LINEAR ALGEBRA

TOPIC : COMPLEX VECTOR SPACE

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COMPLEX VECTOR SPACE

1. If n is a positive integer, then a complex n-tuple is a sequence of n complex numbers (v 1 ,v 2 ,...,v n) . The set of all complex n -tuples Is called complex n-space and is denoted by Cn
2. For example

u = (1 + i,− 4 i, 3 + 2 i),

v = (0 ,i, 5),

$$w = (6 - \sqrt{2}i, 9 + \frac{1}{2}i, \pi i)$$

EXAMPLE 02: Real and Imaginary Parts of Vectors and Matrices

$$v = (3 + i, -2i, 5) \quad and \quad A = \begin{bmatrix} 1+i & -i \\ 4 & 6-2i \end{bmatrix}$$

Sol:

The real and imaginary parts of v

The real and imaginary parts of $\bar{v}$

The real and imaginary parts of A

DEFINITION 2 If $\mathbf{u} = (u_1, u_2, \ldots, u_n)$ and $\mathbf{v} = (v_1, v_2, \ldots, v_n)$ are vectors in C^n, then the ***complex Euclidean inner product*** of $\mathbf{u}$ and $\mathbf{v}$ (also called the ***complex dot product***) is denoted by $\mathbf{u} \cdot \mathbf{v}$ and is defined as

$$\mathbf{u} \cdot \mathbf{v} = u_1\overline{v}_1 + u_2\overline{v}_2 + \cdots + u_n\overline{v}_n \quad (3)$$

We also define the ***Euclidean norm*** on C^n to be

$$\|\mathbf{v}\| = \sqrt{\mathbf{v} \cdot \mathbf{v}} = \sqrt{|v_1|^2 + |v_2|^2 + \cdots + |v_n|^2} \quad (4)$$

EXAMPLE 2 Complex Euclidean Inner Product and Norm

Find $\mathbf{u} \cdot \mathbf{v}$, $\mathbf{v} \cdot \mathbf{u}$, $\|\mathbf{u}\|$, and $\|\mathbf{v}\|$ for the vectors

$$\mathbf{u} = (1 + i, i, 3 - i) \quad \text{and} \quad \mathbf{v} = (1 + i, 2, 4i)$$

Solution

$$\mathbf{u} \cdot \mathbf{v} = (1 + i)(\overline{1 + i}) + i(\overline{2}) + (3 - i)(\overline{4i}) = (1 + i)(1 - i) + 2i + (3 - i)(-4i) = -2 - 10i$$

$$\mathbf{v} \cdot \mathbf{u} = (1 + i)(\overline{1 + i}) + 2(\overline{i}) + (4i)(\overline{3 - i}) = (1 + i)(1 - i) - 2i + 4i(3 + i) = -2 + 10i$$

$$\|\mathbf{u}\| = \sqrt{|1 + i|^2 + |i|^2 + |3 - i|^2} = \sqrt{2 + 1 + 10} = \sqrt{13}$$

$$\|\mathbf{v}\| = \sqrt{|1 + i|^2 + |2|^2 + |4i|^2} = \sqrt{2 + 4 + 16} = \sqrt{22}$$