

Linear Algebra

Numerical Method

Best Approximation; Least Squares

step:01

Given a linear system $Ax = b$

Step:02

$$A^T A$$

Step :03

$$A^T b$$

Step:04

$$A^T Ax = A^T b$$

Step:05

least square method $b - Ax$

Step:06 $|b - Ax|$

EXAMPLE 01: Find the least squares solution, the least squares error vector, and the least squares error of the linear system.

$$\begin{aligned}x_1 - x_2 &= 4 \\3x_1 + 2x_2 &= 1 \\-2x_1 + 4x_2 &= 3\end{aligned}$$

Sol:

$$\begin{bmatrix} 1 & -1 \\ 3 & 2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 3 & -2 \\ -1 & 2 & 4 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 1 & 3 & -2 \\ -1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 3 & 2 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 14 & -3 \\ -3 & 21 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 1 & 3 & -2 \\ -1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \end{bmatrix}$$

Now

$$A^T A x = A^T b$$

$$\begin{bmatrix} 14 & -3 \\ -3 & 21 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \end{bmatrix}$$

$$14x_1 - 3x_2 = 1$$

$$-3x_1 + 21x_2 = 10$$

$$x_1 = \frac{17}{96}, \quad x_2 = \frac{143}{285}$$

$$\mathbf{b} - \mathbf{A}\mathbf{x} = \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 3 & 2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} \frac{17}{96} \\ \frac{143}{285} \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} - \begin{bmatrix} \frac{-92}{285} \\ \frac{439}{285} \\ \frac{95}{57} \end{bmatrix} = \begin{bmatrix} \frac{1232}{285} \\ -\frac{154}{285} \\ \frac{4}{3} \end{bmatrix}$$

$$|\mathbf{b} - \mathbf{A}\mathbf{x}| = 4.556$$

THANK YOU

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