ONE STEP AT A TIME | **Completing Step 5 of the Five-Step Model: Making a Decision and Interpreting Results**

Step	*Operation*
1.	Compare the *t*(obtained) to your *t*(critical). If *t*(obtained) is *in* the critical region, *reject* the null hypothesis. If *t*(obtained) is *not in* the critical region, *fail to reject* the null hypothesis.
2.	Interpret the decision to reject or fail to reject the null hypothesis in terms of the original question. For example, our conclusion for the example problem was "There is a significant difference between the size of center-city and suburban families."

Hypothesis Testing With Sample Proportions (Large Samples)

Testing for the significance of the difference between two sample proportions is analogous to testing sample means. The null hypothesis states that there is no difference between the populations from which the samples are drawn. We compute a test statistic in step 4, which is then compared with the critical region. When sample sizes are large (combined *N*s of more than 100), the *Z* distribution may be used to find the critical region. In this text, we will not consider tests of significance for proportions based on small samples.

In order to find the value of the test statistics, several preliminary equations must be solved. Formula 8.7 uses the values of the two sample proportions (P_s) to give us an estimate of the population proportion (P_u)—the proportion of cases in the population that have the trait under consideration assuming the null hypothesis is true:

FORMULA 8.7

$$P_u = \frac{N_1 P_{s1} + N_2 P_{s2}}{N_1 + N_2}$$

The value of P_u is then used to compute the standard deviation of the sampling distribution of the difference in sample proportions in Formula 8.8:

FORMULA 8.8

$$\sigma_{p-p} = \sqrt{P_u(1 - P_u)}\sqrt{\frac{N_1 + N_2}{N_1 N_2}}$$

This value is then substituted into the formula for computing the test statistic, presented as Formula 8.9:

FORMULA 8.9

$$Z\,(\text{obtained}) = \frac{(P_{s1} - P_{s2}) - (P_{u1} - P_{u2})}{\sigma_{p-p}}$$

where $(P_{s1} - P_{s2})$ = the difference between the sample proportions
$(P_{u1} - P_{u2})$ = the difference between the population proportions
(σ_{p-p}) = the standard deviation of the sampling distribution of the difference between sample proportions

As was the case with sample means, the second term in the numerator is assumed to be zero by the null hypothesis. Therefore, the formula reduces to:

FORMULA 8.10

$$Z(\text{obtained}) = \frac{(P_{s1} - P_{s2})}{\sigma_{p-p}}$$

Remember to solve these equations in order, starting with Formula 8.7 (and skipping Formula 8.9).

Illustrating a Test of Hypothesis Between Two Sample Proportions (Large Samples) An example will make these procedures clearer. Suppose we are researching social networks among senior citizens and wonder if blacks and whites differ in their number of memberships in clubs and other organizations. Random samples of black and white senior citizens have been selected and classified as high or low in terms of their number of memberships in voluntary associations. Is there a statistically significant difference in the participation patterns of black and white elderly? The proportions of each group classified as “high” in participation and the sample size for both groups are:

Sample 1 (Black Senior Citizens)	Sample 2 (White Senior Citizens)
$P_{s1} = 0.34$	$P_{s2} = 0.25$
$N_1 = 83$	$N_2 = 103$

Step 1. Making Assumptions and Meeting Test Requirements.

Model: Independent random samples
Level of measurement is nominal
Sampling distribution is normal

Step 2. Stating the Null Hypothesis. Because no direction has been predicted, this will be a two-tailed test.

$$H_0: P_{u1} = P_{u2}$$

$$H_1: P_{u1} \neq P_{u2}$$

Step 3. Selecting the Sampling Distribution and Establishing the Critical Region. Because the sample size is large, the Z distribution will be used to establish the critical region. Setting alpha at 0.05, we have:

$$\text{Sampling distribution} = Z \text{ distribution}$$

$$\text{Alpha} = 0.05, \text{two-tailed}$$

$$Z(\text{critical}) = \pm 1.96$$

Step 4. Computing the Test Statistic. Solve Formula 8.7 first, substitute the resultant value into Formula 8.8, and then solve for Z(obtained) with Formula 8.10.

$$P_u = \frac{N_1 P_{s1} + N_2 P_{s2}}{N_1 + N_2}$$

$$P_u = \frac{(83)(0.34) + (103)(0.25)}{83 + 103}$$

$$P_u = 0.29$$

$$\sigma_{p-p} = \sqrt{P_u(1 - P_u)}\sqrt{\frac{N_1 + N_2}{N_1 N_2}}$$

$$\sigma_{p-p} = \sqrt{(0.29)(0.71)}\sqrt{\frac{83 + 103}{(83)(103)}}$$

$$\sigma_{p-p} = (0.45)(0.15)$$

$$\sigma_{p-p} = 0.07$$

$$Z(\text{obtained}) = \frac{(P_{s1} - P_{s2})}{\sigma_{p-p}}$$

$$Z(\text{obtained}) = \frac{0.34 - 0.25}{0.07}$$

$$Z(\text{obtained}) = 1.29$$

ONE STEP AT A TIME **Completing Step 4 of the Five-Step Model: Computing *Z*(obtained)**

Solve Formulas 8.7, 8.8, and 8.10—in that order—to find the test statistic.

Step	***Operation***
To Solve Formula 8.7	
1.	Add N_1 and N_2.
2.	Multiply P_{s1} by N_1.
3.	Multiply P_{s2} by N_2.
4.	Add the quantity you found in step 3 to the quantity you found in step 2.
5.	Divide the quantity you found in step 4 by the quantity you found in step 1. This is P_u.
To Solve Formula 8.8	
1.	Multiply P_u by $(1 - P_u)$.
2.	Take the square root of the quantity you found in step 1.
3.	Multiply N_1 by N_2.
4.	Add N_1 and N_2. (Note: You already found this value when solving Formula 8.7. See step 1.)
5.	Divide the quantity you found in step 4 by the quantity you found in step 3.
6.	Take the square root of the quantity you found in step 5.
7.	Multiply the quantity you found in step 6 by the quantity you found in step 2.

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ONE STEP AT A TIME ***(continued)***

Step	Operation
To Solve Formula 8.10	
1.	Find the difference between the sample proportions.
2.	Divide the quantity you found in step 1 by the quantity you found in step 7 of "To Solve Formula 8.8." This is *Z*(obtained).

Step 5. Making a Decision and Interpreting the Results of the Test. Because the test statistic—Z(obtained) = 1.29—does not fall into the critical region as marked by the Z(critical) of ±1.96, we fail to reject the null hypothesis. The difference between the sample proportions is no greater than what would be expected if the null hypothesis were true and only random chance were operating. Black and white senior citizens are not significantly different in terms of participation patterns as measured in this test. *(For practice in testing the significance of the difference between sample proportions, see problems 8.10 to 8.14, 8.15a to c, and 8.16a to c.)*

ONE STEP AT A TIME **Completing Step 5 of the Five-Step Model: Making a Decision and Interpreting Results**

Step	Operation
1.	Compare *Z*(obtained) to *Z*(critical). If *Z*(obtained) is *in* the critical region, *reject* the null hypothesis. If *Z*(obtained) is *not in* the critical region, *fail to reject* the null hypothesis.
2.	Interpret the decision to reject or fail to reject the null hypothesis in terms of the original question. For example, our conclusion for the example problem was "There is no significant difference between the participation patterns of black and white senior citizens."

Applying Statistics 8.2 Testing the Significance of the Difference Between Sample Proportions

Do attitudes toward sex vary by gender? The proportion of each sex that feels that premarital sex is always wrong is:

Females	Males
$P_{s1} = 0.35$	$P_{s2} = 0.32$
$N_1 = 450$	$N_2 = 417$

Females are more likely to say that premarital sex is always wrong. Is the difference significant? The table presents all the information we will need to conduct a test of the null hypothesis following the familiar five-step model with alpha set at .05, two-tailed test.

Step 1. Making Assumptions and Meeting Test Requirements.

Model: Independent random samples
Level of measurement is nominal
Sampling distribution is normal

Step 2. Stating the Null Hypothesis.

$$H_0: P_{u1} = P_{u2}$$
$$H_1: P_{u1} \neq P_{u2}$$

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Applying Statistics 8.2 *(continued)*

Step 3. Selecting the Sampling Distribution and Establishing the Critical Region.

$$\text{Sampling distribution} = Z \text{ distribution}$$
$$\text{Alpha} = 0.05, \text{ two-tailed}$$
$$Z(\text{critical}) = \pm 1.96$$

Step 4. Computing the Test Statistic. Remember to start with Formula 8.7, substitute the value for P_u into Formula 8.8, and then substitute that value into Formula 8.10 to solve for Z(obtained):

$$P_u = \frac{N_1 P_{s1} + N_2 P_{s2}}{N_1 + N_2}$$

$$P_u = \frac{(450)(0.35) + (417)(0.32)}{450 + 417}$$

$$P_u = \frac{290.94}{867}$$

$$P_u = 0.34$$

$$\sigma_{p-p} = \sqrt{P_u(1 - P_u)}\sqrt{\frac{N_1 + N_2}{N_1 N_2}}$$

$$\sigma_{p-p} = \sqrt{(0.34)(0.66)}\sqrt{\frac{450 + 417}{(450)(417)}}$$

$$\sigma_{p-p} = \sqrt{0.2244}\sqrt{0.0046}$$

$$\sigma_{p-p} = (0.47)(0.068)$$

$$\sigma_{p-p} = 0.032$$

$$Z(\text{obtained}) = \frac{(P_{s1} - P_{s2})}{\sigma_{p-p}}$$

$$Z(\text{obtained}) = \frac{0.35 - 0.32}{0.032}$$

$$Z(\text{obtained}) = \frac{0.030}{0.032}$$

$$Z(\text{obtained}) = 0.94$$

Step 5. Making a Decision and Interpreting the Results of the Test. With an obtained Z score of 0.94, we would fail to reject the null hypothesis. Females are not significantly more likely to feel that premarital sex is always wrong.

STATISTICS IN EVERYDAY LIFE

According to the Gallup polls, Republicans and Democrats have become increasingly polarized over the abortion issue. In 1975, two years after the landmark *Roe v. Wade* Supreme Court decision, 18% of Republicans and 19% of Democrats agreed that abortion should be legal "under any circumstances." In 2011, the percentage of Republicans approving abortion under any circumstances had fallen to 13%, while the percentage of Democrats approving had risen to 38%. The one percentage point difference in 1975 might have been statistically significant, but it was clearly not important. The 25% difference in 2011 is statistically significant, important, and extremely consequential for the possibilities of mutual understanding and civil discourse in American politics. These results are based on random samples of about 1,000 adult Americans.

How could we explain these trends? One possibility is that the Republican Party has lost many moderate, pro-choice members since the 1970s, leaving the party smaller and more ideologically homogenous. What information would you need to investigate this possibility?

Source: Saad, Lydia. 2010. "Republicans', Dems' abortion views grow more polarized." Available at http://www.gallup.com/poll/126374/Republicans-Dems-Abortion-Views-Grow-Polarized.aspx?version=print.

Saad, Lydia. 2011. "Americans Still Split Along 'Pro-Choice,' 'Pro-Life' Lines. Available at http://www.gallup.com/poll/147734/Americans-Split-Along-Pro-Choice-Pro-Life-Lines.aspx.