Linear Algebra

LU DECOMPOSITION NUMERICAL METHOD

LU – DECOMPOSITION METHOD

- LU Decomposition method to solve a set of simultaneous linear equations.
- In Linear Algebra, the LU decomposition method decomposition which write a matrix as the product of a lower triangular matrix and an upper triangular matrix. The product sometimes includes a permutation matrix as well. The decomposition is used in numerical analysis to solve system of linear equations or calculate the determinat.

METHOD

- For most non-singular matrix [A] that one could conduct Gauss Elimination forward elimination steps.
- [A] =[L][U]

Where
[L]= lower triangular Matrix
[U] = upper triangular Matrix

Way to use

Given [A][X]=[C]

1. Decomposition [A]=[L][U]
2. Solve [L][Z]=C
3. Solve [U][X]=Z

$$A=LU=\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{21} & u_{22} \\ 0 & 0 & u_{33} \end{bmatrix}$$

EXAMPLE:01

$$Find\ an\ LU\ Decomposition\ of\ \begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix}$$

A= $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix}$ =LU

Where

L = $\begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix}$, U = $\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$

LU= $\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ L_{21}U_{11} & L_{21}U_{12} + U_{12} & L_{21}U_{13} + U_{23} \\ L_{31}U_{11} & L_{31}U_{12} + L_{32}U_{22} & L_{31}U_{13} + L_{32}U_{23} + U_{33} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix}$

$$U_{11} = 1; U_{12} = 2; U_{13} = 4;$$

Now consider the second row

$$L_{21}U_{11} = 3 \quad \therefore L_{21} \times 1 = 3 \quad \therefore \boxed{L_{21} = 3},$$

$$L_{21}U_{12} + U_{22} = 8 \quad \therefore 3 \times 2 + U_{22} = 8 \quad \therefore \boxed{U_{22} = 2},$$

$$L_{21}U_{13} + U_{23} = 14 \quad \therefore 3 \times 4 + U_{23} = 14 \quad \therefore \boxed{U_{23} = 2}.$$

Notice how, at each step, the equation in hand has only one unknown in it, and other quantities that we have already found. This pattern continues on the last row

$$L_{31}U_{11} = 2 \quad \therefore L_{31} \times 1 = 2 \quad \therefore \boxed{L_{31} = 2},$$

$$L_{31}U_{12} + L_{32}U_{22} = 6 \quad \therefore 2 \times 2 + L_{32} \times 2 = 6 \quad \therefore \boxed{L_{32} = 1},$$

$$L_{31}U_{13} + L_{32}U_{23} + U_{33} = 13 \quad \therefore (2 \times 4) + (1 \times 2) + U_{33} = 13 \quad \therefore \boxed{U_{33} = 3}.$$

We have shown that

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

and this is an LU decomposition of A.

$$Find\ an\ LU\ Decomposition\ of\ \begin{bmatrix} 3 & 1 \\ -6 & -4 \end{bmatrix}$$