreviate 'therefore'.

In order to obtain the argument form associated with (a1), we can simply replace each descriptive term by its initial letter; we can do this because the descriptive terms in (a1) all have different initial letters. this yields the following argument form. An alternative version of the form, using X,Y,Z, is given to the right.

(f1)	all L are P	all X are Y
	some L are R	some X are Z
	/ some P are R	/ some Y are Z

By a similar procedure we can convert concrete argument (a2) into an associated argument form.

(f2)	all L are P	all X are Y
	some P are R	some Y are Z
	/ some L are R	/ some X are Z

Observe that argument (a2) is obtained from argument (a1) simply by interchanging the conclusion and the second premise. In other words, these two arguments which are different, consist of precisely the same statements. They are different because their conclusions are different. As we will later see, they are different in that one is a *valid* argument, and the other is an *invalid* argument. Do you know which one is which? In which one does the truth of the premises guarantee the truth of the conclusion?

In deriving an argument form from a concrete argument care must be taken in assigning letters to the descriptive terms. First of all different letters must be assigned to different terms: we cannot use 'L' for both 'Lutherans' and 'Protestants'. Secondly, we cannot use two different letters for the same term: we cannot use 'L' for Lutherans in one statement, and use 'Z' in another statement.

8. DEMONSTRATING INVALIDITY USING THE METHOD OF COUNTEREXAMPLES

Earlier we discussed some of the basic ideas of logic, including the notions of validity and invalidity. In the present section, we attempt to get a better idea about these notions.

We begin by making precise definitions concerning statement forms and argument forms.

> A ***substitution instance*** of an argument/statement form is a concrete argument/statement that is obtained from that form by substituting appropriate descriptive terms for the letters, in such a way that each occurrence of the same letter is replaced by the same term.

> A ***uniform substitution instance*** of an argument/statement form is a substitution instance with the additional property that distinct letters are replaced by distinct (non-equivalent) descriptive terms.

In order to understand these definitions let us look at a very simple argument form (since it has just one premise it is not a syllogistic argument form):

(F) all X are Y
/ some Y are Z

Now consider the following concrete arguments.

(1) all cats are dogs
/ some cats are cows

(2) all cats are dogs
/ some dogs are cats

(3) all cats are dogs
/ some dogs are cows

These examples are not chosen because of their intrinsic interest, but merely to illustrate the concepts of substitution instance and uniform substitution instance.

First of all, (1) is not a substitution instance of (F), and so it is not a uniform substitution instance either (why is this?). In order for (1) to be a substitution instance to (F), it is required that each occurrence of the same letter is replaced by the same term. This is not the case in (1): in the premise, Y is replaced by 'dogs', but in the conclusion, Y is replaced by 'cats'. It is accordingly not a substitution instance.

Next, (2) is a substitution instance of (F), but it is *not* a uniform substitution instance. There is only one letter that appears twice (or more) in (F) – namely, Y. In each occurrence, it is replaced by the same term – namely, 'dogs'. Therefore, (2) is a substitution instance of (F). On the other hand, (2) is not a *uniform* substitution instance since distinct letters – namely, X and Z – are replaced by the same descriptive term – namely, 'cats'.

Finally, (3) is a *uniform* substitution instance and hence a substitution instance, of (F). Y is the only letter that is repeated; in each occurrence, it is replaced by the same term – namely, 'dogs'. So (3) is a substitution instance of (F). To see whether it is a uniform substitution instance, we check to see that the same descriptive term is not used to replace different letters. The only descriptive term that is repeated is 'dogs', and in each case, it replaces Y. Thus, (3) is a uniform substitution instance.

The following is an argument form followed by three concrete arguments, one of which is not a substitution instance, one of which is a non-uniform substitution instance, and one of which is a uniform substitution instance, in that order.

(F) no X are Y
no Y are Z
/ no X are Z

(1) no cats are dogs
no cats are cows
/ no dogs are cows

(2) no cats are dogs
no dogs are cats
/ no cats are cats

(3) no cats are dogs
no dogs are cows
/ no cats are cows

Check to make sure you agree with this classification.

Having defined (uniform) substitution instance, we now define the notion of *having the same form*.

> Two arguments/statements have the ***same form***
> if and only if
> they are both **uniform substitution instances** of the same argument/statement form.

For example, the following arguments have the same form, because they can both be obtained from the argument form that follows as uniform substitution instances.

(a1) all Lutherans are Republicans
some Lutherans are Democrats
/ some Republicans are Democrats

(a2) all cab drivers are maniacs
some cab drivers are Democrats
/ some maniacs are Democrats

The form common to (a1) and (a2) is:

(F) all X are Y
some X are Z
/ some Y are Z

As an example of two arguments that do not have the same form consider arguments (2) and (3) above. They cannot be obtained from a common argument form by *uniform* substitution.

Earlier, we gave two intuitive definitions of *validity*. Let us look at them again.

An argument is **valid**
if and only if
it is **impossible** for
the conclusion to be **false**
while the premises are **all true**.

To say that an argument is ***valid***
is to say that
if the premises **were** true,
then the conclusion **would necessarily** also be true.

Although these definitions may give us a general idea concerning what 'valid' means in logic, they are difficult to apply to specific instances. It would be nice if we had some methods that could be applied to specific arguments by which to decide whether they are valid or invalid.

In the remainder of the present section, we examine a method for showing that an argument is invalid (if it is indeed invalid) – the method of *counterexamples*. Note however, that this method cannot be used to prove that a *valid* argument is in fact valid.

In order to understand the method of counterexamples, we begin with the following fundamental principle of logic.

FUNDAMENTAL PRINCIPLE OF LOGIC

Whether an argument is valid or invalid is determined entirely by its form; in other words:

VALIDITY IS A FUNCTION OF FORM.

This principle can be rendered somewhat more specific, as follows.

FUNDAMENTAL PRINCIPLE OF LOGIC
(REWRITTEN)

If an argument is valid, then every argument *with the same form* is also valid.

If an argument is invalid, then every argument *with the same form* is also invalid.

There is one more principle that we need to add before describing the method of counterexamples. Since the principle almost doesn't need to be stated, we call it the Trivial Principle, which is stated in two forms.

THE TRIVIAL PRINCIPLE

No argument with all true premises but a false conclusion is valid.

If an argument has all true premises but has a false conclusion, then it is invalid.

The Trivial Principle follows from the definition of validity given earlier: an argument is valid if and only if it is impossible for the conclusion to be false while the premises are all true. Now, if the premises are all true, and the conclusion is in fact false, then it is possible for the conclusion to be false while the premises are all true. Therefore, if the premises are all true, and the conclusion is in fact false, then the argument is not valid that is, it is invalid.

Now let's put all these ideas together. Consider the following concrete argument, and the corresponding argument form to its right.

(A)	all cats are mammals some mammals are dogs / some cats are dogs	(F)	all X are Y some Y are Z / some X are Z

First notice that whereas the premises of (A) are both true, the conclusion is false. Therefore, in virtue of the Trivial Principle, argument (A) is invalid. But if (A) is invalid, then in virtue of the Fundamental Principle (rewritten), *every* argument with the *same form* as (A) is also invalid.

In other words, every argument with form (F) is invalid. For example, the following arguments are invalid.

(a2) all cats are mammals
some mammals are pets
/ some cats are pets

(a3) all Lutherans are Protestants
some Protestants are Democrats
/ some Lutherans are Democrats

Notice that the premises are both true and the conclusion is true, in both arguments (a2) and (a3). Nevertheless, both these arguments are *invalid*.

To say that (a2) (or (a3)) is invalid is to say that the truth of the premises does not guarantee the truth of the conclusion – the premises do not support the conclusion. For example, it is possible for the conclusion to be false even while the premises are both true. Can't we imagine a world in which all cats are mammals, some mammals are pets, but no cats are pets. Such a world could in fact be easily brought about by a dastardly dictator, who passed an edict prohibiting cats to be kept as pets. In this world, all cats are mammals (that hasn't changed!), some mammals are pets (e.g., dogs), yet no cats are pets (in virtue of the edict proclaimed by the dictator).

Thus, in argument (a2), it is *possible* for the conclusion to be false while the premises are both true, which is to say that (a2) is *invalid*.

In demonstrating that a particular argument is invalid, it may be difficult to *imagine* a world in which the premises are true but the conclusion is false. An easier method, which does not require one to imagine unusual worlds, is the method of counterexamples, which is based on the following definition and principle, each stated in two forms.

A. A ***counterexample*** to an **argument form** is any substitution instance (not necessarily uniform) of that form having true premises but a false conclusion.

B. A ***counterexample*** to a **concrete argument** $\mathcal{A}$ is any concrete argument that

(1) has the **same form** as $\mathcal{A}$
(2) has all true premises
(3) has a false conclusion

PRINCIPLE OF COUNTEREXAMPLES

A. An argument (form) is **invalid** if it admits a counterexample.

B. An argument (form) is **valid** only if it does not admit any counterexamples.

The Principle of Counterexamples follows our earlier principles and the definition of the term 'counterexample'. One might reason as follows:

> Suppose argument $\mathcal{A}$ admits a counterexample. Then there is another argument $\mathcal{A}$* such that:
> (1) $\mathcal{A}$* has the same form as $\mathcal{A}$,
> (2) $\mathcal{A}$* has all true premises, and
> (3) $\mathcal{A}$* has a false conclusion.
> Since $\mathcal{A}$* has all true premises but a false conclusion, $\mathcal{A}$* is invalid, in virtue of the Trivial Principle. But $\mathcal{A}$ and $\mathcal{A}$* have the same form, so in virtue of the Fundamental Principle, $\mathcal{A}$ is invalid also.

According to the Principle of Counterexamples, one can demonstrate that an argument is *invalid* by showing that it admits a counterexample. As an example, consider the earlier arguments (a2) and (a3). These are both invalid. To see this, we merely look at the earlier argument (A), and note that it is a counterexample to both (a2) and (a3). Specifically, (A) has the same form as (a2) and (a3), it has all true premises, and it has a false conclusion. Thus, the existence of (A) demonstrates that (a2) and (a3) are *invalid*.

Let us consider two more examples. In each of the following, an invalid argument is given, and a counterexample is given to its right.

(a4)	no cats are dogs no dogs are apes / no cats are apes	(c4)	no men are women no women are fathers / no men are fathers
(a5)	all humans are mammals no humans are reptiles / no mammals are reptiles	(c5)	all men are humans no men are mothers / no humans are mothers

In each case, the argument to the right has the same form as the argument to the left; it also has all true premises and a false conclusion. Thus, it demonstrates the *invalidity* of the argument to the left.

In (a4), as well as in (a5), the premises are true, and so is the conclusion; nevertheless, the conclusion does *not follow* from the premises, and so the argument is invalid. For example, if (a4) were valid, then (c4) would be valid also, since they have exactly the same form. But (c4) is not valid, because it has a false conclusion and all true premises. So, (c4) is not valid either. The same applies to (a5) and (c5).

If all we know about an argument is whether its premises and conclusion are true or false, then usually we cannot say whether the argument is valid or invalid. In fact, there is only one case in which we can say: when the premises are all true, and the conclusion is false, the argument is definitely *invalid* (by the Trivial Principle). However, in all other cases, we cannot say, one way or the other; we need *additional* information about the *form* of the argument.

This is summarized in the following table.

PREMISES	CONCLUSION	VALID OR INVALID?
all true	true	can't tell; need more info
all true	false	**definitely invalid**
not all true	true	can't tell; need more info
not all true	false	can't tell; need more info

9. EXAMPLES OF VALID ARGUMENTS IN SYLLOGISTIC LOGIC

In the previous section, we examined a few examples of invalid arguments in syllogistic logic. In each case of an invalid argument we found a counterexample, which is an argument with the same form, having all true premises but a false conclusion.

In the present section, we examine a few examples of valid syllogistic arguments (also called valid *syllogisms*). At present we have no method to demonstrate that these arguments are in fact valid; this will come in later sections of this chapter.

Note carefully: if we cannot find a counterexample to an argument, it does not mean that no counterexample exists; it might simply mean that we have not looked hard enough. Failure to find a counterexample is not proof that an argument is valid.

Analogously, if I claimed "all swans are white", you could refute me simply by finding a swan that isn't white; this swan would be a counterexample to my claim. On the other hand, if you could not find a non-white swan, I could not thereby say that my claim was proved, only that it was *not disproved yet*.

Thus, although we are going to examine some examples of valid syllogisms, we do not presently have a technique to prove this. For the moment, these merely serve as examples.

The following are all valid syllogistic argument *forms*.

(f1) all X are Y
all Y are Z
/ all X are Z

(f2) all X are Y
some X are Z
/ some Y are Z

(f3) all X are Z
no Y are Z
/ no X are Y

(f4) no X are Y
some Y are Z
/ some Z are not X

To say that (f1)-(f4) are valid argument forms is to say that every argument obtained from them by substitution is a valid argument.

Let us examine the first argument form (f1), since it is by far the simplest to comprehend. Since (f1) is valid, every substitution instance is valid. For example the following arguments are all valid.

(1a)	all cats are mammals	T
	all mammals are vertebrates	T
	/ all cats are vertebrates	T
(1b)	all cats are reptiles	F
	all reptiles are vertebrates	T
	/ all cats are vertebrates	T
(1c)	all cats are animals	T
	all animals are mammals	F
	/ all cats are mammals	T
(1d)	all cats are reptiles	F
	all reptiles are mammals	F
	/ all cats are mammals	T
(1e)	all cats are mammals	T
	all mammals are reptiles	F
	/ all cats are reptiles	F
(1f)	all cats are reptiles	F
	all reptiles are cold-blooded	T
	/ all cats are cold-blooded	F
(1g)	all cats are dogs	F
	all dogs are reptiles	F
	/ all cats are reptiles	F
(1h)	all Martians are reptiles	?
	all reptiles are vertebrates	T
	/ all Martians are vertebrates	?

In the above examples, a number of possibilities are exemplified. It is possible for a valid argument to have all true premises and a true conclusion – (1a); it is possible for a valid argument to have some false premises and a true conclusion – (1b)-(1c); it is possible for a valid argument to have all false premises and a true conclusion – (1d); it is possible for a valid argument to have all false premises and a false conclusion – (1g).

On the other hand, it is not possible for a valid argument to have all true premises and a false conclusion – no example of this.

In the case of argument (1h), we don't know whether the first premise is true or whether it is false. Nonetheless, the argument is valid; that is, if the first premise *were* true, then the conclusion *would necessarily* also be true, since the second premise is true.

The truth or falsity of the premises and conclusion of an argument is not crucial to the validity of the argument. To say that an argument is valid is simply to say that the conclusion *follows from* the premises.

The truth or falsity of the premises and conclusion may not even arise, as for example in a fictional story. Suppose I write a science fiction story, and suppose this story involves various classes of people (human or otherwise!), among them being Gargatrons and Dacrons. Suppose I say the following about these two classes.

(1) all Dacrons are thieves
(2) no Gargatrons are thieves

(the latter is equivalent to: no thieves are Gargatrons).

What could the reader immediately conclude about the relation between Dacrons and Gargatrons?

(3) no Dacrons are Gargatrons (or: no Gargatrons are Dacrons)

I (the writer) would not have to say this explicitly for it to be true in my story; I would not have to say it for you (the reader) to know that it is true in my story; it follows from other things already stated. Furthermore, if I (the writer) were to introduce a character in a later chapter call it Persimion (unknown gender!), and if I were to say that Persimion is both a Dacron and a Gargatron, then I would be guilty of *logical inconsistency* in the story.

I would be guilty of inconsistency, because it is not possible for the first two statements above to be true without the third statement also being true. The third statement *follows from* the first two. There is no world (real or imaginary) in which the first two statements are true, but the third statement is false.

Thus, we can say that statement (3) follows from statements (1) and (2) without having any idea whether they are true or false. All we know is that in any world (real or imaginary), if (1) and (2) are true, then (3) must also be true.

Note that the argument from (1) and (2) to (3) has the form (F3) from the beginning of this section.

10. EXERCISES FOR CHAPTER 1

EXERCISE SET A

For each of the following say whether the statement is true (T) or false (F).

1. In any valid argument, the premises are all true.
2. In any valid argument, the conclusion is true.
3. In any valid argument, if the premises are all true, then the conclusion is also true.
4. In any factually correct argument, the premises are all true.
5. In any factually correct argument, the conclusion is true.
6. In any sound argument, the premises are all true.
7. In a sound argument the conclusion is true.
8. Every sound argument is factually correct.
9. Every sound argument is valid.
10. Every factually correct argument is valid.
11. Every factually correct argument is sound.
12. Every valid argument is factually correct.
13. Every valid argument is sound.
14. Every valid argument has a true conclusion.
15. Every factually correct argument has a true conclusion.
16. Every sound argument has a true conclusion.
17. If an argument is valid and has a false conclusion, then it must have at least one false premise.
18. If an argument is valid and has a true conclusion, then it must have all true premises.
19. If an argument is valid and has at least one false premise then its conclusion must be false.
20. If an argument is valid and has all true premises, then its conclusion must be true.

EXERCISE SET B

In each of the following, you are given an argument to analyze. In each case, answer the following questions.

(1) Is the argument **factually correct**?
(2) Is the argument **valid**?
(3) Is the argument **sound**?

Note that in many cases, the answer might legitimately be “can't tell”. For example, in certain cases in which one does not know whether the premises are true or false, one cannot decide whether the argument is factually correct, and hence on cannot decide whether the argument is sound.

1. all dogs are reptiles
all reptiles are Martians
/ all dogs are Martians

2. some dogs are cats
all cats are felines
/ some dogs are felines

3. all dogs are Republicans
some dogs are flea-bags
/ some Republicans are flea-bags

4. all dogs are Republicans
some Republicans are flea-bags
/ some dogs are flea-bags

5. some cats are pets
some pets are dogs
/ some cats are dogs

6. all cats are mammals
all dogs are mammals
/ all cats are dogs

7. all lizards are reptiles
no reptiles are warm-blooded
/ no lizards are warm-blooded

8. all dogs are reptiles
no reptiles are warm-blooded
/ no dogs are warm-blooded

9. no cats are dogs
no dogs are cows
/ no cats are cows

10. no cats are dogs
some dogs are pets
/ some pets are not cats

11. only dogs are pets
some cats are pets
/ some cats are dogs

12. only bullfighters are macho
Max is macho
/ Max is a bullfighter

13. only bullfighters are macho
Max is a bullfighter
/ Max is macho

14. food containing DDT is dangerous
everything I cook is dangerous
/ everything I cook contains DDT

15. the only dogs I like are collies
Sean is a dog I like
/ Sean is a collie

16. the only people still working these exercises are masochists
I am still working on these exercises
/ I am a masochist

EXERCISE SET C

In the following, you are given several syllogistic arguments (some valid, some invalid). In each case, attempt to construct a counterexample. A valid argument does not admit a counterexample, so in some cases, you will not be able to construct a counterexample.

1. all dogs are reptiles
   all reptiles are Martians
   / all dogs are Martians

2. all dogs are mammals
   some mammals are pets
   / some dogs are pets

3. all ducks waddle
   nothing that waddles is graceful
   / no duck is graceful

4. all cows are eligible voters
   some cows are stupid
   / some eligible voters are stupid

5. all birds can fly
   some mammals can fly
   / some birds are mammals

6. all cats are vertebrates
   all mammals are vertebrates
   / all cats are mammals

7. all dogs are Republicans
   some Republicans are flea-bags
   / some dogs are flea-bags

8. all turtles are reptiles
   no turtles are warm-blooded
   / no reptiles are warm-blooded

9. no dogs are cats
   no cats are apes
   / no dogs are apes

10. no mammals are cold-blooded
    some lizards are cold-blooded
    / some mammals are not lizards

11. ANSWERS TO EXERCISES FOR CHAPTER 1

EXERCISE SET A

1.	False	11.	False
2.	False	12.	False
3.	True	13.	False
4.	True	14.	False
5.	False	15.	False
6.	True	16.	True
7.	True	17.	True
8.	True	18.	False
9.	True	19.	False
10.	False	20.	True

EXERCISE SET B

1.	factually correct?	NO	9.	factually correct?	YES
	valid?	YES		valid?	NO
	sound?	NO		sound?	NO
2.	factually correct?	NO	10.	factually correct?	YES
	valid?	YES		valid?	YES
	sound?	NO		sound?	YES
3.	factually correct?	NO	11.	factually correct?	NO
	valid?	YES		valid?	YES
	sound?	NO		sound?	NO
4.	factually correct?	NO	12.	factually correct?	NO
	valid?	NO		valid?	YES
	sound?	NO		sound?	NO
5.	factually correct?	YES	13.	factually correct?	NO
	valid?	NO		valid?	NO
	sound?	NO		sound?	NO
6.	factually correct?	YES	14.	factually correct?	can't tell
	valid?	NO		valid?	NO
	sound?	NO		sound?	NO
7.	factually correct?	YES	15.	factually correct?	can't tell
	valid?	YES		valid?	YES
	sound?	YES		sound?	can't tell
8.	factually correct?	NO	16.	factually correct?	can't tell
	valid?	YES		valid?	YES
	sound?	NO		sound?	can't tell

EXERCISE SET C

	Original Argument	Counterexample
1.	all dogs are reptiles all reptiles are Martians / all dogs are Martians	valid; admits no counterexample
2.	all dogs are mammals some mammals are pets / some dogs are pets	all dogs are mammals some mammals are cats / some dogs are cats
3.	all ducks waddle nothing that waddles is graceful / no duck is graceful	valid; admits no counterexample
4.	all cows are eligible voters some cows are stupid / some eligible voters are stupid	valid; admits no counterexample
5.	all birds can fly some mammals can fly / some birds are mammals	all birds lay eggs some mammals lay eggs (the platypus) / some birds are mammals
6.	all cats are vertebrates all mammals are vertebrates / all cats are mammals	all cats are vertebrates all reptiles are vertebrates / all cats are reptiles
7.	all dogs are Republicans some Republicans are flea-bags / some dogs are flea-bags	all dogs are mammals some mammals are cats / some dogs are cats
8.	all turtles are reptiles no turtles are warm-blooded / no reptiles are warm-blooded	all turtles are reptiles no turtles are lizards / no reptiles are lizards
9.	no dogs are cats no cats are apes / no dogs are apes	no dogs are cats no cats are poodles / no dogs are poodles
10.	no mammals are cold-blooded some lizards are cold-blooded / some mammals are not lizards	no mammals are cold-blooded some vertebrates are cold-blooded / some mammals are not vertebrates