INTEGRAL EQUATION

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VOLTERRA-INTEGRAL EQUATION OF IST KIND

Q.1

.

$$e^x - \sin x - \cos x = \int_0^x 2e^{x-t} u(t)dt$$

SOL:-

$$\mathcal{L}\{e^x\} - \mathcal{L}\{\sin x\} - \mathcal{L}\{\cos x\} = \mathcal{L}\{\int_0^x 2e^{x-t} u(t)dt\}$$

$$\frac{1}{s-1} - \frac{1}{s^2+1} - \frac{s}{s^2+1} = \frac{2}{s-1} U(s)$$

$$\frac{2}{(s-1)(s^2+1)} = \frac{2}{s-1} U(s)$$

$$U(s) = \frac{1}{s^2+1}$$

→

$$\mathcal{L}^{-1}\{U(s)\} = \mathcal{L}^{-1}\{\frac{1}{s^2+1}\}$$

$$u(x) = \sin x$$

Q.2 Solve the Volterra integral equation of the first kind by using the Laplace transform method

Sol:-

$$1 + x - e^{x} = \int_0^x (t - x)u(t)dt$$

$$\mathcal{L}\{1\} + \mathcal{L}\{x\} - \mathcal{L}\{e^x\} = \mathcal{L}\left\{\int_0^x (t - x)u(t)dt\right\}$$

$$\frac{1}{s} + \frac{1}{s^2} - \frac{1}{s-1} = -\frac{1}{s^2}U(s)$$

$$\frac{s(s-1) + s - 1 - s^2}{s^2(s-1)} = -\frac{1}{s^2}U(s)$$

$$U(s) = \frac{1}{(s-1)}$$

$$\mathcal{L}^{-1}\{U(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{(s-1)}\right\}$$

$$u(x) = e^{x}$$

Exercises 3.3.2

Use the *Laplace transform method* to solve the Volterra integral equations of the first kind:

1. $x - \sin x = \int_0^x (x-t)u(t)dt$
2. $e^x + \sin x - \cos x = \int_0^x 2e^{x-t}u(t)dt$
3. $1 + \frac{1}{3!}x^3 - \cos x = \int_0^x (x-t)u(t)dt$
4. $1 + x - \sin x - \cos x = \int_0^x (x-t)u(t)dt$
5. $x = \int_0^x (1+2(x-t))u(t)dt$
6. $\sinh x = \int_0^x e^{x-t}u(t)dt$
7. $x = \int_0^x (x-t+1)u(t)dt$
8. $1 - x - e^{-x} = \int_0^x (t-x)u(t)dt$
9. $1 + x - \frac{1}{3!}x^3 - e^x = \int_0^x (t-x)u(t)dt$
10. $1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 - \sin x - \cos x = \int_0^x (x-t+1)u(t)dt$
11. $3 - 7x + x^2 + \sinh x - 3\cosh x = \int_0^x (x-t-3)u(t)dt$
12. $1 - \cos x = \int_0^x \cos(x-t)u(t)dt$

10. $$1 + x + x^2 + \frac{1}{3!}x^3 - \sin x - \cos x = \int_0^x (x - t + 1)u(t)dt.$$

$$\mathcal{L}\{1\} + \mathcal{L}\{x\} + \frac{1}{2}\mathcal{L}\{x^2\} + \frac{1}{6}\mathcal{L}\{x^3\} - \mathcal{L}\{\sin x\} - \mathcal{L}\{\cos x\} = \mathcal{L}\left\{\int_0^x (x-t+1)u(t)dt\right\}$$

$$\frac{1}{s} + \frac{1}{s^2} + \frac{1}{2}\frac{2}{s^3} + \frac{1}{6}\frac{6}{s^4} - \frac{1}{s^2+1} - \frac{s}{s^2+1} = \mathcal{L}\{x+1\}U(s)$$

$$\frac{1}{s} + \frac{1}{s^2} + \frac{1}{s^3} + \frac{1}{s^4} - \frac{1}{s^2+1} - \frac{s}{s^2+1} = \left(\mathcal{L}\{x\} + \mathcal{L}\{1\}\right)U(s)$$

$$\frac{1}{s} + \frac{1}{s^2} + \frac{1}{s^3} + \frac{1}{s^4} - \frac{1}{s^2+1} - \frac{s}{s^2+1} = \left(\frac{1}{s^2} + \frac{1}{s}\right)U(s)$$

$$\frac{1}{s} + \frac{1}{s^2} + \frac{1}{s^3} + \frac{1}{s^4} - \frac{s+1}{s^2+1} = \left(\frac{1+s}{s^2}\right)U(s)$$

$$s + 1 + \frac{1}{s} + \frac{1}{s^2} - \left(\frac{s^3+s^2}{s^2+1}\right) = (1+s)U(s)$$

$$(s+1) + \frac{s+1}{s^2} - \left(\frac{s^2(s+1)}{s^2+1}\right) = (1+s)U(s)$$

$$1 + \frac{1}{s^2} - \frac{s^2}{s^2+1} = U(s)$$

$$U(s) = \frac{s^2+1-s^2}{s^2(s^2+1)} = \frac{1}{s^2(s^2+1)} = \frac{1}{s^2} + \frac{1}{s^2+1}$$

$$\mathcal{L}^{-1}\{U(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2} + \frac{1}{s^2+1}\right\}$$

$$u(x) = x + \sin x$$

$2. \sinh x = \int_0^x e^{x-t} u(t)\,dt$

Volterra integro-differential equation

$$\mathcal{L}\{\sinh x\} = \mathcal{L}\left\{\int_0^x e^{x-t} u(t)\,dt\right\}$$

$$\frac{1}{s^2 - 1} = \mathcal{L}\{e^x\} U(s)$$

$$\frac{1}{s^2 - 1} = \frac{1}{s-1} U(s)$$

$$U(s) = \frac{(s-1)}{(s^2-1)} = \frac{(s-1)}{(s-1)(s+1)} = \frac{1}{s+1}$$

$$\mathcal{L}^{-1}\{U(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}$$

$$u(x) = e^{-x}$$