$+k^2(c_3 + c_4k) = k + 1$

$k^3(4c_4 + 4c_4 + c_4) + k^2(4c_3 + 8c_4 + 8c_4 + 4c_3 + 8c_4 + 16c_4 + c_3)$

$+k(16c_3 + 32c_4 + 8c_3 + 16c_4 + 16c_4 + c_4)$$+16c_3 + 32c_4 + 4c_3 + 8c_4 = k + 1$

$9k^3c_4 + k^2(9c_3 + 40c_4) + k(24c_3 + 68c_4)$ $+20c_3 + 40c_4 = k + 1$

Comparing like terms,

$24c_3 + 68c_4 = 1$ .....(3)

$20c_3 + 40c_4 = 1$ ...(4)

these both equations gives,

$c_3 = \frac{7}{100}$

$c_4 = \frac{-1}{100}$

So,

P.I=$k^2(\frac{7}{100} - \frac{k}{100})$

General solution is,

$y_k = (c_1 + c_2k)(\frac{-1}{2})^k + k^2(\frac{7}{100} - \frac{k}{100})$

8.3.5 TYPE 5

When the right hand side of nonhomogeneous difference equation has the form as,

$f(x) = g(x)$

Where g(x) is the polynomial in'x'. In order to find particular solution, we shall make the substitution

$y_x = (a_0 + a_1x + a_2x^2 + ...)$

upto the highest power of 'x' defined in the difference equation, the evaluates the value of a_0, a_1, a_2,...

EXAMPLE

Solve $y_{k+2} - 13y_{k+1} + 36y_k = 2^k(k^2 + 1)$ ....(1)

Sol:

consider the corresponding homogeneous equation, we have

$y_{k+2} - 13y_{k+1} + 36y_k = 0$ ....(2)

Let $y_k = Ab^k$ be a nontrivial solution of (2) then it satisfies(2), So

$Ab^k + 2 - 13Ab^k + 1 + 36Ab^k = 0$

$Ab^k(b^2 - 13b + 36) = 0$

$b^2 - 13b + 36 = 0$

$b^2 - 9b - 4b + 36 = 0$

$b(b-9) - 4(b-9) = 0$

$(b-9)(b-4) = 0$

$(b-9) = 0$,$(b-4) = 0$

$b = 9$,$b = 4$

$C.F = C_1 4^k + C_2 9^k$

To find particular integration substitute $y_k = 2^k(a_0 + a_1 k + a_2 k^2)$ in (1), we have

$2^k + 2[a_0 + a_1(k+2) + a_2(k+2)^2] - 13[2^k + 1(a_0 + a_1(k+1)$

$+a_2((k+1)^2)] + 36[2^k(a_0 + a_1 k + a_2 k^2] = 2^k(k^2+1)$

$2^k[4(a_0 + a_1 k + 2a_1 + a_2 k^2 + 4a_2 k + 4a_2) - (13)(2)(a_0$

$+ a_1 k + a_1 + a_2 k^2 + 2a_2 k + a_2) + 36(a_0 + a_1 k + a_2 k^2)] = 2^k(k^2+1)$

$14a_2 k^2 + k(14a_1 - 36a_2) + 14a_0 - 18a_1 - 20a_2 = k^2 + 1$

Comparing, we have

$14a_2 = 1$

$a_2 = \frac{1}{14}$

$14a_1 - 36a_2 = 0$

$14a_0 - 18a_1 - 10a_2 = 1$

$14a_0 = \frac{246}{49}$

$a_0 = \frac{123}{343}$

General Solution is

$Y_k = C_1 4^k + C_2 9^k + 2^k(\frac{123}{343} + \frac{9}{49}k + \frac{1}{14}k^2)$

EXERCISE

1:$y_{k+2} - 9y_{k+1} + 20y_k = 3^k(k^2-1)$

Sol:

$y_{k+2} - 6y_{k+1} + 20y_k = 3^k(k^2-1)$ ....(1)

Consider the coressponding homogeneous difference equation, we have,

$y_{k+2} - 9y_{k+1} + 20y_k = 0$ ....(2)

Let $y_k = Ab^k$ be the nontrivial solution of equation (2), So, it satisfy (2).

$Ab^k + 1 - 9Ab^k + 1 + 20Ab^k = 0$

$Ab^k(b^2 - 9b + 20) = 0$

$b^2 - 9b + 20 = 0$

$b^2 - 5b - 4b + 20 = 0$

$b(b - 5) - 4(b - 5) = 0$

$(b - 5)(b - 4) = 0$

$(b - 5) = 0$,$(b - 4) = 0$

$b = 5$,$b = 4$

$C.F = y_k = C_1 4^k + C_2 5^k$

To find particular integration substitute , $y_k = 3^k(a_0 + a_1 k + a_2) inequation(1)$.

$3^k + 2[a_0 + a_1(k+2) + a_2((k+2)^2)] - 93^k + 1[a_0 + a_1(k+1)$

$+ a_2((k+1)^2)] + (20)(3^k)[a_0 + a_1k + a_2k^2] = 3^k(k^2-1)$

$3^k[9(a_0 + a_1k + 2a_1 + a_2k^2 + 4a_2k + 4a_2) - 27(a_0 + a_1k + a_1$

$+ a_2k^2 + 2ka_2 + a_2) + 20(a_0 + a_1k + a_2k^2)] = 3^k(k^2-1)$

$2a_2k^2 + k(2a_1 - 18a_2) + 2a_0 - 9a_1 + 9a_2 = k^2 - 1$

Comparing like terms.

$2a_2 = 1$

$a_2 = \frac{1}{2}$

$2a_1 - 18a_2 = 0$

$2a_1 = 9$

$a_1 = \frac{9}{2}$

$2a_0 - 9a_1 - 9a_2 = -1$

$a_0 = 22$

General solution is,

$Y_k = C_1 4^k + C_2 5^k + 3^k(22 + \frac{9}{2}k + \frac{1}{2}k^2)$

2:$y_{k+2} - 4y_{k+1} + 4y_k = 2^k(k^2+k+1)$

Sol:

$y_{k+2} - 4y_{k+1} + 4y_k = 2^k(k^2+k+1)$ ....(1)

Consider the coressponding homogeneous equation is,

$y_{k+2} - 4y_{k+1} + 4y_k = 0$ ....(2)

Let $y_k = Ab^k$ be the nontrivial solution of (2).So, it satisfy (2).

$Ab^k + 2 - 4Ab^k + 1 + 4Ab^k = 0$

$Ab^k(b^2 - 4b + 4) = 0$

$b^2 - 4b + 4 = 0$

$(b-2)^2 = 0$

$(b-2)(b-2) = 0$

$b = 2$, $b = 2$

$C.F = y_k = (C_1 + C_2k)2^k$

To find the particular integration, substitute

$y_k = 2^k(C_3 + C_4k + C_5k^2)k^2$ in equation (1),

$2^k + 2((k+2)^2)[C_3 + C_4(k+2) + C_5(k+2)^2] - 4[2^k + 1(k+1)^2(C_3+$

$C_4(k+1) + C_5((k+1)^2))] + (4)(2^k)k^2(C_3 + C4k + C_5k^2) = 2^k(k^2 + k + 1)$

$2^k[4(k^2 + 4k + 4)(C_3 + C_4k + 2C_4 + C_5k^2 + 4C_5k + 4C_5) + (-8)(k^2 + 2k + 1)(C_3$

$+ C_4k + C4 + C_5k^2 + 2C_5k + C_5) + 4k^2(C_3 + C_4k + C5k^2)] = 2^k(k^2 + k + 1)$

$4k^2C_3 + 4k^3C_4 + 8k^2C_4 + 4C_5k^4 + 16C_5k^3$

$+ 16C_5k^2 + 16kC_3 + 16k^2C_4 + 32kC_4 + 16C_5k^3 + 64C_5k^2$

$+ 64C_5k + 16C_3 + 16C_4 + 32C_4 + 16C_5k^2 + 64C_5k + 64C_5 - 8k^2C_3 - 8C_4k^3 - 8C_4k^2$

$- 8C_5k^4 - 16C_5k^3 - 8C_5k^2 - 16kC_3 - 16C_4k^2 - 16C_4k - 16C_5k^3 - 32C_5k^2 - 16C_5k -$

$8C_3 - 8C_4k - 8C_4 - 8C_5 - 16C_5k - 8C_5 + 4C_3k^2 + 4C_4k^3 + 4C_5k^4 = k^2 + k + 1(4C_5 -$

$8C_5 + 4C_5)k^4 + (4C_4 + 16C_5 + 16C_5 - 8C_4 - 16C_5$

-$16C_5 + 4C_4)k^3 + (4C_3 + 8C_4 + 16C_5 + 16C_4 + 64C_5 + 16C_5$

+$16C_5 - 8C_3 - 8C_4 - 8C_5 - 16C_4 - 32C_5 - 8C_5 + 4C_3)k^2 + (16C_3 + 32C_4$

+$64C_5 + 16C_4 + 64C_5 - 16C_3 - 16C_4 - 8C_4 - 16C_5 - 16C_3 + 32C_4$

+$64C_5 - 8C_3 - 8C_4 - 8C_5 = k^2 + k + 1$

$48C_5k^2 + (24C_4 + 96C_5)k + 8C_3 + 24C_4 + 56C_5 = k^2 + k + 1$

Comparing the like terms.

$48C_5 = 1$

$C_5 = \frac{1}{48}$

$24C_4 + 96C_5 = 1$

$8C_3 + 24C_4 + 56C_5 = 1$

$24C_4 = -1$

$C_4 = \frac{-1}{24}$

$C_3 = \frac{5}{48}$

So, General solution is,

$Y_k = (C_1 + C_2k)2^k + 2^kk^2(\frac{5}{48} - \frac{1}{24}k + \frac{1}{48}k^2)$

3:$y_{n+2} - 9y_{n+1} + 20y_n = 4^n(n^2 + 1)$

Sol:

$y_{n+2} - 9y_{n+1} + 20y_n = 4^n(n^2 + 1)$ ....(1)

Consider the coressponding homogeneous equation, we have

$y_{n+2} - 9y_{n+1} + 20y_n = 0$ .....(2)

Let $y_n = Ab^n$ be the nontrivial solution of (2).So, it satisfy .(2)

$Ab^n + 2 - 9Ab^n + 1 + 20Ab^n = 0$

$Ab^n(b^2 - 9b + 20) = 0$

$$b^2 - 9b + 20 = 0$$
$$b^2 - 5b - 4b + 20 = 0$$
$$b(b - 5) - 4(b - 5) = 0$$
$$(b - 5)(b - 4) = 0$$

$b = 5$, $b = 4$

$C.F = y_n = C_1 4^n + C_2 5^n$

To find particular integration substitute $y_n = 4^n(n)(C_3 + C_4 n + C_5 n^2)$ in equation (1).

$4^n + 2(n+2)[C_3 + C_4(n+2) + C_5((n+2)^2)] - 94^n + 1(n+1)[C_3$

$+ C_4(n+1) + C_5((n+1)^2)] + 204^n(n)(C_3 + C_4 n + C_5 n^2 = 4^n(n^2+1)$

$4^n[16(n+2)(C_3 + C_4 n + 2C_4 + C_5 n^2 + 4nC_5) - 36(n+1)(C_3$

$+ C_4 n + C_4 + C_5) + 20n(C_3 + C_4 n + C_5 n^2)] = 4^n(n^2+1)$

$(16C_5 - 36C_5 + 20C_5)n^3 + (16C_4 + 64C_5 + 32C_5 - 36C_4 - 72C_5$

$-36C_5 + 20C_4)n^2 + (16C_3 + 32C_4 + 64C_5 + 32C_4 + 128C_5 - 36C_3 - 36C_4 - 36C_5 - 36C_4$

$-72C_5 + 20C_3) + 32C_3 + 64C_4 + 128C_5 - 36C_3 - 36C_4 - 36C_5 = n^2 + 1$

Comparing the like terms.

$-12C_5 = 1$

$C_5 = \frac{-1}{12}$

$-8C_4 + 84C_5 = 0$

$C_4 = \frac{-7}{8}$

$C_3 = \frac{-199}{24}$

General solution is,

$Y_n = C_1 4^n + C_2 5^n + 4^n n(-\frac{199}{24} - \frac{7}{8}n - \frac{1}{12}n^2)$

4:$y_{n+2} - 7y_{n+1} - 8y_n = 2^n(n^2 - n)$

Sol:

$y_{n+2} - 7y_{n+1} - 8y_n = 2^n(n^2 - n)$ ....(1)

Consider the coresspondin homogeneous

difference equation is,

$y_{n+2} - 7y_{n+1} - 8y_n = 0$ ....(2)

Let $y_n = Ab^n$ be the nontrivial solution of equation (2). So, it satisfies equation (2).

$Ab^n + 2 - 7Ab^n + 1 - 8Ab^n = 0$

$Ab^n(b^2 - 7b - 8) = 0$

$b^2 - 7b - 8 = 0$

$b^2 - 8b + b - 8 = 0$

$b(b - 8) + 1(b - 8) = 0$

$(b - 8)(b + 1) = 0$

$b = 8, b = -1$

$C.F = y_n = C_1(-1)^n + C_2 8^n$

To find the particular integration substitute $y_n = 2^n(C_3 + C_4 n + C5)$ in equation (1).

$2^n + 2[C_3 + C_4(n+2) + C_5(n+2)^2] - 72^n + 1[C_3 + C_4(n+1)$

$+C_5(n+1)^2] - 82^n[C_3 + C_4 n + C_5 n^2] = 2^n(n^2 - n)$

Comparing the same terms, we have

$-18C_5 = 1$

$C_5 = -\frac{1}{18}$

$-10C_4 - 12C_5 = -1$

$C_4 = \frac{1}{6}$

$-18C_3 - 6C_4 - 2C_5 = 0$

$C_3 = -\frac{4}{81}, -\frac{1}{27}$

General solution is,

$Y_n = C_{(-1)^n} + C_2 8^n + 2^n(-\frac{4}{81} + \frac{1}{6}n - \frac{1}{18}n^2)$

8.3.6 TYPE 6

When the right hand side of nonhomogeneous difference equation has the form as,

$f(x) = Sin(Ax)$ or $f(x) = Cos(Bx)$

Where 'A' and 'B' are constants, then in order to find particular integration, we shall make the substitution, $y_x = C_1 Sin(Ax) + C_2 Cos(Ax)$

OR

$y_x = C_3 Sin(Bx) + Cos(Bx)$

Then evaluate the values of C_1, C_2, C_3, C_4.

EXERCISE

1:Solve

$y_{k+2} - 7y_{k+1} + 12y_k = Sin(3k)$

Sol:

$y_{k+2} - 7y_{k+1} + 12y_k = Sin(3k)$ .....(1)

Consider the coressponding homogeneous equation,

$y_{k+2} - 7y_{k+1} + 12y_k = 0$ ....(2)