Modified Decomposition Method

Eq. a Volterra Integral Equation of 2^{nd} kind

$$u(x) = f(x) + \lambda \int_0^x K(x,t)\, u(t)\, dt$$

Solution: $f(x) = f_1(x) + f_2(x)$

where

$$u_0(x) = f_1(x)$$

$$u_1(x) = f_2(x) + \lambda \int_0^x K(x,t)\, u_0(t)\, dt$$

$$u_{n+1}(x) = \lambda \int_0^x K(x,t)\, u_n(t)\, dt \quad ; \quad n \geq 1$$

Example: Solve the Volterra Integral Equation by using Modified Decomposition Method.

$$u(x) = \sec x \tan x + (e^{\sec x} - e) - \int_0^x e^{\sec t}\, u(t)\, dt$$

Sol:

$$f(x) = \sec x \tan x + (e^{\sec x} - e)$$

$$f_1(x) = \sec x \tan x$$

$$f_2(x) = e^{\sec x} - e$$

$$u_0(x) = \sec x \tan x = f_1(x)$$

$$u_1(x) = f_2(x) + \lambda \int_0^x K(x,t)\, u_0(t)\, dt$$

$$u_1(x) = (e^{\sec x} - e) - \int_0^x e^{\sec t} (\sec t \tan t)\, dt$$

let $u = \sec t$

$du = \sec t \tan t \, dt$

when $t \to 1$ then $u \to 1$

when $t \to x$ then $u \to \sec x$

So.

$$u_1(x) = (e^{\sec x} - e) - \int_1^{\sec x} e^u \, du$$

$$u_1(x) = e^{\sec x} - e - \left| e^u \right|_1^{\sec x}$$

$$u_1(x) = e^{\sec x} - e - e^{\sec x} + e$$

$$u_1(x) = 0$$

$$u_2(x) = -\int_0^x K(x,t)\, u_1(t)\, dt$$

$$u_2(x) = -\int_0^x e^{\sec t}(0)\, dt$$

$$u_2(x) = 0$$

It is obvious that each component of u_j, $j \geq 1$ is zero

$$u(x) = u_0(x) + u_1(x) + u_2(x) + \dots$$

$$u(x) = \sec x \tan x + 0 + 0 + \dots$$

$$\boxed{u(x) = \sec x \tan x}$$ Ans.

Example: Solve the Volterra Integral Equation by using Modified Decomposition Method.

(25)

(a) $u(x) = 2x + \sin x + x^2 - \cos x + 1 - \int_0^x u(t)\,dt$

(b) $u(x) = 1 + x^2 + \cos x - x - \frac{1}{3}x^3 - \sin x + \int_0^x u(t)\,dt$

Sol: (a) $u(x) = 2x + \sin x + x^2 - \cos x + 1 - \int_0^x u(t)\,dt$

$f(x) = 2x + \sin x + x^2 - \cos x + 1$

$f_0(x) = 2x + \sin x$

$f_1(x) = x^2 - \cos x + 1$

$u_0(x) = 2x + \sin x = f_0(x)$

$u_0(x) = 2x + \sin x$

$u_1(x) = f_1(x) + \lambda \int_0^x K(x,t)\, u_0(t)\,dt$

$u_1(x) = x^2 - \cos x + 1 - \int_0^x (2t + \sin t)\,dt$

$u_1(x) = x^2 - \cos x + 1 - \left[t^2\right]_0^x - \left[-\cos t\right]_0^x$

$u_1(x) = x^2 - \cos x + 1 - x^2 + \cos x - 1$

$u_1(x) = 0$

$u_2(x) = \lambda \int_0^x K(x,t)\, u_1(t)\,dt$

$u_2(x) = -\int_0^x (0)\,dt$

$u_2(x) = 0$

It is obvious that each component of u_j, $j \geq 1$ is zero

$u(x) = u_0(x) + u_1(x) + u_2(x) + \dots$

$u(x) = 2x + \sin x + 0 + 0 + \dots$

$\boxed{u(x) = 2x + \sin x}$ Ans

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(b) $u(x) = 1+x^2+\cos x - x - \frac{1}{3}x^3 - \sin x + \int_0^x u(t)\,dt$

Sol. $f(x) = 1+x^2+\cos x - x - \frac{1}{3}x^3 - \sin x$

$f_1(x) = 1+x^2+\cos x$

$f_2(x) = -x-\frac{1}{3}x^3-\sin x$

$$u_0(x) = 1+x^2+\cos x = f_1(x)$$

$$u_0(x) = 1+x^2+\cos x$$

$$u_1(x) = f_2(x) + \lambda \int_0^x k(x,t)\, u_0(t)\,dt$$

$$u_1(x) = -x-\frac{1}{3}x^3-\sin x + \int_0^x (1+t^2+\cos t)\,dt$$

$$u_1(x) = -x-\frac{1}{3}x^3-\sin x + |t|_0^x + \left|\frac{t^3}{3}\right|_0^x + |\sin t|_0^x$$

$$u_1(x) = \cancel{-x} - \cancel{\frac{1}{3}x^3} - \cancel{\sin x} + \cancel{x} + \cancel{\frac{x^3}{3}} + \cancel{\sin x}$$

$$u_1(x) = 0$$

$$u_2(x) = \lambda \int_0^x k(x,t)\, u_1(t)\,dt$$

$$u_2(x) = \int_0^x (0)\,dt$$

$$u_2(x) = 0$$

It is obvious that each component of u_j, $j \geq 1$ is zero

So

$$u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots$$

$$u(x) = 1+x^2+\cos x + 0 + 0 + \cdots$$

$$\boxed{u(x) = 1+x^2+\cos x}$$ Ans.

Exercise 3.2.2: Use the modified decomposition method to solve the following Volterra Integral Equations.

127

(1) $u(x) = \cos x + \sin x - \int_0^x u(t)\,dt$

Sol. $u(x) = \cos x + \sin x - \int_0^x u(t)\,dt$

$f(x) = \cos x + \sin x$

$f_1(x) = \cos x$

$f_2(x) = \sin x$

$u_0(x) = \cos x = f_1(x)$

$u_0(x) = \cos x$

$u_1(x) = f_2(x) + \lambda \int_0^x K(x,t)\, u_0(t)\,dt$

$u_1(x) = \sin x - \int_0^x \cos t\,dt$

$u_1(x) = \sin x - |\sin t|_0^x$

$u_1(x) = \cancel{\sin x} - \cancel{\sin x}$

$u_1(x) = 0$

$u_2(x) = \lambda \int_0^x K(x,t)\, u_1(t)\,dt$

$u_2(x) = -\int_0^x (0)\,dt$

$u_2(x) = 0$

It is obvious that each component of u_j, $j \geq 1$ is zero

So

$u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots$

$u(x) = \cos x + 0 + 0 + \cdots$

$\boxed{u(x) = \cos x}$ Ans.

(2) $u(x) = \sinh x + \cosh x - 1 - \int_0^x u(t)\,dt$

Sol: $u(x) = \sinh x + \cosh x - 1 - \int_0^x u(t)\,dt$

$f(x) = \sinh x + \cosh x - 1$

$f_1(x) = \sinh x$

$f_2(x) = \cosh x - 1$

$u_0(x) = \sinh x = f_1(x)$

$u_0(x) = \sinh x$

$u_1(x) = f_2(x) + \lambda \int_0^x K(x,t)\, u_0(t)\, dt$

$u_1(x) = \cosh x - 1 - \int_0^x \sinh t \, dt$

$u_1(x) = \cosh x - 1 - \left| \cosh t \right|_0^x$

$u_1(x) = \cancel{\cosh x} - \cancel{1} - \cancel{\cosh x} + \cancel{1}$

$u_1(x) = 0$

$u_2(x) = \lambda \int_0^x K(x,t)\, u_1(t)\, dt$

$u_2(x) = -\int_0^x (0)\, dt$

$u_2(x) = 0$

It is obvious that each component of u_j, $j \geq 1$ is zero

So, $u(x) = u_0(x) + u_1(x) + u_2(x) + \ldots$

$u(x) = \sinh x + 0 + 0 + \ldots$

$\boxed{u(x) = \sinh x}$ Ans.

(3) $u(x) = 2x + 3x^3 + (e^{x^2+x^3} - 1) - \int_0^x e^{(t^2+t^3)} u(t)\, dt$

Sol: $u(x) = 2x + 3x^3 + (e^{x^2+x^3} - 1) - \int_0^x e^{(t^2+t^3)} u(t)\, dt$

$f(x) = 2x + 3x^3 + (e^{x^2+x^3} - 1)$

$f_1(x) = 2x + 3x^3$

$f_2(x) = e^{x^2+x^3} - 1$

$$u_0(x) = 2x + 3x^3 = f_1(x)$$

$$u_0(x) = 2x + 3x^3$$

$$u_1(x) = f_2(x) + \lambda \int_0^x K(x,t)\, u_0(t)\, dt$$

$$u_1(x) = e^{x^2+x^3} - 1 - \int_0^x e^{(t^2+t^3)} (2t + 3t^2)\, dt$$

$$u_1(x) = e^{x^2+x^3} - 1 - e^{t^2+t^3} \Big|_0^x$$

$$u_1(x) = \cancel{e^{x^2+x^3}} \cancel{-1} - \cancel{e^{x^2+x^3}} \cancel{+1}$$

$$u_1(x) = 0$$

$$u_2(x) = \lambda \int_0^x K(x,t)\, u_1(t)\, dt$$

$$u_2(x) = -\int_0^x e^{(t^2+t^3)} (0)\, dt$$

$$u_2(x) = 0$$

It is obvious that each component of u_j, $j \geq 1$ is zero

So $$u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots$$

$$u(x) = 2x + 3x^3 + 0 + 0 + \cdots$$

$$\boxed{u(x) = 2x + 3x^3}$$ Ans.

(4) $$u(x) = 3x^2 + (1 - e^{-x^3}) - \int_0^x e^{-x^3+t^3}\, u(t)\, dt$$

Sol:

$$u(x) = 3x^2 + (1 - e^{-x^3}) - \int_0^x e^{-x^3+t^3}\, u(t)\, dt$$

$f(x) = 3x^2 + (1 - e^{-x^3})$

$f_1(x) = 3x^2$

$f_2(x) = 1 - e^{-x^3}$

$u_0(x) = 3x^2 = f_1(x)$

$u_0(x) = 3x^2$

$u_1(x) = f_2(x) + \lambda \int_0^x K(x,t)\, u_0(t)\, dt$

$u_1(x) = 1 - e^{-x^3} - \int_0^x e^{-x^3+t^3} (3x^2)\, dt$

$u_1(x) = 1 - e^{-x^3} - \int_0^x e^{-x^3} . e^{t^3} (3x^2)\, dt$

$u_1(x) = 1 - e^{-x^3} - e^{-x^3} \int_0^x e^{t^3} (3x^2)\, dt$

$u_1(x) = 1 - e^{-x^3} - e^{-x^3} \left| e^{t^3} \right|_0^x$

$u_1(x) = 1 - e^{-x^3} - e^{-x^3} \left(e^{x^3} - e^{0} \right)$

$u_1(x) = 1 - \cancel{e^{-x^3}} - e^{0} + \cancel{e^{-x^3}}$

$u_1(x) = \cancel{1} - \cancel{1}$

$u_1(x) = 0$

$u_2(x) = \lambda \int_0^x K(x,t)\, u_1(t)\, dt$

$u_2(x) = - \int_0^x e^{-x^3+t^3} (0)\, dt$

$u_2(x) = 0$

It is obvious that each component of u_j, $j \geq 1$ is zero

So $u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots$

$u(x) = 3x^2 + 0 + 0 + \cdots$

(130)

$\boxed{u(x) = 3x^2}$ Ans.

(5) $u(x) = 2x - (1 - e^{-x^2}) + \int_0^x e^{-x^2+t^2} u(t)\,dt$

Sol. $u(x) = 2x + (1 - e^{-x^2}) + \int_0^x e^{-x^2+t^2} u(t)\,dt$

$f(x) = 2x + (1 - e^{-x^2})$

$f_1(x) = 2x$

$f_2(x) = -1 + e^{-x^2}$

$u_0(x) = 2x = f_1(x)$

$u_0(x) = 2x$

$u_1(x) = f_2(x) + \lambda \int_0^x K(x,t)\, u_0(t)\,dt$

$u_1(x) = -1 + e^{-x^2} + \int_0^x e^{-x^2+t^2}\, 2t\,dt$

$u_1(x) = -1 + e^{-x^2} + \int_0^x e^{-x^2} \cdot e^{t^2}\,(2t)\,dt$

$u_1(x) = -1 + e^{-x^2} + e^{-x^2} \int_0^x e^{t^2}(2t)\,dt$

$u_1(x) = -1 + e^{-x^2} + e^{-x^2} \left| e^{t^2} \right|_0^x$

$u_1(x) = -1 + e^{-x^2} + e^{-x^2}(e^{x^2} - e^0)$

$u_1(x) = -1 + e^{-x^2} + (e^0 - e^{-x^2})$

$u_1(x) = \cancel{-1} + \cancel{e^{-x^2}} + \cancel{1} - \cancel{e^{-x^2}}$

$u_1(x) = 0$

$u_2(x) = \lambda \int_0^x K(x,t)\, u_1(t)\,dt$

$u_2(x) = \int_0^x e^{-x^2+t^2}\,(0)\,dt$

$u_2(x) = 0$

It is obvious that each component of u_j, $j \geq 1$ is zero

So $u(x) = u_0(x) + u_1(x) + u_2(x) + \dots$

$u(x) = 2x + 0 + 0 + \dots$

$\boxed{u(x) = 2x}$ Ans.

(6) $u(x) = e^{-x^2} - \frac{x}{2}(1 - e^{-x^2}) + \int_0^x xt\, u(t)\, dt.$

Sol: $u(x) = e^{-x^2} - \frac{x}{2}(1 - e^{-x^2}) + \int_0^x xt\, u(t)\, dt$

$f(x) = e^{-x^2} - \frac{x}{2}(1 - e^{-x^2})$

$f_1(x) = e^{-x^2}$

$f_2(x) = -\frac{x}{2}(1 - e^{-x^2})$

$u_0(x) = e^{-x^2} = f_1(x)$

$\boxed{u_0(x) = e^{-x^2}}$

$u_1(x) = f_2(x) + \lambda \int_0^x k(x,t)\, u_0(t)\, dt$

$u_1(x) = -\frac{x}{2}(1 - e^{-x^2}) + \int_0^x xt\, e^{-t^2}\, dt$

$u_1(x) = -\frac{x}{2}(1 - e^{-x^2}) + x\left(-\frac{1}{2}\right) \int_0^x e^{-t^2}(-2t)\, dt$

$u_1(x) = -\frac{x}{2}(1 - e^{-x^2}) - \frac{x}{2} \left| e^{-t^2} \right|_0^x$

$u_1(x) = -\frac{x}{2}(1 - e^{-x^2}) - \frac{x}{2}(e^{-x^2} - 1)$

$u_1(x) = \cancel{-\frac{x}{2}(1-e^{-x^2})} + \cancel{\frac{x}{2}(1-e^{-x^2})}$

$u_1(x) = 0$

$u_2(x) = \lambda \int_0^x K(x,t)\,u_1(t)\,dt$

$u_2(x) = \int_0^x xt\,(0)\,dt$

$u_2(x) = 0$

It is obvious that each component of u_j, $j \geq 1$ is zero.

So $u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots$

$u(x) = e^{-x^2} + 0 + 0 + \ldots$

$\boxed{u(x) = e^{-x^2}}$ Ans.

(7) $u(x) = \cosh x + x \sinh x - \int_0^x x\,u(t)\,dt$.

Sol:

$u(x) = \cosh x + x \sinh x - \int_0^x x\,u(t)\,dt$

$f(x) = \cosh x + x \sinh x$

$f_1(x) = \cosh x$

$f_2(x) = x \sinh x$

$u_0(x) = \cosh x = f_1(x)$

$u_0(x) = \cosh x$

$u_1(x) = f_2(x) + \lambda \int_0^x K(x,t)\,u_0(t)\,dt$

$u_1(x) = x \sinh x - \int_0^x x \cosh x\,dt$

$u_1(x) = x \sinh x - x \int_0^x \cosh t\,dt$

$u_1(x) = x \sinh x - x\,|\sinh t|_0^x$

$u_1(x) = \cancel{x \sinh x} - \cancel{x \sinh x}$

$u_1(x) = 0$

$$u_2(x) = \lambda \int_0^x K(x,t)\, u_1(t)\, dt$$

$$u_2(x) = -\int_0^x x\,(0)\, dt$$

$$u_2(x) = 0$$

It is obvious that each component of u_j, $j \geq 1$ is zero

So $u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots$

$$u(x) = \cosh x + 0 + 0 + \cdots$$

$$\boxed{u(x) = \cosh x}$$ Ans.

(8) $u(x) = e^x + xe^x - x - \int_0^x x\, u(t)\, dt$

Sol:

$$u(x) = e^x + xe^x - x - \int_0^x x\, u(t)\, dt$$

$$f(x) = e^x + xe^x - x$$

$$f_1(x) = e^x$$

$$f_2(x) = xe^x - x$$

$$u_0(x) = e^x = f_1(x)$$

$$u_0(x) = e^x$$

$$u_1(x) = f_2(x) + \lambda \int_0^x K(x,t)\, u_0(t)\, dt$$

$$u_1(x) = xe^x - x - \int_0^x x e^t\, dt$$

$$u_1(x) = xe^x - x - x\int_0^x e^t\, dt$$

$$u_1(x) = xe^x - x - x\left|e^t\right|_0^x$$

$$u_1(x) = xe^x - x - x\left(e^x - 1\right)$$

$$u_1(x) = \cancel{xe^x} - \cancel{x} - \cancel{xe^x} + \cancel{x}$$

$$u_1(x) = 0$$

$$u_2(x) = \lambda \int_0^x K(x,t)\, u_1(t)\, dt$$

(134)

$$u_2(x) = -\int_0^x x(0)\,dt$$

$$u_2(x) = 0$$

It is obvious that each component of u_j, $j \geq 1$ is zero.

So, $u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots$

$$u(x) = e^x + 0 + 0 + \cdots$$

$$\boxed{u(x) = e^x}$$ Ans.

(9) $u(x) = 1 + \sin x + x + x^2 - x\cos x - \int_0^x x\,u(t)\,dt.$

Sol:

$$u(x) = 1 + \sin x + x + x^2 - x\cos x - \int_0^x x\,u(t)\,dt$$

$$f(x) = 1 + \sin x + x + x^2 - x\cos x$$

$$f_1(x) = 1 + \sin x$$

$$f_2(x) = x + x^2 - x\cos x$$

$$u_0(x) = 1 + \sin x = f_1(x)$$

$$u_0(x) = 1 + \sin x$$

$$u_1(x) = f_2(x) + \lambda\int_0^x k(x,t)\,u_0(t)\,dt$$

$$u_1(x) = x + x^2 - x\cos x - \int_0^x x(1+\sin t)\,dt$$

$$u_1(x) = x + x^2 - x\cos x - x\int_0^x (1+\sin t)\,dt$$

$$u_1(x) = x + x^2 - x\cos x - x\left[\,|t|_0^x + |-\cos t|_0^x\right]$$

$$u_1(x) = x + x^2 - x\cos x - x\left[x - \cos x + 1\right]$$

$$u_1(x) = \cancel{x} + \cancel{x^2} - \cancel{x\cos x} - \cancel{x^2} + \cancel{x\cos x} - \cancel{x}$$

$$u_1(x) = 0$$

$$u_2(x) = \lambda\int_0^x k(x,t)\,u_1(t)\,dt$$

$$u_2(x) = -\int_0^x x(0)\,dt$$

$u_2(x) = 0$

It is obvious that each component of u_j, $j \geq 1$ is zero.

So, $u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots$

$u(x) = 1 + \sin x + 0 + 0 + \cdots$

$\boxed{u(x) = 1 + \sin x}$ Ans.

(10) $u(x) = e^x - xe^x + \sin x + x\cos x + \int_0^x x\, u(t)\, dt.$

Sol: $u(x) = e^x - xe^x + \sin x + x\cos x + \int_0^x x\, u(t)\, dt$

$f(x) = e^x + \sin x - xe^x + x\cos x$

$f_1(x) = e^x + \sin x$

$f_2(x) = -xe^x + x\cos x$

$u_0(x) = e^x + \sin x = f_1(x)$

$u_0(x) = e^x + \sin x$

$u_1(x) = f_2(x) + \lambda \int_0^x K(x,t)\, u_0(t)\, dt$

$u_1(x) = -xe^x + x\cos x + \int_0^x x(e^t + \sin t)\, dt$

$u_1(x) = -xe^x + x\cos x + x\int_0^x (e^t + \sin t)\, dt$

$u_1(x) = -xe^x + x\cos x + x\left[|e^t|_0^x + |-\cos t|_0^x \right]$

$u_1(x) = -xe^x + x\cos x + x\left[e^x - e^0 - \cos x + \cos 0 \right]$

$u_1(x) = \cancel{-xe^x} + \cancel{x\cos x} + \cancel{xe^x} - \cancel{x} - \cancel{x\cos x} + \cancel{x}$

$u_1(x) = 0$

It is obvious that each component of u_j, $j \geq 1$ is zero

So, $u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots$

$u(x) = e^x + \sin x + 0 + 0 + \cdots$

$\boxed{u(x) = e^x + \sin x}$ Ans.

(136)

(11) $1+x+x^2+\frac{1}{2}x^3+\cosh x+x\sinh x-\int_0^x x\,u(t)\,dt$

Sol: $u(x)=1+x+x^2+\frac{1}{2}x^3+\cosh x+x\sinh x-\int_0^x x\,u(t)\,dt$

$$f(x)=1+x+x^2+\frac{1}{2}x^3+\cosh x+x\sinh x$$

$$f_1(x)=1+x+\cosh x$$

$$f_2(x)=x^2+\frac{1}{2}x^3+x\sinh x$$

$$u_0(x)=1+x+\cosh x=f_1(x)$$

$$u_0(x)=1+x+\cosh x$$

$$u_1(x)=f_2(x)+\lambda\int_0^x K(x,t)\,u_0(t)\,dt$$

$$u_1(x)=x^2+\frac{1}{2}x^3+x\sinh x-\int_0^x x(1+t+\cosh t)\,dt$$

$$u_1(x)=x^2+\frac{1}{2}x^3+x\sinh x-x\int_0^x(1+t+\cosh t)\,dt$$

$$u_1(x)=x^2+\frac{1}{2}x^3+x\sinh x-x\left[|t|_0^x+\left|\frac{t^2}{2}\right|_0^x+|\sinh t|_0^x\right]$$

$$u_1(x)=x^2+\frac{1}{2}x^3+x\sinh x-x\left[x+\frac{x^2}{2}+\sinh x\right]$$

$$u_1(x)=\cancel{x^2}+\cancel{\frac{1}{2}x^3}+\cancel{x\sinh x}-\cancel{x^2}-\cancel{\frac{x^3}{2}}-\cancel{x\sinh x}$$

$$u_1(x)=0$$

$$u_2(x)=\lambda\int_0^x K(x,t)\,u_1(t)\,dt$$

$$u_2(x)=-\int_0^x x(0)\,dt$$

$$u_2(x)=0$$

It is obvious that each component of u_j, $j\geq 1$ is zero

So $u(x)=u_0(x)+u_1(x)+u_2(x)+\cdots$

$$u(x)=1+x+\cosh x+0+0+\cdots$$

$$\boxed{u(x)=1+x+\cosh x}$$ Ans.

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(12) $u(x) = \cos x - (1 - e^{\sin x})x - x\int_0^x e^{\sin t} u(t)\,dt$

Sol:

$$u(x) = \cos x - (1 - e^{\sin x})x - x\int_0^x e^{\sin t} u(t)\,dt$$

$$f(x) = \cos x - (1 - e^{\sin x})x$$

$$f_1(x) = \cos x$$

$$f_2(x) = -(1 - e^{\sin x})x$$

$$u_0(x) = \cos x = f_1(x)$$

$$u_0(x) = \cos x$$

$$u_1(x) = f_2(x) + \lambda\int_0^x K(x,t)\,u_0(t)\,dt$$

$$u_1(x) = -(1 - e^{\sin x})x - x\int_0^x e^{\sin t} \cos t\,dt$$

$$u_1(x) = -x + x e^{\sin x} - x \int_0^x e^{\sin t}(+\cos t)\,dt$$

$$u_1(x) = -x + x e^{\sin x} - x \left[e^{\sin t} \right]_0^x$$

$$u_1(x) = -x + x e^{\sin x} - x \left[e^{\sin x} - 1 \right]$$

$$u_1(x) = \cancel{-x} + \cancel{x e^{\sin x}} - \cancel{x e^{\sin x}} + \cancel{x}$$

$$u_1(x) = 0$$

$$u_2(x) = \lambda\int_0^x K(x,t)\,u_1(t)\,dt$$

$$u_2(x) = -x\int_0^x e^{\sin t}(0)\,dt$$

$$u_2(x) = 0$$

It is obvious that each component of u_j, $j \geq 1$ is zero.

So $\quad u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots$

$$u(x) = \cos x + 0 + 0 + \cdots$$

$$\boxed{u(x) = \cos x}$$ Ans.

(138)

(13) $u(x) = \sec^2 x - (1 - e^{\tan x})x - x\int_0^x e^{\tan t} u(t)\,dt$

Sol:

$$u(x) = \sec^2 x - (1 - e^{\tan x})x - x\int_0^x e^{\tan t} u(t)\,dt$$

$$f(x) = \sec^2 x - (1 - e^{\tan x})x$$

$$f_1(x) = \sec^2 x$$

$$f_2(x) = -(1 - e^{\tan x})x$$

$$u_0(x) = \sec^2 x = f_1(x)$$

$$u_0(x) = \sec^2 x$$

$$u_1(x) = f_2(x) + \lambda\int_0^x K(x,t)\,u_0(t)\,dt$$

$$u_1(x) = -x + x e^{\tan x} - x\int_0^x e^{\tan t} \sec^2 t\,dt$$

$$u_1(x) = -x + x e^{\tan x} - x\left| e^{\tan t} \right|_0^x$$

$$u_1(x) = -x + x e^{\tan x} - x\left[e^{\tan x} - 1 \right]$$

$$u_1(x) = -\cancel{x} + \cancel{x e^{\tan x}} - \cancel{x e^{\tan x}} + \cancel{x}$$

$$u_1(x) = 0$$

$$u_2(x) = \lambda\int_0^x K(x,t)\,u_1(t)\,dt$$

$$u_2(x) = x\int_0^x e^{\tan t}(0)\,dt$$

$$u_2(x) = 0$$

It is obvious that each component of u_j, $j \geq 1$ is zero

So $u(x) = u_0(x) + u_1(x) + u_2(x) + \ldots$

$$u(x) = \sec^2 x + 0 + 0 + \ldots$$

$$\boxed{u(x) = \sec^2 x}$$ Ans.

(4) $u(x) = \cosh x + \frac{x}{2}(1 - e^{\sinh x}) + \frac{x}{2}\int_0^x e^{\sinh t} u(t)\,dt.$

Sol:

$$u(x) = \cosh x + \frac{x}{2}(1 - e^{\sinh x}) + \frac{x}{2}\int_0^x e^{\sinh t} u(t)\,dt$$

$$f(x) = \cosh x + \frac{x}{2}(1 - e^{\sinh x})$$

$$f_1(x) = \cosh x$$

$$f_2(x) = \frac{x}{2}(1 - e^{\sinh x})$$

$$u_0(x) = \cosh x = f_1(x)$$

$$u_0(x) = \cosh x$$

$$u_1(x) = f_2(x) + \lambda \int_0^x K(x,t)\,u_0(t)\,dt$$

$$u_1(x) = \frac{x}{2} - \frac{x}{2}e^{\sinh x} + \frac{x}{2}\int_0^x e^{\sinh t}\cos t\,dt$$

$$u_1(x) = \frac{x}{2} - \frac{x}{2}e^{\sinh x} + \frac{x}{2}\left| e^{\sinh t} \right|_0^x$$

$$u_1(x) = \frac{x}{2} - \frac{x}{2}e^{\sinh x} + \frac{x}{2}\,\sinh\!\left(e^{\sinh x} - e^{0}\right)$$

$$u_1(x) = \cancel{\frac{x}{2}} - \cancel{\frac{x}{2}e^{\sinh x}} + \cancel{\frac{x}{2}e^{\sinh x}} - \cancel{\frac{x}{2}}$$

$$u_1(x) = 0$$

$$u_2(x) = \lambda \int_0^x e^{\sinh t}(0)\,dt$$

$$u_2(x) = 0$$

It is obvious that each component of u_j, $j \geq 1$ is zero

So $u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots$

$$u(x) = \cosh x + 0 + 0 + \cdots$$

$$\boxed{u(x) = \cosh x}$$ Ans.

(15) $u(x) = \sinh x + \frac{1}{10}(e - e^{\cosh x}) + \frac{1}{10}\int_0^x e^{\cosh t} u(t)\, dt$

Sol: $u(x) = \sinh x + \frac{1}{10}(e - e^{\cosh x}) + \frac{1}{10}\int_0^x e^{\cosh t} u(t)\, dt$

$f(x) = \sinh x + \frac{1}{10}(e - e^{\cosh x})$

$f_1(x) = \sinh x$

$f_2(x) = \frac{1}{10}(e - e^{\cosh x})$

$u_0(x) = \sinh x = f_1(x)$

$u_0(x) = \sinh x$

$u_1(x) = f_2(x) + \lambda \int_0^x K(x,t)\, u_0(t)\, dt$

$u_1(x) = \frac{1}{10}(e - e^{\cosh x}) + \frac{1}{10}\int_0^x e^{\cosh t} \sinh t\, dt$

$u_1(x) = \frac{1}{10}(e - e^{\cosh x}) + \frac{1}{10}\int_0^x e^{\cosh t} (+\sinh t)\, dt$

$u_1(x) = \frac{1}{10}(e - e^{\cosh x}) + \frac{1}{10} \left| e^{\cosh t} \right|_0^x$

$u_1(x) = \frac{1}{10}(e - e^{\cosh x}) + \frac{1}{10}(e^{\cosh x} - e)$

$u_1(x) = \cancel{\frac{1}{10}(e - e^{\cosh x})} - \cancel{\frac{1}{10}(e - e^{\cosh x})}$

$u_1(x) = 0$

$u_2(x) = \lambda \int_0^x K(x,t)\, u_1(t)\, dt$

$u_2(x) = \frac{1}{10}\int_0^x e^{\cosh t} (0)\, dt$

$u_2(x) = 0$

It is obvious that each component of u_j, $j \geq 1$ is zero

So $u(x) = u_0(x) + u_1(x) + u_2(x) + \dots$

$u(x) = \sinh x + 0 + 0 + \dots$

$\boxed{u(x) = \sinh x}$ Ans.

(16) $u(x) = x^3 - x^5 + 5 \int_0^x t\, u(t)\, dt$

Sol:

$$u(x) = x^3 - x^5 + 5 \int_0^x t\, u(t)\, dt$$

$$f(x) = x^3 - x^5$$

$$f_1(x) = x^3$$

$$f_2(x) = -x^5$$

$$u_0(x) = x^3 = f_1(x)$$

$$u_0(x) = x^3$$

$$u_1(x) = f_2(x) + \lambda \int_0^x k(x,t)\, u_0(t)\, dt$$

$$u_1(x) = -x^5 + 5 \int_0^x t(t^3)\, dt$$

$$u_1(x) = -x^5 + 5 \int_0^x t^4\, dt$$

$$u_1(x) = -x^5 + \cancel{5} \left| \frac{t^5}{\cancel{5}} \right|_0^x$$

$$u_1(x) = \cancel{-x^5} + \cancel{x^5}$$

$$u_1(x) = 0$$

$$u_2(x) = \lambda \int_0^x k(x,t)\, u_1(t)\, dt$$

$$= 5 \int_0^x t\,(0)\, dt$$

$$u_2(x) = 0$$

It is obvious that each component of u_j, $j \geq 1$ is zero

So $u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots$

$$u(x) = x^3 + 0 + 0 + \cdots$$

$$\boxed{u(x) = x^3} \quad \text{Ans}$$

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