

The background features a dark blue gradient with a subtle starry pattern. On the left side, there is a large, semi-circular scale with tick marks and numerical labels from 140 to 260. Several circular and semi-circular lines, some solid and some dashed, are scattered across the page, some with arrows indicating direction.

# LINEAR ALGEBRA

TOPIC : COMPLEX VECTOR SPACE

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# COMPLEX VECTOR SPACE

1. If  $n$  is a positive integer, then a complex  $n$ -tuple is a sequence of  $n$  complex numbers  $(v_1, v_2, \dots, v_n)$ . The set of all complex  $n$ -tuples is called complex  $n$ -space and is denoted by  $C^n$

2. For example

$$u = (1 + i, -4i, 3 + 2i),$$

$$v = (0, i, 5),$$

$$w = (6 - \sqrt{2}i, 9 + \frac{1}{2}i, \pi i)$$

## EXAMPLE 02: Real and Imaginary Parts of Vectors and Matrices

$$v = (3 + i, -2i, 5) \quad \text{and} \quad A = \begin{bmatrix} 1 + i & -i \\ 4 & 6 - 2i \end{bmatrix}$$

Sol:

The real and imaginary parts of  $v$

The real and imaginary parts of  $\bar{v}$

The real and imaginary parts of  $A$

**DEFINITION 2** If  $\mathbf{u} = (u_1, u_2, \dots, u_n)$  and  $\mathbf{v} = (v_1, v_2, \dots, v_n)$  are vectors in  $C^n$ , then the *complex Euclidean inner product* of  $\mathbf{u}$  and  $\mathbf{v}$  (also called the *complex dot product*) is denoted by  $\mathbf{u} \cdot \mathbf{v}$  and is defined as

$$\mathbf{u} \cdot \mathbf{v} = u_1 \bar{v}_1 + u_2 \bar{v}_2 + \cdots + u_n \bar{v}_n \quad (3)$$

We also define the *Euclidean norm* on  $C^n$  to be

$$\|\mathbf{v}\| = \sqrt{\mathbf{v} \cdot \mathbf{v}} = \sqrt{|v_1|^2 + |v_2|^2 + \cdots + |v_n|^2} \quad (4)$$

► **EXAMPLE 2 Complex Euclidean Inner Product and Norm**

Find  $\mathbf{u} \cdot \mathbf{v}$ ,  $\mathbf{v} \cdot \mathbf{u}$ ,  $\|\mathbf{u}\|$ , and  $\|\mathbf{v}\|$  for the vectors

$$\mathbf{u} = (1 + i, i, 3 - i) \quad \text{and} \quad \mathbf{v} = (1 + i, 2, 4i)$$

**Solution**

$$\mathbf{u} \cdot \mathbf{v} = (1 + i)(\overline{1 + i}) + i(\bar{2}) + (3 - i)(\overline{4i}) = (1 + i)(1 - i) + 2i + (3 - i)(-4i) = -2 - 10i$$

$$\mathbf{v} \cdot \mathbf{u} = (1 + i)(\overline{1 + i}) + 2(\bar{i}) + (4i)(\overline{3 - i}) = (1 + i)(1 - i) - 2i + 4i(3 + i) = -2 + 10i$$

$$\|\mathbf{u}\| = \sqrt{|1 + i|^2 + |i|^2 + |3 - i|^2} = \sqrt{2 + 1 + 10} = \sqrt{13}$$

$$\|\mathbf{v}\| = \sqrt{|1 + i|^2 + |2|^2 + |4i|^2} = \sqrt{2 + 4 + 16} = \sqrt{22} \quad \blacktriangleleft$$

