$$u(x) = f(x) + \lambda \int_{g(x)}^{h(x)} K(x,t)\, u(t)\, dt \quad \text{--- (2)}$$

is called singular if the limits of integrations are infinite.

- Fredholm integral equation of 1^{st} and 2^{nd} kind

$$u(x) = \lambda \int_{g(x)}^{h(x)} K(x,t)\, u(t)\, dt \quad \text{--- (3)}$$

or

$$u(x) = f(x) + \lambda \int_{g(x)}^{h(x)} K(x,t)\, u(t)\, dt \quad \text{--- (4)}$$

is called singular if limit of integrations are infinite.

(Exercise 2.1)

Problems

Question # 1: For each of the following integral equation, classify as Fredholm, volteera, Volteera - fredholm, 1^{st} and 2^{nd} kind also classify the equation is singular or not.

(i) $u(t) = 1 + \int_0^x u(t)\, dt$

Sol: This is volteera integral equation, because of the limit of integration is not constant, it is 2^{nd} Kind because the unknown function appear inside/outside the integral, it is not homogeneous because of 1, it is not singular since neither limits of integration are infinite.

$f(x) = 1$
$\lambda = 1$
$K(x,t) = 1$
$g(x) = 0$
$h(x) = x$

Ans.

(ii) $u(x) = \int (1+x-t)\,u(t)\,dt$

Sol: This is volterra integral equation because of the limit of integration is not constant, it is 1st kind because the unknown function appears inside the integral, it is homogeneous because $f(x)=0$, it is not singular since neither limits of integrations are infinite.

Ans.

$f(x)=0$
$\lambda=1$
$K(x,t)=1+x-t$
$g(x)=0$
$h(x)=x$

(iii) $u(x) = e^x + e - 1 - \int_0 u(t)\,dt$

Sol: This is fredholm integral equation because of the limits of integration are constant, it is 2nd kind because the unknown function appear inside / outside the integral, it is non-homogeneous because of the $f(x) = e^x+e-1$, it is not singular since neither limits of integration are infinite. Ans.

$f(x)=e^x+e-1$
$\lambda=-1$
$K(x,t)=1$
$g(x)=0$
$h(x)=1$

(iv) $x+1-\frac{\pi}{2} = \int_0^{\pi/2} (x-t)\,u(t)\,dt$

Sol: This is fredholm integral equation because of the limit of integrations are constant, it is 1st kind because the un-known function appears inside the integral, it is homogeneous because $f(x)=0$, it is not singular since neither limits of integration are infinite. Ans.

$f(x)=0$
$\lambda=1$
$K(x,t)=x-t$
$g(x)=0$
$h(x)=\pi/2$

(7)

(v) $u(x) = \frac{3}{2}x - \frac{1}{3} - \int_0^1 (x-t)\, u(t)\, dt$

Sol: This is fredholm integral equation because of the limits of integration are constant, it is 2nd kind because the unknown function appears inside / outside the integral, it is non-homogeneous because $f(x) = \frac{3}{2}x - \frac{1}{3}$, it is not singular since neither limits of integration are infinite. Ans.

$f(x) = \frac{3}{2}x - \frac{1}{3}$
$\lambda = -1$
$k(x,t) = x-t$
$g(x) = 0$
$h(x) = 1$

(vi) $u(x) = x + \frac{1}{6}x^3 - \int_0^x (x-t)\, u(t)\, dt$

Sol: This is volteera integral equation because of the limits of integration are not constant, it is 2nd kind because the unknown function appears inside / outside the integral, it is non-homogeneous because $f(x) = x + \frac{1}{6}x^3$, it is not singular since neither limits of integration are infinite. Ans.

$f(x) = x + \frac{1}{6}x^3$
$\lambda = -1$
$k(x,t) = x-t$
$g(x) = 0$
$h(x) = x$

(vii) $\frac{1}{6}x^3 = \int_0^x (x-t)\, u(t)\, dt$

Sol: This is volteera integral equation because of the limits of integration are not constant, it is 1st kind because the unknown function appears inside the integral, it is homogeneous because $f(x) = 0$, it is not singular since neither limits of integration are infinite. Ans.

$f(x) = 0$
$\lambda = 1$
$k(x,t) = x-t$
$g(x) = 0$
$h(x) = x$

(viii) $\frac{1}{2}x^2 - \frac{2}{3}x + \frac{1}{1} = \int_0^1 (x-t)\,u(t)\,dt$

Sol: This is fredholm integral equation because of the limits of integration are constant, it is 1^{st} kind because the unknown function appears under the integral, it is homogeneous because $f(x)=0$, it is not singular since neither limits of integration are infinite. Ans

$f(x)=0$
$\lambda=1$
$K(x,t)=x-t$
$g(x)=0$
$h(x)=1$

(ix) $u(x) = \frac{3}{2}x + \frac{1}{6}x^2 - \int_0^x (x-t)\,u(t)\,dt - \int_0^1 x\,u(t)\,dt$

Sol: This is volterra-fredholm integral equation because of the limits of integration are not constant | constant, it is 2^{nd} kind because unknown function appears inside/outside the integral, it is non-homogeneous because $f(x) = \frac{3}{2}x + \frac{1}{6}x^2$, it is not singular since neither the limits of integration are infinite. Ans.

$f(x)=\frac{3}{2}x+\frac{1}{6}x^2$
$\lambda=-1$
$K(x,t)=(0,t)$
$g(x)=0,0$
$h(x)=x,1$

(X) $u(x,t) = x + t^3 + \frac{1}{2}t^2 - \frac{1}{2}t - \int_0^t \int_0^1 (\tau - \xi)\,d(\xi)\,dt$

Sol: This is volterra-fredholm integral equation because of the limits of integration are not constant | constant, it is 2^{nd} kind because unknown function appear inside/outside the integral, it is non-homogeneous because $f(x) = x + t^3 + \frac{1}{2}t^2 - \frac{1}{2}t$, it is not singular since neither

$f(x)=x+t^3+\frac{1}{2}t^2-\frac{1}{2}t$
$\lambda=-1$
$K(x,t)=(\tau-\xi)$
$g(x)=(0,0)$
$h(x)=(t,1)$

the limits of integration are infinite. Ans.

(ii) $u(x) = 1 + x^2 - \int_0^1 \frac{1}{\sqrt{1-xt}} u(t)\,dt$

Sol: This is fredholm integral equation because of the limits of integration are constant, it is 2^{nd} kind because the unknown function appears inside/outside the integral, it is non-homogeneous because $f(x) = 1+x^2$, it is not singular since neither the limits of integration are infinite. Ans.

$f(x) = 1+x^2$
$\lambda = -1$
$K(x,t) = \frac{1}{\sqrt{1-xt}}$
$g(x) = 0$
$h(x) = 1$

Lecture #2 (Monday) 20-01-2020

Linearity (linear or non-linear) & homogeneous: Integral equation and integral-differential equation has two types:

1) Linearity 2) Homogeneous

1) Linearity: If the exponent of unknown function $u(x)$ inside the integral sign is one, the integral equation or integral differential equation is called linear

Example:

(i) $u(x) = 1 - \int_0^x (x-t)\,u(t)\,dt$

Sol: The above equation is volterra integral equation and linear