and the general ...

$$Y_x = c_1 3^x + c_2 4^x + \frac{l_1}{l_1^2 + l_2^2} \sin 3x - \frac{l_2}{l_1^2 + l_2^2} \cos 3x$$

where

$$l_1 = \cos 6 - 7\cos 3 + 12$$

and $l_2 = \sin 6 - 7\sin 3$

- **Type-VI:** When the R.H.S of the given non-homogeneous difference equation has the form as

$$f(x) = a^x \sin Ax \quad \text{or} \quad f(x) = a^x \cos Bx$$

Then in order ~~the~~ to find particular solution we make substitution as.

$$y_x = a^x [c_1 \sin Ax + c_2 \cos Ax] \quad \text{or}$$

$$y_x = a^x [c_3 \sin Bx + c_4 \cos Bx].$$

and then find values of c_1, c_2, c_3 & c_4.

Exercise.

Q. Solve $y_{k+2} + 13y_{k+1} + 3y_k = 3^k \cos 4k$ — ①

Sol: Consider the homogeneous difference equation corresponding to ① i.e

$$y_{k+2} + 13y_{k+1} + 3y_k = 0 \quad \text{— ②}$$

Let

$y_k = Ab^k$ be the non-trivial solution to eq ② so it must satisfy it i.e.

$$Ab^{k+2} + 13Ab^{k+1} + 13Ab^k = 0.$$

$$Ab^k[b^2 + 13b + 13] = 0$$

$\because Ab^k \neq 0$,

$$b^2 + 13b + 13 = 0.$$

$$b = \frac{-13 \pm \sqrt{169-12}}{2}$$

$$b = \frac{-13 \pm \sqrt{157}}{2}$$

So C.F. is

$$y_k = c_1\left(\frac{-13+\sqrt{157}}{2}\right)^k + c_2\left(\frac{-13-\sqrt{157}}{2}\right)^k$$

To find P.I, Substitute

$$y_k = 3^k[c_3 \cos 4k + c_4 \sin 4k] \text{ in ①.}$$

① $\Rightarrow$ $3^{k+2}[c_3 \cos 4(k+2) + c_4 \sin 4(k+2)] + 13 \cdot 3^{k+1}[c_3 \cos 4(k+1) + c_4 \sin 4(k+1)] + 3 \cdot 3^k[c_3 \cos 4k + c_4 \sin 4k] = 3^k \cos 4k$

$\Rightarrow 3^k[9c_3(\cos 4k\cos 8 - \sin 4k\sin 8) + 9(\sin 4k\cos 8 + \cos 4k\sin 8) + 39c_3(\cos 4k\cos 4 - \sin 4k\sin 4) + 39c_4(\sin 4k\cos 4 + \cos 4k\sin 4) + 3c_3\cos 4k + 3c_4\sin 4k] = 3^k\cos 4k.$

By comparing Like Terms,

$\Rightarrow c_3[9\cos 8 + 39\cos 4 + 3] + c_4[9\sin 8 + 39\sin 4] = 1$ — (i)

$-c_3[9\sin 8 + 39\sin 4] + c_4[9\cos 8 + 39\cos 4 + 3] = 0$ — (ii)

Let

$l_1 = 9\cos 8 + 39\cos 4 + 3$

$l_2 = 9\sin 8 + 39\sin 4$

So (i) and (ii) becomes as,

$c_3 l_1 + c_4 l_2 = 1$ — (iii)

$-c_3 l_2 + c_4 l_1 = 0.$ — (iv)

$\Rightarrow \dfrac{c_3}{l_1} = \dfrac{c_4}{l_2} = \dfrac{1}{l_1^2 + l_2^2}$

$\Rightarrow c_3 = \dfrac{l_1}{l_1^2 + l_2^2}, \quad c_4 = \dfrac{l_2}{l_1^2 + l_2^2}$

So P.I is,

$y_k^* = 3^k\left[\dfrac{l_1}{l_1^2 + l_2^2}\cos 4k + \dfrac{l_2}{l_1^2 + l_2^2}\sin 4k\right]$

and the General Sol. is,

$y_k = c_1\left(\dfrac{-13 + \sqrt{157}}{2}\right)^k + c_2\left(\dfrac{-13 - \sqrt{157}}{2}\right)^k + 3^k\left[\dfrac{l_1}{l_1^2 + l_2^2}\cos 4k + \dfrac{l_2}{l_1^2 + l_2^2}\sin 4k\right]$ Ans.