the limits of integration are infinite. Ans.

(ii) $u(x) = 1+x^2 - \int_0^1 \frac{1}{\sqrt{1-xt}} u(t)\, dt$

Sol: This is fredholm integral equation because of the limits of integration are constant, it is 2^{nd} kind because the unknown function appears inside/outside the integral. it is non-homogeneous because $f(x) = 1+x^2$, it is not singular since neither the limits of integration are infinite. Ans.

$f(x) = 1+x^2$
$\lambda = -1$
$K(x,t) = \frac{1}{\sqrt{1-xt}}$
$g(x) = 0$
$h(x) = 1$

Lecture #2 (Monday) 20-01-2020

Linearity (linear or non-linear) & homogeneous: Integral equation and integral differential equation has two types:

1) Linearity 2) Homogeneous

1) Linearity: If the exponent of unknown function $u(x)$ inside the integral sign is one, the integral equation or integral differential equation is called linear

Example:

(i) $u(x) = 1 - \int_0^x (x-t)u(t)\, dt$

Sol: The above equation is volterra integral equation and linear because the exponent of the unknown function inside the integral is one.

(ii) $u(x) = 1 - \int_0^1 (x-t)\, u(t)\, dt$

Sol: The above equation is Fredholm integral equation because the limit of integrations are constant and equation is linear because the exponent of unknown function inside the integral is one.

(iii) $u(x) = 1 - \int_0^x (x-t)\, u^4(t)\, dt$

Sol: The above equation is Volterra integral equation because the limit of integration is not constant and equation is non-linear because the exponent of unknown function inside the integral is not one.

(iv) $u(x) = 1 + \int_0^x u\, t\, e^{u(t)}\, dt$

Sol: The above equation is Volterra integral equation because the limit of integration is not constant and equation is non-linear because dependent variable is multiply by the unknown function inside the integral.

2) *Homogeneous*: Integral equation and integral differential equation of the 2^{nd} kind as homogeneous if $f(x)$ absolutely zero

Example.

(i) $u(x) = \sin x + \int_0^x (x-t)\, u(t)\, dt$

The above equation is Volteera integral equation because limits of integration is not constant and equation is non-homogeneous because $f(x) = \sin x$.

(ii) $u(x) = x + \int_0^1 (x-t)\, u(t)\, dt$

Sol. The above equation is Fredholm integral equation because limits of integration are not constant and equation is non-homogeneous because $f(x) = x$.

(iii) $u(x) = \int_0^x (x-t)\, u(t)\, dt$

Sol. The above equation is Volteera integral equation because limits of integration are not constant and equation is homogeneous because $f(x) = 0$.

(iv) $u(x) = x - \int_0^1 (x-t)\, u(t)\, dt$

Sol. The above equation is Fredholm integral equation because limits of integration are constant and equation is non-homogeneous because $f(x) = x$.

Exercise 2.2: For each of the following integro-differential equatio[ns] classify as Fredholm, Volteera or Volteera-fredholm integro-differen[tial] equation:

(i) $u'(x) = 1 + \int_0^x x\, u(t)\, dt \quad ; \quad u(0) = 0$

Sol. This is Volteera integro-differential equation because limits of

$f(x) = 1$
$\lambda = 1$
$k(x,t) =$

integration are not constant. Ans.

$g(x) = 0$
$h(x) = x$

(ii) $u'(x) = x + \int_0^1 (1+x-t)\, u(t)\, dt \quad ; \quad u(0) = 1, \ u'(0) = 0$

Sol: This is Fredholm integro-differential equation because the limits of integration are constant.

Ans.

$f(x) = x$
$\lambda = 1$
$K(x,t) = (1+x-t)$
$g(x) = 0$
$h(x) = 1$

(iii) $u''(x) + u(x) = x + \int_0^x t\, u(t)\, dt + \int_0^1 u(t)\, dt \quad ; \quad u(0) = 0, \ u'(0) = 1$

Sol: This is Volterra-fredholm integro-differential equation because limits of integration are not constant / constant.

Ans.

$f(x) = x$
$\lambda = 1$
$K(x,t) = (t, 1)$
$g(x) = (0, 0)$
$h(x) = (x, 1)$

(iv) $u'(x) + u(x) = x + \int_0^1 t\, u(t)\, dt \quad ; \quad u(0) = 1$

Sol. This is Fredholm integro-differential equation because limits of integration are constant.

Ans.

$f(x) = x$
$\lambda = 1$
$K(x,t) = t$
$g(x) = 0$
$h(x) = 1$

(v) $u''(x) = 1 + \int_0^x t\, u(t)\, dt \quad ; \quad u(0) = 0, \ u'(0) = 1$

Sol. This is Volterra integro-differential equation because limits of integration are not constant.

Ans.

$f(x) = 1$
$\lambda = 1$
$K(x,t) = t$
$g(x) = 0$
$h(x) = x$

Exercise 2.3: Classify the following equation as Fredholm or Volterra, linear or non-linear, homogeneous and non-homogeneous:

(i) $u(x) = 1 + \int_0 (x-t)^2 u(t)\, dt$

Sol. This is Volterra integral equation because limits of integration are not constant. equation is linear because exponent of unknown function inside the integral is one. equation is non-homogeneous because $f(x) = 1$.

Ans.

$f(x) = 1$
$\lambda = 1$
$K(x,t) = (x-t)^2$
$g(x) = 0$
$h(x) = x$

(ii) $u(x) = \cos x + \int_0 (x-t)\, u(t)\, dt$

Sol. This is Fredholm integral equation because limits of integration are constant, equation is linear because exponent of the unknown function inside the integral is one and equation is non-homogeneous because $f(x) = \cos x$.

Ans.

$f(x) = \cos x$
$\lambda = 1$
$K(x,t) = (x-t)$
$g(x) = 0$
$h(x) = 1$

(iii) $u(x) = \int_0 (2+x-t)\, u(t)\, dt$

Sol. This is Volterra integral equation because limits of integration are not constant, equation is linear because exponent of the unknown function inside the integral is one, equation is homogeneous because $f(x) = 0$.

Ans.

$f(x) = 0$
$\lambda = 1$
$K(x,t) = (2+x-t)$
$g(x) = 0$
$h(x) = x$

(iv) $u(x) = \lambda \int_0 t^2\, u(t)\, dt$

Sol: This is Fredholm integral equation because limits of integration are constant, equation is non-linear because dependent variable is multiply with unknown function

$f(x) = 0$
$\lambda = \lambda$
$K(x,t) = t^2$
$g(x) = 0$
$h(x) = 1$

inside the integral, equation is homogeneous because $f(x)=0$.

Ans.

(v) $u(x) = 1 + \int_0^1 (x-t)\, u(t)\, dt \; ; \; u(0)=1$

Sol: This is Fredholm integral equation because limits of integration are constant. equation is linear because exponent of the unknown function inside the integral is one. equation is non-homogeneous because $f(x)=1$.

Ans.

$f(x)=1$
$\lambda=1$
$K(x,t)=x-t$
$g(x)=0$
$h(x)=1$

(vi) $u(x) = 1 + \int_0^1 u^3(t)\, dt$

Sol: This is Fredholm integral equation because limits of integration are constant, equation is non-linear because exponent of the unknown function inside the integral is not one, equation is non-homogeneous because $f(x)=1$

Ans.

$f(x)=1$
$\lambda=1$
$K(x,t)=1$
$g(x)=0$
$h(x)=1$

(vii) $u(x) = \int_0^x (x-t)\, u(t)\, dt \; ; \; u(0)=0$

Sol. This is Volterra integral equation because limits of integration are not constant, equation is linear because exponent of the unknown function inside the integral is one, equation is homogeneous because $f(x)=0$.

Ans.

$f(x)=0$
$\lambda=1$
$K(x,t)=x-t$
$g(x)=0$
$h(x)=x$