# Linear Algebra

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Topic : Linear Transformations

LINEAR TRANSFORMATION:

DEF:

A linear transformation is a transformation T: $R^n \to R^m$ satisfying

1. $T(u+v) = T(u) + T(v)$
2. $T(cu) = cT(u)$

For all vectors $u, v\ inR^n$ and all scalar c

Or

$T(ax+by) = T(ax) + T(by) = aT(x) + bT(b)$

For all $x, y\ in\ R^n$

Properties of Linear Transformation

Theorem: Let V and W be two vector spaces. Suppose $T: V \to W$ is a linear transformation. Then

1. T(0) = 0
2. T(-v) = -T(v) for all v in V
3. $T(u - v) = T(u) - T(v), for\ all\ u, v\ in\ V$
4. $if\ v = c_1 v_1 + c_2 v_2 + \ldots + c_n v_n, then$

$$T(v) = T(c_1 v_1 + c_2 v_2 + \ldots + c_n v_n) = c_1 T(v_1) + c_2 T(v_2) + c_3 T(v_3) + \ldots + c_n T(v_n)$$

Example : 01 Matrix Transformation

T : $M_{22} \rightarrow R$

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = 3a - 4b + c - d$$

Sol

$$T(u - v) = T(u) - T(v)$$

$$T(v) = T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = 3a - 4b + c - d$$

$$T(u) = T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = 3a - 4b + c - d$$

$$T(u - v) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 30 - 40 + 0 - 0 = 0$$

Example : 02 Matrix Transformation

T : $M_{22} \rightarrow R$

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a^2 + b^2$$

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EXAMPLE :03 The Zero Transformation

The Mapping $T: V \rightarrow W$ is zero transformation such that T(v)=0

Sol:

1. $T(u+v) = T(u) + T(v) = 0 + 0 = 0$
2. $T(k\,u) = kT(u) = k(0) = 0$

Example 04: The Identity Operator

The Mapping I: $V \rightarrow W$ is Identity Operator such as $I(v) = v$

Sol:

$$T(u+v) = T(u) + T(v)$$
$$T(k\,u) = kT(u)$$

Example 05: A Linear Transformation from $P_n$ to $P_{n+1}$

Let $p = p(x) = c_0 + c_1x + c_2x^2 + \cdots + c_nx^n$ *and transformation is defined by*

$$T\big(p(x)\big) = T(c_0 + c_1x + c_2x^2 + \cdots + c_nx^n) = xp(x)$$

Sol:

1. $\mathrm{T}(p_1 + p_2) = T\big(p_1(x) + p_2(x)\big) = x\big(p_1(x) + p_2(x)\big)$

$= \mathrm{x}\big(p_1(x)\big) + x\big(p_2(x)\big) = \mathrm{T}(p_1) + T(p_2)$

2. $T(kp) = T(kp(x)) = \mathrm{x}\big(kp(x)\big) = k\big(xp(x)\big) = kT(p)jhh$