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TRANSPOSE MATRIX

Definition :-

The new matrix obtained by interchanging the rows and colums of original matrix is called transpose of the matrix.
If $A = [a_{ij}]$ be an matrix having order $m \times n$, then the matrix obtained by interchanging the rows and colums of A would be the transpose of A.
It is denoted by (A^T) or A'

$$A = [a_{ij}]_{m \times n}$$

$$A^T = [a_{ji}]_{m \times n}$$

Example :-

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

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Properties of Transpose

1. Transpose of Transpose

The transpose of transpose of matrix is the matrix itself

$$(A^T)^T = A$$

Example And verification

If

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 3 & 2 \end{bmatrix}$$

We have to find Transpose of given matrix so row ↔ colums

$$A^T = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 3 & 2 \end{bmatrix}^T$$

$$A^T = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 3 & 2 \end{bmatrix}^T$$

$$A^T = \begin{bmatrix} 3 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}$$

Again Taking Transpose

$$(A^T)^T = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 3 & 2 \end{bmatrix} = A$$

Hence 1st property proved

$$(A^T)^T = A$$

2. Transpose of scalar multiple

The transpose of a matrix time a scalar (K) is equal to constant time the transpose of the matrix.

$$(KA)^T = KA^T$$

Example And verification

Let

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \;;\; A^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

$$K = 2$$

$$KA = 2\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$KA = \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix}$$

$$(KA)^T = \begin{bmatrix} 2a & 2c \\ 2b & 2d \end{bmatrix}$$

$$= 2\begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

$$= 2(A^T)$$

Hence

$$(KA)^T = KA^T$$

Proved

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3. Transpose of product

The Transpose of the product of two matrices is equilent to the product of ~~two~~ of their transposes. in reversed order ; Also valid for multiple matrices (More than two matrices)

$$(AB)^T = B^T A^T$$

$$(AB)^T \neq A^T B^T$$

Example and verification.

let

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} ; \quad B = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$A^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix} ; \quad B^T = \begin{bmatrix} A & C \\ B & D \end{bmatrix}$$

$$AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$AB = \begin{bmatrix} aA + bC & aB + bD \\ cA + dC & cB + dD \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} aA+bC & cA+dC \\ aB+bD & cB+dD \end{bmatrix} \rightarrow (A)$$

$$B^T A^T = \begin{bmatrix} A & C \\ B & D \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} aA + bC & cA + cC \\ aB + bD & cB + dD \end{bmatrix} \rightarrow (B)$$

using A and (B)

$$(AB)^T = B^T A^T$$

Hence proved

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4. Transpose of sum

The transpose of the sum of two matrices is equivalent to sum of their transpose

$$(A+B)^T = A^T + B^T$$

Example and verification

Let

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$A^T = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix} ; \quad B^T = \begin{bmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{bmatrix}$$

$$A+B = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$A+B = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{bmatrix}$$

Taking Transpose

$$(A+B)^T = \begin{bmatrix} a_{11}+b_{11} & a_{21}+b_{21} \\ a_{12}+b_{12} & a_{22}+b_{22} \end{bmatrix} \rightarrow (A)$$

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$$A^T + B^T = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} a_{11}+b_{11} & a_{21}+b_{21} \\ a_{12}+b_{12} & a_{22}+b_{22} \end{bmatrix} \longrightarrow (B)$$

From (A) and (B)

Hence proved

$$(A+B)^T = A^T + B^T$$

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RECTAGULAR MATRIX

Rectangular matrix is a type of of matrix and element are arr-anged in the matrix as number of rows and number of colums The arrangement of element in matrix is in rectangular shape Thus it is called rectangular matrix.

The rectangular matrix can be expressed in general forma as follow The element of the matrix are arranged in m row and n columns Number of rows and coloumns should not be equal. So for rectangular Matrix it is an Basic condition

(i) $m \neq n$

(ii) $m > n$

(iii) $n > m$

So Rectangular shape only possible when it follows above condition

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Example

$$M = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & g & K & m \end{bmatrix}$$

So order of this matrix is 3×4 which is fulfill condition require for Rectangular Matrices.

TYPES

1 - Row Matrix

A matrix having one and only row known as Row matrix, Columns should greater than one. It is Rectangular Matrix

Example

$$A = \begin{bmatrix} a & b & c \end{bmatrix}_{1 \times 2}$$

So it is row matrix

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2. Column Matrix

Those matrixes having only one column and row should be greater then one known as column Matrix

Example

$$A = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$m \neq n$
$\Rightarrow m > n$
$\Rightarrow n = 1$

So it is an rectangular Matrices as well

3. Sum of Rectangular Matrices is an rectangular Matrices

Let consider two Matrix A and B Both are rect-angular matrix.

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3 & 4 & 4 \\ 0 & 2 & 5 \end{bmatrix}$$

So it is also a rectangular matrix

So sum of Rectangular Matrix is an rectangular matrix

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General Rectangular Matrices

All those matrices whose number of rows does not equals to number of columns are rectangular Matrix

Example :

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

It is a null vector but due its order it is also a rectangular matrix

Identity Matrix never be a rectangular matrix because it could not fulfill the condition for rectangular matrices

Examples of Rectangular Matrices:

$$M = \begin{bmatrix} 1 & 2 & 3 & 7 \\ 4 & 5 & 6 & 8 \\ 9 & 2 & 3 & 4 \end{bmatrix}_{3 \times 4}$$

$$M = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}_{4 \times 3} \longrightarrow$$

$$M = \begin{bmatrix} 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}_{2 \times 3}$$

$$M = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{3 \times 4}$$

Null or zero Matrix and rectangular Matrix as well