Matrix:

Matrix theory is branch of mathematics which is focused on study of matrix it was sub-branch of linear algebra
Linear → line
Algebra → sign and letters represent numbers.

Defination:

A rectangular array of numbers elements enclosed by a set of square brackets is called matrix

→ Usually denoted by a capital letter e.g (A, B, X)
→ Number of Rows and Column are present in it

Row:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \end{bmatrix} \rightarrow \text{Row}$$

$1 \times n$ one Row is present called Row vector

Column:

$$\begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} \rightarrow \text{column}$$

$n \times 1$ A matrix with one column is present is called column matrix.

Element / Entries:

Each value in a matrix either a number or a constant

Dimension:

Number of rows and number of columns of a matrix.

→ A matrix is named by its dimension.
→ Dimension / order of matrix is same

E.g:

$$A = \begin{bmatrix} 2 & -1 & 5 \\ 0 & 5 & 9 \end{bmatrix}_{2\times 3}$$

Rows ← 2, 3 → columns

Notation:~

matrices are commonly written in box brackets or parentheses.

$$B = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & & a_{mn} \end{bmatrix} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & & a_{mn} \end{pmatrix}$$

$$= (a_{ij}) \in \mathbb{R}^{m\times n}$$

$1 \leq i \leq m \longrightarrow$ rows

$1 \leq j \leq n \longrightarrow$ columns

Properties of Matrix

⇒ Properties of Matrix Addition:

we restrict attention to the set of all $m \times n$ matrix.

- **Associative property of Addition:**

  $A + (B + C) = (A + B) + C.$

- **Commutative property of Addition:**

  $A + B = B + A$

- **Additive Identity:**

  $A + O = A = O + A$

The Zero matrix O, the same size as A, is the additive identity for matrix the same A.

- **Additive Inverse property of Addition:**

  $A + (-A) = O = (-A) + A.$

The matrix $-A$ is the unique additive Inverse of A.

Thus. Matrix addition has the same property as the addition of real numbers. apart from the fact that the sum of two matrix is only defined when they have the same size.

Properties of Matrix Multiplication:

- **Associative property of multiplication:**

$$(AB)C = A(BC)$$

- **Multiplicative Identity:**

$$1A = A$$

- **Distributive property:**

$$A(B+C) = AB + AC$$
$$(B+C)A = BA + CA$$

Thus. apart from the fact that it is not always defined, matrix multiplication has the same properties as the multiplication of real numbers except for the following three major differences.

1: Matrix multiplication is not commutative.
For Example:

$$\text{if} \quad A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad \& \quad B = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

then $AB \neq BA$

2. The product of two matrix can be a zero matrix without either matrix being a zero matrix.

For Example.

if $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ & $B = \begin{pmatrix} -2 & -6 \\ 1 & 3 \end{pmatrix}$

then $AB = 0$.

3. Cancellation of matrices is not allowed in general.

For Example,

if $A = \begin{pmatrix} 0 & 2 \\ 0 & 1 \end{pmatrix}$ & $B = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ & $C = \begin{pmatrix} -1 & 1 \\ 1 & 4 \end{pmatrix}$

then $A \neq 0$ & $AB = AC$ but $B \neq C$.

⇒ Properties Of Scalar Multiplication:

- $1A = A$ and $-1A = -A$
- $0A = 0$
- $\lambda(A + B) = \lambda A + \lambda B$ where λ is a scalar
- $\lambda\mu(A) = \lambda(\mu A)$
- $(\lambda + \mu)A = \lambda A + \mu A$
- $(\lambda A)B = A(\lambda B) = \lambda(AB)$.

Rectangular Matrix:

Let M be a $m \times n$ Matrix. If $m \neq n$ then M is called rectangular matrix.

Example

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

Types of Square Matrix

1) Diagonal Matrix 2) Triangular Matrix

Diagonal Matrix:

1) Scalar Matrix 2) Identical Matrix

Scalar Matrix:

A diagonal Matrix said to be Scalar matrix if all its diagonal elements are same.

Example

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Idempotent Matrix:-

An idempotent matrix is a matrix which, when multiplied by itself, yields itself.

Example:-

1)
$$A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$A^2 = A \cdot A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \cdot \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} (2)(2)+(-2)(-1)+(-4)(1) & (2)(-2)+(-2)(3)+(-4)(-2) & (2)(-4)+(-2)(4)+(-4)(-3) \\ (-1)(2)+(3)(-1)+(4)(1) & (-1)(-2)+(3)(3)+(4)(-2) & (-1)(-4)+(3)(4)+(4)(-3) \\ (1)(2)+(-2)(-1)+(-3)(1) & (1)(-2)+(-2)(3)+(-3)(-2) & (1)(-4)+(-2)(4)+(-3)(-3) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4+2-4 & -4-6+8 & -8-8+12 \\ -2-3+4 & 2+9-8 & 4+12-12 \\ 2+2-3 & -2-6+6 & -4-8+9 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

2)
$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Transpose Matrix :-

Let $A = [a_{ij}]$ be an $m \times n$ matrix. The transpose of A, denoted by A^t is the $n \times m$ matrix obtained by interchanging the rows and columns of A.

e.g; $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

$A^t = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

Zero Matrix :-

A zero matrix or null matrix is a matrix all of whose entries are zero.

$A_{2\times2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$B_{2\times3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Matrix Multiplication:

Two matrices A and B comfortable for multiplication giving product AB. if the number of columns of A is equal to number of rows of B.

Example:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}_{2\times2}, \quad B = \begin{bmatrix} 4 \\ 1 \end{bmatrix}_{2\times1}$$

$$AB = \begin{bmatrix} (1\times4) + (2\times1) \\ (3\times4) + (0\times1) \end{bmatrix}_{2\times2}$$

$$= \begin{bmatrix} 4+2 \\ 12+0 \end{bmatrix}_{2\times1}$$

$$= \begin{bmatrix} 6 \\ 12 \end{bmatrix}_{2\times1}$$

Defination:

If A is a "$m\times r$" matrix and B is "$r\times n$" matrix then the product $m\times n$ matrix whose entries are determined as follow.

To find the entries in row "i" and column "j" of AB (i.e a_{ij}) single out row "i" from matrix A and column "j" from matrix B. Multiply the corresponding entries from row and column together and then add up the resulting product.

Example:

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix}_{2\times3} \quad B = \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix}_{2\times4}$$

$a_{23} = (2)(4) + 6(3) + 0(5)$ $[2 \ 6 \ 0] \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$

$= 8 + 18$

$= 26$

$a_{11} = [1 \ 2 \ 4] \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

$= 1\times4 + 2\times0 + 4\times2$

$= 4 + 0 + 8$

$= 12$

$\therefore A_{m\times r} \; B_{r\times n} = AB_{m\times n}$ $\because$ Inside equal

inside

outside

Q $[8 \ -4 \ 5]_{1\times3} \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}_{3\times1}$ $\because B_{2\times3}$ Row Column

$= \begin{bmatrix} 8(3) \\ -4(2) \\ 5(-1) \end{bmatrix} \Rightarrow \begin{bmatrix} 24 \\ -8 \\ -5 \end{bmatrix}$

$= 8(3) + (-4)2 + 5\times(-1)$

$= 24 - 8 - 5$

$= 11$

$[3 \ 8 \ -2 \ 4]_{1\times4} \begin{bmatrix} 5 \\ -1 \\ 6 \end{bmatrix}_{3\times1}$ As number of rows of B matrix not equal to number of column of A matrix

not possible

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$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 2 \\ -1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(0)+(2)(-1)+(3)(1) & (1)(1)+(2)(2)+(3)(2) & (1)(2)+(2)(3)+(3)(3) \\ (-1)(0)+(2)(-1)+(3)(1) & (-1)(1)+(2)(2)+(3)(2) & (-1)(2)+(2)(3)+(3)(3) \\ (0)(0)+(1)(-1)+(2)(1) & (0)(1)+(1)(2)+(2)(2) & (0)(2)+(1)(3)+(2)(3) \end{bmatrix}$$

$$= \begin{bmatrix} 0-2+3 & 1+4+6 & 2+6+9 \\ 0-2+3 & -1+4+6 & -2+6+9 \\ 0-1+2 & 0+2+4 & 0+3+6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 11 & 17 \\ 1 & 9 & 13 \\ 1 & 6 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 3/2 & -4/2 \\ -1/2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$$

$$\frac{6}{2} - \frac{4}{2} \qquad \frac{12}{2} - \frac{12}{2}$$

$$\frac{-2}{2} + 1 \qquad \frac{-4}{2} + 3$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 3/2 & -4/2 \\ -1/2 & 1 \end{bmatrix}$$

$$\begin{pmatrix} \frac{3}{\cancel{2}} \times \cancel{2} + \frac{-4}{2} & -\frac{8}{2} + 4 \\ \frac{3}{2} - \frac{3}{2} & -4/2 + 3 \end{pmatrix}$$

$$\begin{pmatrix} \frac{6-4}{2} & \frac{-8+8}{2} \\ \frac{3-3}{2} & \frac{-4+6}{2} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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Matrix Addition and Subtraction

Matrix Addition:-

The matrix addition can be done when the dimensions of matrices are same.

Matrix addition is the operation of adding two matrices by adding the corresponding entries together.

Example:-

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 9 & 8 & 7 \end{bmatrix}, \quad B = \begin{bmatrix} 6 & 5 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

Soln:-

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 9 & 8 & 7 \end{bmatrix}_{2\times 3}$$

$$B = \begin{bmatrix} 6 & 5 & 4 \\ 3 & 4 & 5 \end{bmatrix}_{2\times 3}$$

$$A + B = \begin{bmatrix} 0 & 1 & 2 \\ 9 & 8 & 7 \end{bmatrix} + \begin{bmatrix} 6 & 5 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 0+6 & 1+5 & 2+4 \\ 9+3 & 8+4 & 7+5 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 6 & 6 \\ 12 & 12 & 12 \end{bmatrix}_{2\times 3} \quad \underline{\text{Ans.}}$$

Matrix Subtraction:-

The matrix subtraction can be done when the dimensions of matr[ices] are same.

Subtracting one matrices by another

$$A + 3B = \begin{bmatrix} 1 & 0 & 4 \\ 3 & 8 & 2 \\ -2 & -3 & 0 \end{bmatrix} + 3\begin{bmatrix} 2 & 4 & 4 \\ -3 & 2 & 3 \\ -5 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 4 \\ 3 & 8 & 2 \\ -2 & -3 & 0 \end{bmatrix} + \begin{bmatrix} 3\times 2 & 3\times 4 & 3\times 4 \\ 3\times -3 & 3\times 2 & 3\times 3 \\ 3\times -5 & 3\times 0 & 3\times 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 4 \\ 3 & 8 & 2 \\ -2 & -3 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 12 & 12 \\ -9 & 6 & 9 \\ -15 & 0 & 15 \end{bmatrix}$$

$$\therefore A + 3B = \begin{bmatrix} 1+6 & 0+12 & 4+12 \\ 3+(-9) & 8+6 & 2+9 \\ -2+(-15) & -3+0 & 0+15 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 12 & 16 \\ -6 & 14 & 11 \\ -17 & -3 & 15 \end{bmatrix} \text{ Ans.}$$

ii) $2A - 3B = ?$

$$2A - 3B = 2\begin{bmatrix} 1 & 0 & 4 \\ 3 & 8 & 2 \\ -2 & -3 & 0 \end{bmatrix} - 3\begin{bmatrix} 2 & 4 & 4 \\ -3 & 2 & 3 \\ -5 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times 1 & 2\times 0 & 2\times 4 \\ 2\times 3 & 2\times 8 & 2\times 2 \\ 2\times -2 & 2\times -3 & 2\times 0 \end{bmatrix} - \begin{bmatrix} 3\times 2 & 3\times 4 & 3\times \\ 3\times -3 & 3\times 2 & 3 \\ 3\times -5 & 3\times 0 & 3\times \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 8 \\ 6 & 16 & 4 \\ -4 & -16 & 0 \end{bmatrix} - \begin{bmatrix} 6 & 12 & 12 \\ -9 & 6 & 9 \\ -15 & 0 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 2-6 & 0-12 & 8-12 \\ 6-(-9) & 16-6 & 4-9 \\ -4-(15) & -6-0 & 0-15 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -12 & -4 \\ 15 & 10 & -5 \\ 11 & -6 & -15 \end{bmatrix} \text{ Ans}$$

matrices is obtained by subtracting the corresponding entries of the matrices.

Example:-

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 3 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -4 & 3 \\ 9 & -4 & -3 \end{bmatrix}$$

Soln:-

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 3 & 6 \end{bmatrix}_{2\times 3}$$

$$B = \begin{bmatrix} 0 & -4 & 3 \\ 9 & -4 & -3 \end{bmatrix}_{2\times 3}$$

$$A-B = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 3 & 6 \end{bmatrix} - \begin{bmatrix} 0 & -4 & 3 \\ 9 & -4 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -1-0 & 2-(-4) & 0-3 \\ 0-9 & 3-(-4) & 6-(-3) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2+4 & -3 \\ -9 & 3+4 & 6+3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 6 & -3 \\ -9 & 7 & 9 \end{bmatrix}$$

Problems:-

① $A+3B = ?$ (ii) $2A-3B = ?$

if, $A = \begin{bmatrix} 1 & 0 & 4 \\ 3 & 8 & 2 \\ -2 & -3 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 & 4 \\ -3 & 2 & 3 \\ -5 & 0 & 5 \end{bmatrix}$

Sol:-

(i)

$$A = \begin{bmatrix} 1 & 0 & 4 \\ 3 & 8 & 2 \\ -2 & -3 & 0 \end{bmatrix}_{3\times 3}$$

$$B = \begin{bmatrix} 2 & 4 & 4 \\ -3 & 2 & 3 \\ -5 & 0 & 5 \end{bmatrix}_{3\times 3}$$

Laplacian Matrix:

In mathematical field of graph theory, the laplacian matrix, Sometimes called admittance matrix, kirchhoff matrix are discrete laplacian, is a matrix representation of graph.

Example:-

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Laplacian Matrix $L = D - A$

$$L = \begin{bmatrix} 3 & -1 & 0 & -1 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ -1 & 0 & -1 & 2 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Lower matrix:-

A square matrix is called lower triangular if all entries above the main diagonal are zero.

Examples:-

$$\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 5 & 6 & 3 \end{bmatrix}$$

→ These are the examples of lower matrix

Upper matrix:-

A square matrix is called upper triangular if all entries below the main diagonal are zero.

Examples:-

$$\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

→ These are the examples of upper diagonal ma

Diagonal matrix :-

A diagonal matrix is a matrix in which the entries outside the main diagonal are all zeros the term usually refers to square matrices

Examples:-

diagonal matrix (1, 2, 3) = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

diagonal matrix (3, 3, 4, 2, 1) = $\begin{bmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} a_{11} & 0 & 0 \cdots & 0 \\ 0 & a_{22} & 0 \cdots & 0 \\ 0 & 0 & a_{33} \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & a_{nn} \end{bmatrix}$$

These are the examples of diagonal matrix

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Square matrix:-

A square matrix is a matrix with the same number of rows & columns. Any of two square matrix of the same order can be added or multiplied. Square matrices are often used to represent simple linear transformations, such as shearing or rotation.

Examples:-

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} \qquad \begin{bmatrix} 5 & 8 & 9 \\ 6 & 2 & 3 \\ 7 & 4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

→ These are the examples of square matrix.