SPECIAL FUNCTION

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Topic: Legendre Generating Function

Properties of Legendre Generating Function

- (1) $(1-2xt+t^2)^{-1/2} \to 1, \qquad if\ t \to 0.$
- (2) $P_n(x) = 1.$
- Sol:
- $(1-2xt+t^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)t^n$
- If x =1
- $(1-2t+t^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(1)t^n$
- $[(1-t)^2]^{-1/2} = \sum_{n=0}^{\infty} P_n(1)t^n$

$(1-t)^{-1} = \sum_{n=0}^{\infty} P_n(1)t^n$

$$\sum_{n=0}^{\infty} \frac{(1)_n t^n}{n!} = \sum_{n=0}^{\infty} P_n(1)t^n$$

$P_n(1) = 1$

(3) If we replace x by –x and t by –t in eq (i)

$$(1-2xt+t^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} P_n(-x)(-t)^n$$

Eq (1) becomes

$$\sum_{n=0}^{\infty} P_n(x)t^n = \sum_{n=0}^{\infty} (-1)^n P_n(-x)t^n$$

comparing

$$p_n(x) = (-1)^n p_n(-x)$$

(4) Since

$(1 - 2xt + t^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)t^n$

at x =0

$\sum_{n=0}^{\infty} P_n(x)t^n = (1 + t^2)^{-1/2}$

$\sum_{n=0}^{\infty} P_n(0)t^n = \sum_{n=0}^{\infty} \frac{(-1)^n(1\backslash 2)_n t^{2n}}{n!}$

$$\sum_{n=0}^{\infty} P_{2n}(0)t^{n2} = \sum_{n=0}^{\infty} \frac{(-1)^n(1\backslash 2)_n t^{2n}}{n!}$$

$P_{2n}(0) = \frac{(-1)^n(1\backslash 2)_n}{n!}$