Linear Algebra

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Eigenvalues and Eigenvectors

- Eigenvalues and Eigenvectors
- Diagonalization
- Complex Vector Spaces

- Objective:-
- we will focus on classes of scalars and vectors known as “eigenvalues”
- and “eigenvectors,” terms derived from the German word eigen, meaning “own,” “characteristic,” or “individual.

- Eigen Values:-

The number λ is an eigen values of A (given any matrix), iff (A- λI) is singular, det(λI $- A$)=0 (1)

For each value of λ , solve (λI $- A$)x=0 , to find an eigen vector of x

Some properties:

- Eq (1) is called the characteristic polynomial of A.
- A be the square matrix (n x n) then A (Real & Complex).
- λ is said to be eigen values of Matrix A if their exists a column matrix of order (n x 1) such that

$$Ax = \lambda x$$

$$\lambda x - Ax = 0$$

$$x(\lambda - A) = 0$$

$$x(\lambda \mathrm{I} - A) = 0$$

working rule to find eigen values and eigen vectors.

Step 01:

Characteristic Eq $\det(\lambda I - A) = 0$

Step 02:

Solution of characteristic eqs is called Eigen values $\lambda = \lambda_1, \quad \lambda = \lambda_2, \ldots so\ on$

Step 03:

$(\lambda I - A)x = 0$ (1) $eqs\ of\ E.vectors$

$(\lambda_1 I - A)x = 0$ solve that equation the solution of eq(1) for any values of λ_1 , x is called the Eigen vector of Matrix A corresponding the value of $\lambda = \lambda_1$

- Example :01

Find the Eigen values and Eigen vectors for Matrix A,

$$A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$$

Sol:

<u>Step 01: the characteristic equation for Matrix A.</u>

$|\lambda I - A| = 0,$

$$\left| \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} \lambda - 1 & -2 \\ -5 & \lambda - 4 \end{vmatrix} = 0$$

<u>Step 02: Find the Eigen values.</u>

$(1 - \lambda)(4 - \lambda) - (-5)(-2) = 0$

$\lambda^2 - 5\lambda - 6 = 0$

$\lambda^2 - 6\lambda + \lambda - 6 = 0$

$\lambda(\lambda - 6) + 1(\lambda - 6) = 0$

$$(\lambda - 6)(\lambda + 1) = 0$$

$$\lambda_1 = 6 , \lambda_2 = -1$$

Step 03 : find eigen vectors

Case 01: $\lambda_1 = 6$ and $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$(A - 6\text{I}) \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$\begin{pmatrix} 1-6 & -2 \\ -5 & 4-6 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$\begin{bmatrix} -5 & -2 \\ -5 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$\begin{bmatrix} -5 & -2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$-5x_1 - 2x_2 = 0$

$-5x_1 = 2x_2$

$\frac{x_1}{2} = \frac{x_2}{-5}$ = k (say)

$\frac{x_1}{2}$ = k , $x_1 = 2k$

$\frac{x_2}{-5}$ = k , $x_2 = 2k$ (infinite many solution

Case 02 : $\lambda_2 = -1$

Do students

Example 02:

$$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$$

Sol :

<u>Step 01:</u>

$$\left|\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 8 & 1 \end{bmatrix}\right| = 0$$

$$\left|\begin{bmatrix} \lambda - 3 & 0 \\ 8 & \lambda - 1 \end{bmatrix}\right| = 0$$

$(\lambda - 3)(\lambda + 1) - 0 = 0$

$-3 - 3\lambda + \lambda + \lambda^2 = 0$

$\lambda^2 - 2\lambda - 3 = 0$

$\lambda^2 - 3\lambda + \lambda - 3 = 0$

$\lambda(\lambda - 3) + 1(\lambda - 3) = 0$

$\lambda_1 = 3, \lambda_2 = -1$

Step 02:

$$\left|\begin{bmatrix} \lambda - 3 & 0 \\ 8 & \lambda - 1 \end{bmatrix}\right| \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

Case 01 $\lambda_1 = 3$

$$\begin{bmatrix} 3 - 3 & 0 \\ 8 & 3 - 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & 0 \\ 8 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$8x_1 + 2x_2 = 0$

$8x_1 = -2x_2$

(infinite many solution)

HOME WORK

Exercise Set 5.1

▶ In Exercises 1–4, confirm by multiplication that $\mathbf{x}$ is an eigenvector of A, and find the corresponding eigenvalue. ◀

1. $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix};\ \mathbf{x} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

2. $A = \begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix};\ \mathbf{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

3. $A = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix};\ \mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

4. $A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix};\ \mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

▶ In each part of Exercises 5–6, find the characteristic equation, the eigenvalues, and bases for the eigenspaces of the matrix. ◀

5. (a) $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} -2 & -7 \\ 1 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

6. (a) $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -3 \\ 0 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}$