Introduction to Integral Equations with Applications

Second Edition

ABDUL J. JERRI
Clarkson University

A Wiley-Interscience Publication
JOHN WILEY & SONS, INC.
New York • Chichester • Weinheim • Brisbane • Singapore • Toronto

Contents

Preface *xiii*

Acknowledgments *xvii*

1 Integral Equations, Origin, and Basic Tools *1*

1.1 Various Problems as Integral Equations *3*

Exercises 1.1 *20*

1.2 Classification of Integral Equations *24*

Exercises 1.2 *28*

1.3 Some Important Identities and Basic Definitions *31*

1.3.1 Multiple Integrals Reduced to Single Integrals *31*

1.3.2 Generalized Leibnitz Formula *33*

1.3.3 Convergence of Integrals and Basic Definitions *36*

Exercises 1.3 *40*

1.4 Laplace, Fourier, and Other Transforms *42*

1.4.1 The Laplace Transform *43*

1.4.2 Fourier Transforms *53*

1.4.3 Other Transforms *70*

Exercises 1.4 *73*

1.5 Basic Numerical Integration Formulas *79*

1.5.1 Basic (Elementary) Integration Formulas *80*

1.5.2 *The Smoothing Effect of Integration* 86
1.5.3 *Interpolation of the Numerical Solutions of Integral Equations* 87
1.5.4 *Review of Cramer's Rule* 93
Exercises 1.5 94

2 *Modeling of Problems as Integral Equations* 97
2.1 *Population Dynamics* 98
2.1.1 *Human Population* 98
2.1.2 *Biological Species Living Together* 100
Exercises 2.1 102
2.2 *Control and Other Problems* 104
2.2.1 *Mortality of Equipment and Rate of Replacement* 104
Exercises 2.2 105
2.3 *Mechanics Problems* 107
2.3.1 *Hanging Chain* 107
2.3.2 *Sliding a Bead Along a Wire: Abel's Problem* 110
Exercises 2.3 112
2.4 *Initial Value Problems Reduced to Volterra Integral Equations* 113
Exercises 2.4 116
2.5 *Boundary Value Problems Reduced to Fredholm Integral Equations* 118
Exercises 2.5 122
2.6 *Mixed Boundary Conditions: Dual Integral Equations* 124
2.6.1 *Electrified Infinite Plane* 124
2.6.2 *Electrified Disc* 126
Exercises 2.6 127
2.7 *Integral Equations in Higher Dimensions* 128
2.7.1 *Schrödinger Equation as an Integral Equation in the Three-Dimensional Momentum Space* 129

3 *Volterra Integral Equations* 133
3.1 *Volterra Equations of the Second Kind* 134
3.1.1 *Resolvent Kernel Method: Neumann Series* 134
3.1.2 *Method of Successive Approximations (Iterations)* 141
3.1.3 *Laplace Transform Method: Difference Kernel* 143

Exercises 3.1 146
3.2 Volterra Integral Equations of the First Kind 148
3.2.1 Volterra Integral Equation of the First Kind with a Difference Kernel—Laplace Transform Method 150
Exercises 3.2 154
3.3 Numerical Solution of Volterra Integral Equations 156
3.3.1 Numerical Approximation Setting of Volterra Equations 156
Exercises 3.3 162

4 The Green's Function 165
4.1 Construction of the Green's Function 165
4.1.1 Nonhomogeneous Differential Equations 166
4.1.2 Construction of the Green's Function — Variation of Parameters Method 169
4.1.3 Orthogonal Series Representation of Green's Function 187
4.1.4 Green's Function in Two Dimensions 192
Exercises 4.1 193
4.2 Fredholm Integral Equations and the Green's Function 202
Exercises 4.2 206

5 Fredholm Integral Equations 209
5.1 Fredholm Integral Equations with Degenerate Kernel 211
5.1.1 Nonhomogeneous Fredholm Equations with Degenerate Kernel 211
5.1.2 Fredholm Alternative 215
5.1.3 Approximating a Kernel by a Degenerate One 231
Exercises 5.1 232
5.2 Fredholm Integral Equations with Symmetric Kernel 237
5.2.1 Homogeneous Fredholm Equations with Symmetric Kernel 237
5.2.2 Solution of Fredholm Equations of the Second Kind with Symmetric Kernel 245
Exercises 5.2 251
5.3 Fredholm Integral Equations of the Second Kind 253
5.3.1 Method of Fredholm Resolvent Kernel 253
5.3.2 Method of Iterated Kernels 256

5.3.3 Some Basic Approximate Methods 262
Exercises 5.3 269
5.4 Fredholm Integral Equations of the First Kind 271
5.4.1 Fredholm Equations of the First Kind with Symmetric Kernels 272
5.4.2 Ill-Posed Problems and the Fredholm Equation of the First Kind 277
Exercises 5.4 282
5.5 Numerical Solution of Fredholm Integral Equations 285
5.5.1 Numerical Approximation Setting of Fredholm Integral Equations 286
5.5.2 Homogeneous Fredholm Equations 291
Exercises 5.5 295

6 Existence of the Solutions: Basic Fixed Point Theorems 299
6.1 Preliminaries: Toward a Contractive Mapping 299
6.1.1 Basic Definitions: Complete Metric Spaces 305
6.1.2 Contractive Mapping for Linear Fredholm Equations 308
6.1.3 Contractive Mapping for Linear Volterra Equations 310
6.2 Fixed Point Theorem of Banach 313
6.2.1 Existence of the Solution for Linear Integral Equations 316
6.2.2 Existence of the Solution for Nonlinear Integral Equations 319
6.2.3 Existence of the Solution for Nonlinear Differential Equations 324

7 Higher Quadrature Rules for the Numerical Solutions 327
7.1 Higher Quadrature Rules of Integration with Tables 327
Exercises 7.1 348
7.2 Higher Quadrature Rules for Volterra Equations 350
Exercises 7.2 357
7.3 Higher Quadrature Rules for Fredholm Equations 358
7.3.1 Comments on Higher Quadrature Rules for Some Singular Fredholm Equations 365
Exercises 7.3 370

Appendix A The Hankel Transforms 373

A.1 The Hankel Transform for the Electrified Disc 373
A.2 The Finite Hankel Transform 375
Exercises: Appendix A 376

Appendix B Green's Function for Various Boundary Value Problems 379
B.1 Green's Functions in Terms of Simple Functions 379
B.2 Green's Function in Terms of Special Functions 382

Answers to Exercises 383
Chapter 1 383
Chapter 2 388
Chapter 3 392
Chapter 4 394
Chapter 5 398
Chapter 7 408
Appendix A 419

References 421

Index 425