Measure of dispersion and its types

Measure of dispersion and its types

- Mean deviation for absolute and relative measure(ungroup and group)
- variance and standard for absolute and relative measure(ungroup and group)
- Properties of variance and standard deviation and co-efficient of variation, skewness and kurtosis.

Mean deviation

The mean of the absolute values of the numerical differences between the numbers of a set (such as statistical data) and their mean or median or mode.

Mean deviation

The mean deviation is 16 cappuccinos. That is, the number of cappuccinos sold deviates, on average, by 16 from the mean of 50 cappuccinos.

The following shows the detail of determining the mean deviation for the number of cappuccinos sold at the Ontario Airport.

	A	B	C
1	Calculation of Mean Deviation Ontario		
2	Number Sold	Each Value – Mean	Absolute Deviation
3	20	20 – 50 = -30	30
4	49	49 – 50 = -1	1
5	50	50 – 50 = 0	0
6	51	51 – 50 = 1	1
7	80	80 – 50 = 30	30
8			
9		Total	62

$$MD = \frac{\Sigma|X - \overline{X}|}{n} = \frac{30 + 1 + 0 + 1 + 30}{5} = \frac{62}{5} = 12.4$$

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Finding **mean deviation about Median** = $\frac{\sum f_i|x_i - M|}{\sum f_i}$

Class	Frequency	Cumulative frequency	Mid-point x_i	$\lvert x_i - M\rvert$	$f_i\lvert x_i - M\rvert$
0 – 10	6	6	5	$\lvert 5 - 28\rvert = 23$	$6 \times 23 = 138$
10 – 20	7	7 + 6 = 13	15	$\lvert 15 - 28\rvert = 13$	$7 \times 13 = 91$
20 – 30	15	13 + 15 = 28	25	$\lvert 25 - 28\rvert = 3$	$15 \times 3 = 45$
30 – 40	16	28 + 16 = 44	35	$\lvert 35 - 28\rvert = 7$	$16 \times 7 = 112$
40 – 50	4	44 + 4 = 48	45	$\lvert 45 - 28\rvert = 17$	$4 \times 17 = 68$
50 – 60	2	48 + 2 = 50	55	$\lvert 55 - 28\rvert = 27$	$2 \times 27 = 54$
	$\sum f_i = 50$				$\sum f_i\lvert x_i - M\rvert = 508$

$\sum f_i = 50$ & $\sum f_i |x_i - M| = 508$

$\therefore$ Mean Deviation (M) = $\frac{\sum f_i |x_i - M|}{f_i}$

$= \frac{508}{50}$

= 10.16

Mean Deviation Through mean

Class Interval	f	x	fx	$x-\mu$	$\lvert x-\mu\rvert$	$f\lvert x-\mu\rvert$
40 -- 44	4	42	168	-13	13	52
45 -- 49	7	47	329	-8	8	56
50 -- 54	14	52	728	-3	3	42
55 -- 59	11	57	627	2	2	22
60 -- 64	8	62	496	7	7	56
65 -- 69	6	67	402	12	12	72
	50		2750			300

Mean Deviation from Mean

$$M.D\ (mean) = \frac{\sum f\lvert x-\mu\rvert}{N} = \frac{300}{50} = 6$$

Coefficient of M.D from Mean

$$Coefficient\ of\ M.D\ (mean) = \frac{M.D}{Mean}$$

$$Coefficient\ of\ M.D = \frac{6}{55} = 0.1091$$

$x = Midpoint$

$$x = \frac{L.C.L + U.C.L}{2}$$

$$e.g\ x = \frac{40+44}{2} = 42$$

$$\mu = \frac{\sum fx}{\sum f}$$

$$\mu = \frac{2750}{50} = 55$$

Variance

- *Variance* is defined as the average of the square deviations:

$$\sigma^2 = \frac{\sum (X - \mu)^2}{N}$$

What Does the Variance Formula Mean?

- First, it says to subtract the mean from each of the scores
  - This difference is called a *deviate* or a *deviation score*
  - The deviate tells us how far a given score is from the typical, or average, score
  - Thus, the deviate is a measure of dispersion for a given score

What Does the Variance Formula Mean?

- Why can't we simply take the average of the deviates? That is, why isn't variance defined as:

$$\sigma^2 \neq \frac{\sum (X - \mu)}{N}$$

This is not the formula for variance!

What Does the Variance Formula Mean?

- One of the definitions of the *mean* was that it always made the sum of the scores minus the mean equal to 0
- Thus, the average of the deviates must be 0 since the sum of the deviates must equal 0
- To avoid this problem, statisticians square the deviate score prior to averaging them
  - Squaring the deviate score makes all the squared scores positive

What Does the Variance Formula Mean?

- Variance is the mean of the squared deviation scores
- The larger the variance is, the more the scores deviate, on average, away from the mean
- The smaller the variance is, the less the scores deviate, on average, from the mean

Standard Deviation

- When the deviate scores are squared in variance, their unit of measure is squared as well
  - E.g. If people's weights are measured in pounds, then the variance of the weights would be expressed in pounds^2 (or squared pounds)
- Since squared units of measure are often awkward to deal with, the square root of variance is often used instead
  - The standard deviation is the square root of variance

Standard Deviation

- Standard deviation = $\sqrt{\text{variance}}$
- Variance = standard deviation^2

Computational Formula

- When calculating variance, it is often easier to use a computational formula which is algebraically equivalent to the definitional formula:

$$\sigma^2 = \frac{\sum X^2 - \frac{\left(\sum X\right)^2}{N}}{N} = \frac{\sum (X-\mu)^2}{N}$$

- σ^2 is the population variance, X is a score, μ is the population mean, and N is the number of scores

Computational Formula Example

X	X^2	$X-\mu$	$(X-\mu)^2$
9	81	2	4
8	64	1	1
6	36	-1	1
5	25	-2	4
8	64	1	1
6	36	-1	1
$\Sigma = 42$	$\Sigma = 306$	$\Sigma = 0$	$\Sigma = 12$

Computational Formula Example

$$\sigma^2 = \frac{\sum X^2 - \frac{(\sum X)^2}{N}}{N}$$

$$= \frac{306 - \frac{42^2}{6}}{6}$$

$$= \frac{306 - 294}{6}$$

$$= \frac{12}{6}$$

$$= 2$$

$$\sigma^2 = \frac{\sum (X - \mu)^2}{N}$$

$$= \frac{12}{6}$$

$$= 2$$

Variance

Month	Citations (X)	$X - \mu$	$(X - \mu)^2$
January	19	−10	100
February	17	−12	144
March	22	−7	49
April	18	−11	121
May	28	−1	1
June	34	5	25
July	45	16	256
August	39	10	100
September	38	9	81
October	44	15	225
November	34	5	25
December	10	−19	361
Total	348	0	1,488

1. We begin by determining the arithmetic mean of the population. The total number of citations issued for the year is 348, so the mean number issued per month is 29.

Variance of a Sample

- Because the sample mean is not a perfect estimate of the population mean, the formula for the variance of a sample is slightly different from the formula for the variance of a population:

$$s^2 = \frac{\sum\left(X - \overline{X}\right)^2}{N-1}$$

- s^2 is the sample variance, X is a score, $\overline{X}$ is the sample mean, and N is the number of scores

Properties of variance

- Variance of constant is zero
- Variance is always positive or greater than zero.
- Variance is not affected by change of origin
- Variance is affected by change of scale.

Properties of standard deviation

- The positive square root of variance properties is known as standard deviation properties.

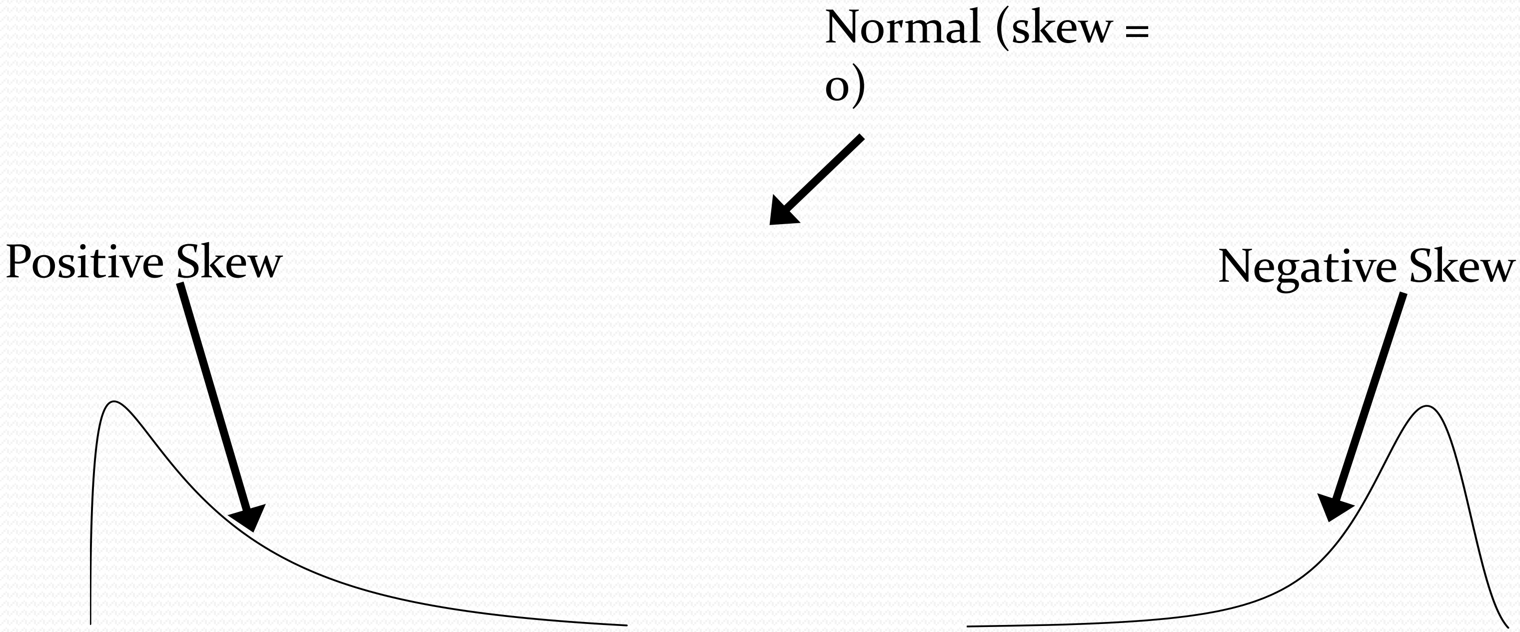
Coefficient of Variation

- Measure of Relative Variation
- Always a %
- Shows Variation Relative to Mean
- Used to Compare 2 or More Groups
- Formula (for Sample):

$$CV = \left(\frac{S}{\overline{X}} \right) \cdot 100\%$$

Measure of Skew

- *Skew* is a measure of symmetry in the distribution of scores



Measure of Skew

- The following formula can be used to determine skew:

$$s^{k} = \frac{\dfrac{\sum\left(X-\overline{X}\right)^{3}}{N}}{\sqrt{\dfrac{\sum\left(X-\overline{X}\right)^{2}}{N}}}$$

Measure of Skew

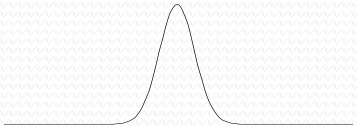
- If $s^k < 0$, then the distribution has a negative skew
- If $s^k > 0$ then the distribution has a positive skew
- If $s^k = 0$ then the distribution is symmetrical
- The more different s^3 is from 0, the greater the skew in the distribution

Kurtosis

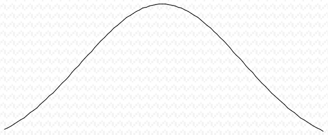
- Kurtosis measures whether the scores are spread out more or less than they would be in a normal (Gaussian) distribution

Mesokurtic ($k = 3$)

Leptokurtic ($k > 3$)



Platykurtic ($k < 3$)



Kurtosis

- When the distribution is normally distributed, its kurtosis equals 3 and it is said to be *mesokurtic*
- When the distribution is less spread out than normal, its kurtosis is greater than 3 and it is said to be *leptokurtic*
- When the distribution is more spread out than normal, its kurtosis is less than 3 and it is said to be *platykurtic*

Measure of Kurtosis

- The measure of kurtosis is given by:

$$k = \frac{\sum \left(\dfrac{X - \overline{X}}{\sqrt{\dfrac{\sum \left(X - \overline{X} \right)^2}{N}}} \right)^4}{N}$$

s^2, s^k, & k

- Collectively, the variance (s^2), skew (sk), and kurtosis (k) describe the shape of the distribution

- For practice Question see book “Statistical techniques in Business and Economics. ”

or

other recommended books mention in the course outline.

- Any Query about lecture contact with me on gmail or whatsapp.