FIGURE 7.8 Sampling Distribution Showing *t*(Obtained) versus *t*(Critical) ($\alpha = 0.05$, two-tailed test, *df* = 29)

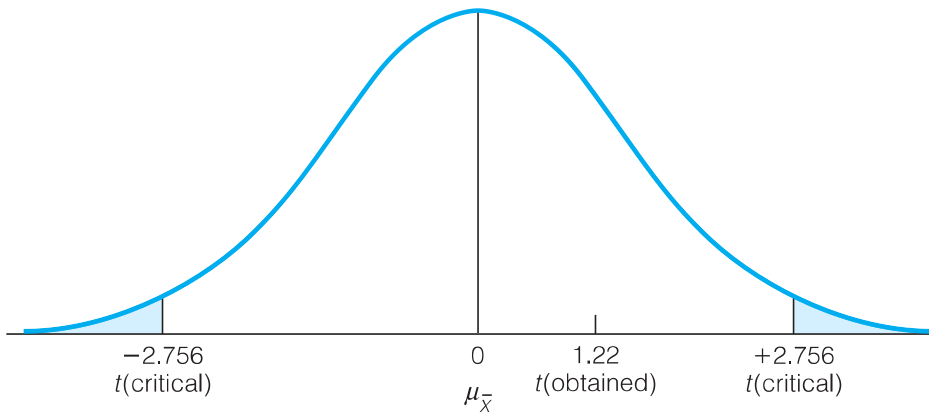

TABLE 7.6 Choosing a Sampling Distribution When Testing Single-Sample Means for Significance

If Population Standard Deviation (σ) Is	Sampling Distribution
Known	*Z* distribution
Unknown and the sample size (*N*) is large	*Z* distribution
Unknown and the sample size (*N*) is small	*t* distribution

will be used. These decisions are summarized in Table 7.6. *(For practice in using the t distribution in a test of hypothesis, see problems 7.8, 7.10 to 7.12, and 7.19 .)*

Tests of Hypotheses for Single-Sample Proportions (Large Samples)

In many cases, we work with variables that are not interval-ratio in level of measurement. One alternative in this situation would be to use a sample proportion (P_s) rather than a sample mean as the test statistic. As we shall see, the overall procedures for testing single-sample proportions are the same as those for testing means. The central question is still "Does the population from which the sample was drawn have a certain characteristic?" We still conduct the test based on the assumption that the null hypothesis is true, and we still evaluate the probability of the obtained sample outcome against a sampling distribution of all possible sample outcomes. Our decision at the end of the test is also the same. If the obtained test statistic falls into the critical region (is unlikely, given the assumption that the H_0 is true), we reject the H_0.

Of course, there are also some important differences in significance tests for sample proportions. These differences are best related in terms of the five-step model. In step 1, we assume that the variable is measured at the nominal level. In step 2, the symbols used to state the null hypothesis are different even though it is still a statement of "no difference."

In step 3, we will use only the standardized normal curve (the Z distribution) to find areas under the sampling distribution and to locate the critical region. This will be appropriate as long as the sample size is large. We will not consider small-sample tests of hypothesis for proportions in this text.

In step 4—computing the test statistic—the form of the formula remains the same. That is, the test statistic—Z(obtained)—equals the sample statistic minus the mean of the sampling distribution divided by the standard deviation of the sampling distribution. However, the symbols will change because we are basing the tests on sample proportions. The formula can be stated as:

FORMULA 7.3

$$Z(\text{obtained}) = \frac{P_s - P_u}{\sqrt{P_u(1 - P_u)/N}}$$

Step 5—making a decision—is exactly the same as before. If the test statistic—Z(obtained)—falls into the critical region, reject the H_0.

Applying Statistics 7.2 Testing Sample Proportions for Significance: The One-sample Case

Seventy-six percent of the respondents in a random sample ($N = 103$) drawn from the most affluent neighborhood in a community voted Republican in the most recent presidential election. For the community as a whole, 66% of the electorate voted Republican. Was the affluent neighborhood significantly more likely to have voted Republican?

Step 1. Making Assumptions and Meeting Test Requirements.

Model: Random sampling
Level of measurement is nominal
Sampling distribution is normal

This is a large sample, so we may assume a normal sampling distribution. The variable "percent Republican" is only nominal in level of measurement.

Step 2. Stating the Null Hypothesis (H_0). The null hypothesis says that the affluent neighborhood is not different from the community as a whole:

$$H_0\text{: } P_u = 0.66$$

The original question ("Was the affluent neighborhood *more* likely to vote Republican?") suggests a one-tailed research hypothesis:

$$(H_1\text{: } P_u > .66)$$

Step 3. Selecting the Sampling Distribution and Establishing the Critical Region.

$$\text{Sampling distribution} = Z \text{ distribution}$$
$$\alpha = 0.05$$
$$Z(\text{critical}) = +1.65$$

The research hypothesis says that we will be concerned only with outcomes in which the neighborhood is *more* likely to vote Republican or with sample outcomes in the upper tail of the sampling distribution.

Step 4. Computing the Test Statistic. The information necessary for a test of the null hypothesis, expressed in the form of proportions, is:

Neighborhood	Community
$P_s = 0.76$ $N = 103$	$P_u = 0.66$

The test statistic—Z(obtained)—would be:

$$Z(\text{obtained}) = \frac{P_s - P_u}{\sqrt{P_u(1 - P_u)/N}}$$

$$Z(\text{obtained}) = \frac{0.76 - 0.66}{\sqrt{(0.66)(1 - 0.66)/103}}$$

$$Z(\text{obtained}) = \frac{0.10}{\sqrt{(0.2244)/103}}$$

(*continued next page*)

Applying Statistics 7.2 (continued)

$$Z(\text{obtained}) = \frac{0.100}{0.047}$$

$$Z(\text{obtained}) = 2.13$$

Step 5. Making a Decision and Interpreting Test Results. With alpha set at 0.05, one-tailed, the critical region begins at Z(critical) $= +1.65$. With an obtained Z score of 2.13, the null hypothesis is rejected. The difference between the affluent neighborhood and the community as a whole is statistically significant and in the predicted direction. Residents of the affluent neighborhood were significantly more likely to have voted Republican in the last presidential election.

A Test of Hypothesis by Using Sample Proportions. An example should clarify these procedures. A random sample of 122 households in a low-income neighborhood revealed that 53 (or a proportion of 0.43) of the households were headed by females. In the city as a whole, the proportion of female-headed households is 0.39. Are households in the lower-income neighborhood significantly different from the city as a whole in terms of this characteristic?

Step 1. Making Assumptions and Meeting Test Requirements.

Model: Random sampling
Level of measurement is nominal
Sampling distribution is normal in shape

Step 2. Stating the Null Hypothesis. The research question, as stated earlier, asks only if the sample proportion is *different from* the population proportion. Because we have not predicted a direction for the difference, a two-tailed test will be used.

$$H_0: P_u = 0.39$$
$$(H_1: P_u \neq 0.39)$$

Step 3. Selecting the Sampling Distribution and Establishing the Critical Region.

$$\text{Sampling distribution} = Z \text{ distribution}$$
$$\alpha = 0.10, \text{ two-tailed test}$$
$$Z(\text{critical}) = \pm 1.65$$

Step 4. Computing the Test Statistic.

$$Z(\text{obtained}) = \frac{P_s - P_u}{\sqrt{P_u(1 - P_u)/N}}$$

$$Z(\text{obtained}) = \frac{0.43 - 0.39}{\sqrt{(0.39)(0.61)/122}}$$

$$Z(\text{obtained}) = +0.91$$

ONE STEP AT A TIME **Completing Step 4 of the Five-Step model: Computing Z(obtained)**

Use Formula 7.3 to compute the test statistic.

Step	***Operation***
1.	Start with the denominator of Formula 7.3 and then substitute in the value for P_u. This value will be given in the statement of the problem.
2.	Subtract the value of P_u from 1.
3.	Multiply the value you found in step 2 by the value of P_u.
4.	Divide the quantity you found in step 3 by N.
5.	Take the square root of the value you found in step 4.
6.	Subtract P_u from P_s.
7.	Divide the quantity you found in step 6 by the quantity you found in step 5. This value is Z(obtained).

Step 5. Making a Decision and Interpreting Test Results. The test statistic—Z(obtained)—does not fall into the critical region. Therefore, we fail to reject the H_0. There is no statistically significant difference between the low-income community and the city as a whole in terms of the proportion of households headed by females. Figure 7.9 displays the sampling distribution, the critical region, and the Z(obtained). *(For practice in tests of significance using sample proportions, see problems 7.1c, 7.13 to 7.16, 7.17a to d, 7.18, and 7.21a and b.)*

ONE STEP AT A TIME **Completing Step 5 of the Five-Step model: Making a Decision and Interpreting Results**

Step	***Operation***
1.	Compare your Z(obtained) to your Z(critical). If Z(obtained) is *in* the critical region, *reject* the null hypothesis. If Z(obtained) is *not in* the critical region, *fail to reject* the null hypothesis.
2.	Interpret the decision in terms of the original question. For example, our conclusion for the example problem used in this section was "There is no significant difference between the low-income community and the city as a whole in the proportion of households that are headed by females."

FIGURE 7.9 Sampling Distribution Showing Z(Obtained) versus Z(Critical) ($\alpha = 0.10$, Two-Tailed Test)