2 Physical properties of magma

2.1 INTRODUCTION

Igneous rocks are formed from molten material known as magma, which usually consists of a solution of the Earth's most abundant elements: oxygen and silicon with smaller amounts of aluminum, calcium, magnesium, iron, sodium, and potassium. Robert Bunsen (1851), of "Bunsen burner" fame, was the first to recognize that magmas were actually "solutions" no different from those of salts in water, except that they were hotter. Most magmas are silicate melts, but rarer ones contain little or no silica and, instead, are composed essentially of calcium carbonate, sulfide, or iron oxide. The major elements of common magmas combine, on cooling, to form the so-called *rock-forming minerals*: quartz, feldspars, feldspathoids, pyroxenes, olivine, and, when water is present, amphiboles, and mica. In addition, common *accessory minerals*, such as iron–titanium oxides, apatite, zircon, and sulfides, form from minor magmatic constituents (Fe^{3+}–Ti, P, Zr, and S, respectively) that do not readily enter the structures of the major rock-forming minerals.

The history of an igneous rock begins with the formation of magma at some depth in the Earth. The composition of the magma is determined by the chemical and mineralogical composition of the rock in the source region and by the process of melting. When sufficient melt has formed and coalesced, buoyancy causes it to rise. Further chemical modifications may occur during this period of transport. The magma may rise to the surface and extrude as lava or, if it contains volatiles, may explode onto the surface to be widely distributed as volcanic ash. Most magma, however, solidifies beneath the Earth's surface, where slow cooling and crystallization allow for further modification of the initial composition. During this final period, a rock develops its characteristic appearance, which we refer to as *texture*.

A discussion of igneous rocks could begin with a treatment of the initial melting processes in the source region, but this is a theoretical topic and is better left to a later chapter (Chapter 23). Instead, we shall begin with a more tangible subject, the physical properties of magma. It is important to have an accurate conception of the physical nature of magma before considering its emplacement and crystallization. Fortunately, active volcanoes provide opportunities to witness first hand the eruption of magma onto the Earth's surface. Here, such important properties as magmatic temperature, density, and viscosity can be measured. These will be discussed first. Other physical properties, such as

heat capacity, thermal conductivity, and compressibility, can be determined only through sophisticated laboratory measurements. These properties will be touched upon in later chapters.

2.2 MAGMATIC TEMPERATURES

During periods of volcanic eruption, magmatic temperatures can be measured directly with optical pyrometers and thermocouples. Many eruptions, however, are too violent for this to be done safely. In these cases, magmatic temperatures can be determined by heating samples of the erupted rock in the laboratory and measuring their melting points. Such experiments were first done in 1790 by the Scottish geologist Sir James Hall (Fig. 2.1). He withheld publication of his results (Hall, 1805) until after the death of his distinguished friend James Hutton in 1797, who, while teaching that nature could be discovered only through observation, considered it rash "to judge of the great operations of the mineral kingdom from having kindled a fire and looked into the bottom of a little crucible" (www.lhl.lib.mo.us/events_exhib/exhibit/exhibits/vulcan/56.shtml). Although such distrust of experimental petrology did not die with Hutton, this approach is accepted today. Modern techniques permit accurate measurements to be made of melting temperatures of rocks under compositionally controlled atmospheres and pressures. Many such determinations have been made on a wide variety of rocks, and most agree well with direct measurements of magmatic temperatures in the field where such measurements can be made.

In discussing magmatic temperatures, it is important to keep in mind that a rock, being a mixture of minerals, does not have a single melting temperature. Instead, it melts over a temperature range that is commonly several hundred degrees. In addition, this melting range gives only the minimum temperature necessary to have liquid present; magmas could have temperatures well above this. However, one line of evidence indicates that magmas rarely exceed this temperature range. Magma that cools rapidly by coming in contact with cold rock or by being extruded on the Earth's surface crystallizes to a fine-grained aggregate of minerals or is quenched to a glass. Suspended in this material are almost always larger crystals known as *phenocrysts* (Fig. 2.2(B)). Because these crystals were growing prior to the quenching of the magma, their presence indicates that
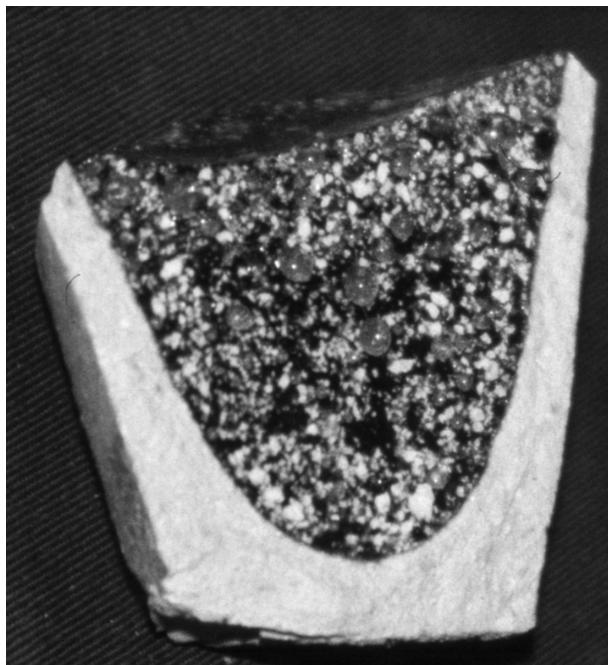
Fig. 2.1 One of James Hall's early experiments from which he tried "to judge of the great operations of the mineral kingdom from having kindled a fire and looked into the bottom of a little crucible." This section shows melted basalt.

Fig. 2.2 The Kilauea Iki crater, Hawaii, was filled with a 100-m deep lava lake by an eruption in 1959. A crust formed on the lava lake within hours. As soon as the crust was strong enough to support the weight of a helicopter and a drill rig, the U.S. Geological Survey started a drilling program to monitor the solidification of the lava lake (see Fig. 2.3). (**A**) Panorama of crater in 2004. The vent from which the eruption occurred is in the right foreground. Inset shows steam rising from one of the drill holes in the lava lake in 1985. A thermocouple is attached to the white cable to measure the temperature in the lava lake. (**B**) Photomicrograph of a 1988 sample of drill core from 77 m below the surface of the lava lake showing large clear phenocrysts of olivine, in a groundmass of intermediate colored pyroxene, opaque plates of ilmenite, clear laths of plagioclase, and interstitial liquid that was quenched to a brown glass during the 1988 drilling. Width of field is 6.7 mm. Plane polarized light.

most magmas have temperatures between the beginning and end of the melting interval. This observation is extremely important, as emphasized by Bowen (1928), for it indicates that magmas are rarely superheated. Most magma is a mixture of crystals and liquid. If the proportion of crystals is small, we describe the magma as having the properties of a suspension, which are predominantly those of a liquid. If the proportion of crystals is large enough that the crystals touch one another and form a three-dimensional network, we describe the magma as a mush, which has properties closer to those of a solid than to a liquid yet is still capable of moving. All magmas pass through the mush stage during cooling.

Extrusion temperatures for basalt, the type of lava commonly erupted from the Hawaiian volcanoes, commonly range from 1100 to 1200 °C. Higher temperatures do occur, but this results from the oxidation of magmatic gases. Most rhyolitic lava, the volcanic equivalent of granite, has temperatures between 800 and 1000 °C. Increased pressure causes melting temperatures to rise, but if water is present, the increased pressure allows water to dissolve in the magma, which in turn significantly lowers the melting temperatures. Basalt and rhyolite can both have their beginning of melting temperatures lowered by several hundred degrees by the addition of water.

Radiation cooling of the surface of lava flows causes solid crusts to form rapidly. The rate of conductive heat transfer through magma and solid crust is, however, so slow that the interior of a flow may remain hot for long periods. For example, in 1959, the eruption of Kilauea Iki
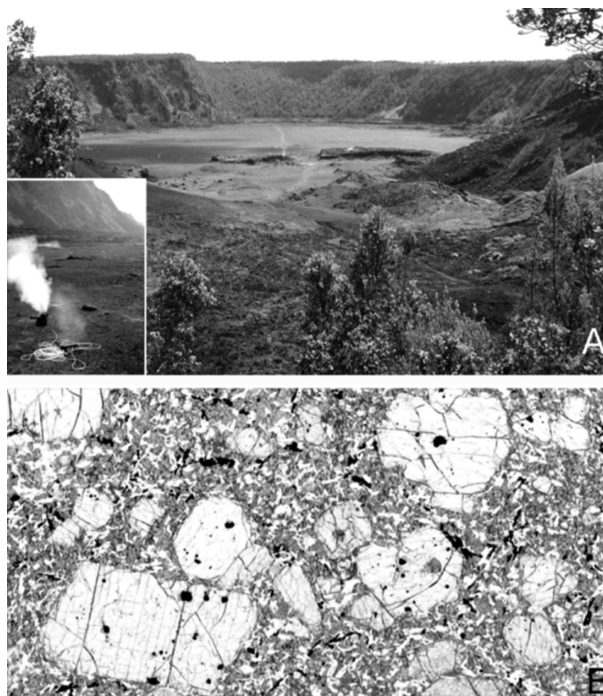
on the Big Island of Hawaii produced a 100-m-deep lava lake on which a crust developed almost immediately (within hours). Over the next 20 years, the U.S. Geological Survey repeatedly drilled into the lava lake to study its solidification (Fig. 2.2(A)). The crust thickened with time (Fig. 2.3), but three years later, the base of the crust, which was at a temperature of 1065 °C, had grown to a depth of only 14 m. Even 19 years later, some liquid still remained amongst the olivine, plagioclase, and pyroxene crystals at depths of 75 to 85 m in the lake (Fig. 2.2(B)). Conductive cooling of magma is obviously a slow process in the lava lake, and in bodies of magma cooling at depth in the Earth, it would be still slower.

Several factors determine the cooling rate of magma, the most important of which are thermal gradients, rate of heat diffusion through magma and rock, heat capacity of magma and surrounding rocks, and the shape of magmatic bodies. A discussion of the relations between these factors is deferred to Chapter 5 to allow us first to consider the forms and mechanisms of emplacement of igneous bodies.
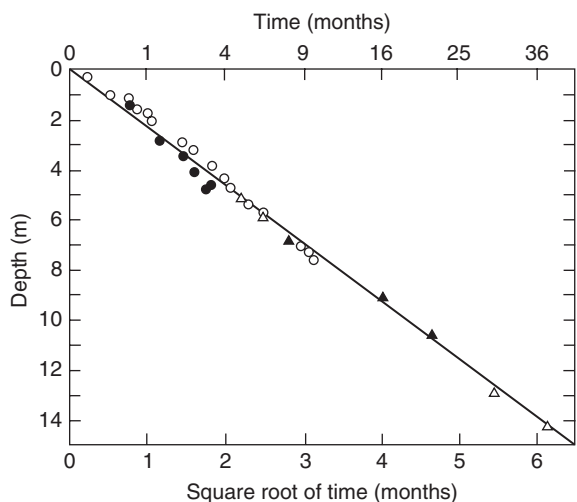
Time (months)

Fig. 2.3 Thickness of crust in the Alae and Kilauea Iki lava lakes, Hawaii. The base of the crust was defined as that depth at which the drill sank into the liquid. Solid circles represent the base of the crust, open circles the 1067 °C isotherm at Alae Crater and solid triangles the base of the crust, and open triangles the 1065 °C isotherm at Kilauea Iki Crater. The base of the crust was at approximately 85 °C below the initial extrusion temperature. The linear relation between the rate of crustal thickening and the square root of the length of time of cooling is discussed in Chapter 5 (after Peck *et al.*, 1964).

2.3 MAGMA DENSITIES

Many important petrologic processes are controlled by the density of magma. For example, buoyancy largely determines the rise of magma from a source region. Indeed, magmas that ascend through the lithosphere may have a limited compositional range that is determined by magma density; the lithosphere may act effectively as a density filter. Crystals may float or sink in a magma, and by so doing they can change the composition of the remaining magma (differentiation); this process is clearly determined by contrasts in crystal and liquid densities. Where magma chambers are periodically replenished with surges of fresh magma, as for example beneath oceanic ridges, mixing of magma may occur if magma densities are closely matched, but if densities are different, the magmas remain separate. Magma mixing, which plays an important role in generating certain rock types, some of which are of economic significance, is thus controlled, in part, by magma densities.

The density of magma can be determined in several ways. First, it can be measured directly at high temperatures. This is technically difficult; so, instead, density measurements are often made at room temperature on glasses formed from the rapid quenching of these liquids. Because glass is a supercooled liquid, the high-temperature density can be calculated from the room temperature measurements if the coefficient of thermal expansion is known. If V_1 is a volume at T_1, the temperature at which the density measurement is made, the volume, after heating to temperature T, is given by

$$V_T = V_1[1 + \alpha(T - T_1)] \tag{2.1}$$

Table 2.1 *Partial molar volumes and coefficients of thermal expansion of common oxide components of silicate liquids containing 40 to 80 mol% SiO_2 at 1400 °C*

Oxide	Molar volume $10^6 \times V_i$ (m^3 mol^{-1})	Coefficient of expansion $10^5 \times \alpha$ (°C^{-1})^a
SiO_2	26.75	0.1
TiO_2	22.45	37.1
Al_2O_3	37.80	2.6
Fe_2O_3	44.40	32.1
FeO	13.94	34.7
MgO	12.32	12.2
CaO	16.59	16.7
Na_2O	29.03	25.9
K_2O	46.30	35.9

^{a} Values in this column have been multiplied by 10^5. For example, the coefficient of expansion of SiO_2 is 0.1×10^{-5} °C^{-1} and its molar volume is 26.75×10^{-6} m^3 mol^{-1}.
From Bottinga and Weill (1970) and Bottinga *et al.* (1982).

where α, the coefficient of thermal expansion at constant pressure, is defined as

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \tag{2.2}$$

The *coefficient of thermal expansion* is the relative increase in volume *($\partial V/V$)* per unit increase in temperature at constant pressure (Table 2.1). The partial derivative $(\partial V/\partial T)_P$ is used instead of the ordinary derivative to show that this relation is restricted to the change at constant pressure.

Because density is mass per unit volume, Eq. (2.1) can be rewritten as

$$\rho_T = \frac{\rho_1}{1 + \alpha(T - T_1)}$$

where ρ_1 and ρ_T are the densities at temperatures T_1 and T, respectively. Multiplying the top and bottom of the right-hand side of this equation by $[1 - \alpha(T - T_1)]$ gives

$$\rho_T = \frac{\rho_1[1 - \alpha(T - T_1)]}{1 - [\alpha(T - T_1)]^2}$$

Because α is very small (about 10^{-5} °C^{-1}), the squared term is negligible, and the denominator is essentially one. Hence, the density at any temperature T is related to the density at a temperature T_1, by

$$\rho_T = \rho_1[1 - \alpha(T - T_1)] \tag{2.3}$$

Another method of determining magma density uses a chemical analysis of the magma, and determines how much each constituent contributes to the total density by using what are known as the *partial molar volumes* of each constituent. These were determined by Bottinga and Weill (1970) and Bottinga and others (1982) from the analysis of density measurements of silicate liquids (Problem 2.1).

In any solution, such as a magma, the partial molar volume of a component i, denoted by the symbol $\bar{V}_i$, is defined as the change in volume resulting from a change in the number of moles of component i (n_i) at constant temperature, pressure, and numbers of moles of all other components. This is expressed mathematically as

$$\bar{V}_i \equiv \left(\frac{\partial V}{\partial n_i} \right)_{T,P,n_{j \neq i}} \tag{2.4}$$

The partial derivative is used to show that this expression is restricted to the volume change at constant temperature, pressure, and number of moles of all constituents except component i. The partial molar volumes of the common oxide constituents of magma are given in Table 2.1 for 1400 °C. To obtain volumes at other temperatures, the coefficient of thermal expansion is used along with Eq. (2.2).

The volume of a magma consisting of many components is given by

$$V = \bar{V}_a n_a + \bar{V}_b n_b + \cdots + \bar{V}_i n_i = \sum_i \bar{V}_i n_i \tag{2.5}$$

Thus, the partial molar volume of a component is that quantity which, when multiplied by the number of moles of the component, gives the contribution of the component to the total volume.

Compositions are often more conveniently expressed in terms of mole fractions than as numbers of moles of constituents. The *mole fraction* of a component i, denoted by X_i, is simply $n_i/(n_a + n_b + \cdots + n_i)$. Consequently, the number of moles of each component in Eq. (2.5) can be converted to mole fractions by dividing both sides of the equation by $(n_a + n_b + \cdots + n_i)$, giving

$$\frac{V}{n_a + n_b + \cdots + n_i} = \bar{V}_a X_a + \bar{V}_b X_b + \cdots + \bar{V}_i X_i$$

The volume of magma per total number of moles of all components is referred to as the *molar volume* and is denoted by $\bar{V}$. Hence,

$$\bar{V} = \sum_i \bar{V}_i X_i \tag{2.6}$$

The density of a magma is obtained by dividing this molar volume into the molecular weight of the magma, which is given by $\sum_i M_i X_i$, where M_i is the molecular weight of component i. Thus

$$\rho = \frac{\sum_i M_i X_i}{\sum_i \bar{V}_i X_i} \tag{2.7}$$

Another way of calculating the density of magma from a chemical analysis is by using the computer programs MELTS (Ghiorso and Sack, 1995; Asimow and Ghiorso, 1998) for low pressure, and pMELTS (Ghiorso *et al*. 2002) for higher pressures between 1 and 3 GPa. These programs, which can be downloaded from http://melts.ofm-research.org, use an internally consistent thermodynamic data set that was calibrated

with a large number of careful experimental results for a wide compositional range of liquids coexisting with common rock-forming minerals. These programs will be discussed in more detail in Chapter 8, but for the moment, these data provide the molar volume of the liquid from which the density is calculated. After a liquid's composition is entered as oxides and the desired temperature, pressure, and oxygen fugacity (approximately the partial pressure of oxygen) selected, the programs return the properties of the liquid, including density, under the selected conditions (Problem 2.4).

A final method of obtaining the approximate density of magma is based on the fact that the volume expansion on melting for most rocks is ~10%. The density of most magmas, therefore, is 90% of the density of the equivalent solid rock.

The densities of common magmas near the surface of the Earth vary from 2.3 to 3.0 Mg m^{-3}, with the more iron-rich varieties having the higher values. Most basaltic magmas have densities of $2.60 - 2.65$ Mg m^{-3}, which is similar to that of plagioclase feldspar ($2.63 - 2.76$ Mg m^{-3}) but less than that of pyroxene and olivine ($3.0 - 3.7$ Mg m^{-3}). Most granitic magmas have a density of ~2.4 Mg m^{-3} and are therefore less dense than any silicates (Problem 2.3).

Because most magmas are mixtures of liquid and crystals, their bulk density will be greater than if they consisted entirely of liquid. To determine the bulk density of a magma it is necessary to know the density and proportions of the liquid fraction and of each of the solid phases present. The MELTS program provides you with all of these values and the calculated bulk density.

Under the higher pressures at depth in the Earth, magmas will be compressed and hence have a higher density. To determine the magnitude of this effect we make use of the *isothermal coefficient of compressibility*, which is denoted by β. This coefficient gives the relative decrease in volume $(-\partial V/V)_T$ per unit increase in pressure at constant temperature. This is expressed mathematically as

$$\beta \equiv -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T \tag{2.8}$$

The inverse of this coefficient is known as the *bulk modulus*, which has units of Pa. Typical β values for magmas are of the order of 5×10^{-11} Pa^{-1} (Bass, 1995), which translates to a density change of only ~0.13 Mg m^{-3} per GPa. For example, experiments show that the density of a Kilauea basaltic magma decreases from 2.743 to 2.610 Mg m^{-3} in rising from a depth of 35 km (Kushiro, 1980). If increased pressures result in higher dissolved water contents, the resulting change in composition results in significantly lower magma densities (Bottinga and Weill, 1970).

Although magmas have low values of β, they are more compressible than solids, and under the extremely high pressures in the mantle their densities increase and may even become greater than that of the solids from which they are formed. Experiments indicate that at pressures greater than 13.5 GPa (equivalent to a depth of 400 km) melts of "basaltic" composition formed from the partial melting of peridotite

have greater densities than do the olivine crystals in the peridotite (Agee and Walker, 1993; Suzuki and Ohtani, 2003). At shallower depths, basaltic magma would rise relative to olivine, but below this depth it would sink, which raises the interesting possibility that magma might sink to the base of the lower mantle and become part of the D″ layer (Section 1.2; Ohtani and Maeda, 2001).

2.4 MAGMA VISCOSITIES

Some lavas are able to flow rapidly over great distances, whereas others barely move, even when erupted on steep slopes. Viscosity is the physical property that describes this resistance to flow (Fig. 2.4). Viscosity is extremely important in determining the rates of emplacement of magma and the shapes of igneous bodies, and in determining whether sinking or floating crystals will separate fast enough from their parent magma to change the magma's bulk composition (Dingwell, 2006).

Like any fluid, magma deforms continuously under the action of a shear stress, but the deformation may be very slow in many magmas. When a fluid is at rest, there should be no shear stresses (see yield strength below). Fluids react to a shear stress by flowing, but some do this more slowly than do others. The property of a fluid that defines the rate at which deformation takes place when a shear stress is applied is known as *viscosity*:

$$\text{viscosity} \equiv \frac{\text{shear stress}}{\text{rate of shear strain}}$$

If a large shear stress applied to a liquid results in slow deformation, the liquid is said to have a high viscosity. Tar and honey are familiar examples of highly viscous liquids, whereas water and gasoline have low viscosities (Table 2.2).

As the magnitude of a shear stress applied to a liquid increases, the rate of strain also increases. For many liquids, a linear relation exists between the applied shear stress and the strain rate (Fig. 2.5). These liquids are referred to as being *Newtonian*. From the definition of viscosity, the slope in Figure 2.5 is clearly the viscosity. *Non-Newtonian* liquids exhibit a nonlinear relation between applied shear stress and strain rate; their viscosities (slopes) vary with shear stress. Most magmas, at least when totally liquid and deformed at a

low strain rate, show near Newtonian behavior. As the percentage of crystals in a magma increases, the bulk viscosity of the mixture also increases. If the fraction of crystals present in the magma is F, the effective bulk viscosity of the magma, η_B (eta), is related to the viscosity of the liquid, η_L, by

$$\eta_B = \eta_L(1 - 1.65F)^{-2.5} \tag{2.9}$$

It will be seen that as F approaches 60%, the bulk viscosity approaches infinity. This is known as the *Einstein limit* in recognition of his work in this field. As a magma approaches this limit, it would behave essentially as a solid, despite the presence of 40% liquid.

Not only do crystals, or bubbles, in a magma increase viscosity, they also commonly cause the magma to exhibit non-Newtonian behavior of the type exhibited by *Bingham liquids* (Shaw, 1969; Petford, 2003). These liquids resist deformation until some minimum shear stress (*yield strength*) is applied, after which strain rates increase linearly with shear stress (Fig. 2.5). Thus, at low shear stresses, they appear to have infinite viscosity, but above the yield strength, they behave as Newtonian liquids. A common example of such a liquid is household paint. When one applies a thin coat of

Table 2.2 *Viscosities of magmas and common substances*

Material	Viscosity (Pa s)	Wt% SiO_2	Temp. (°C)
Water	1.002×10^{-3}	–	20
ASE 30 motor oil	2×10^{-1}	–	20
Kimberlite	10^{-1}–1	30–35	~1000
Komatiite	10^{-1}–10	40–45	1400
Ketchup	$\sim 5 \times 10$	–	20
Basalt	10–10^2	45–52	1200
Peanut butter	$\sim 2.5 \times 10^2$	–	20
Crisco® shortening	2×10^3	–	20
Andesite	$\sim 3.5 \times 10^3$	~58–62	1200
Silly Putty®	$\sim 10^4$		
Tonalite 6% H_2O	$\sim 10^4$	65	950
Rhyolite	$\sim 10^5$	~73–77	1200
Granite 6% H_2O	$\sim 10^5$	75	750
Rhyolite	$\sim 10^8$	~73–77	800
Average mantle	10^{21}	–	–

Magma viscosities from Dingwell (1995) and references therein. Granite and tonalite viscosities from Petford (2003). Mantle viscosity is from King (1995).

Fig. 2.4 (**A**) Basaltic spatter frozen while flowing back into the 1984 fissure vent at the Krafla volcano, Iceland. Basaltic magma is the most fluid of the common magmas, yet from the shape of these dribbles, it can be seen to be a viscous liquid. (**B**) For comparison, ketchup, which has approximately the same viscosity as basalt (Table 2.2), is shown dripping from a bottle.
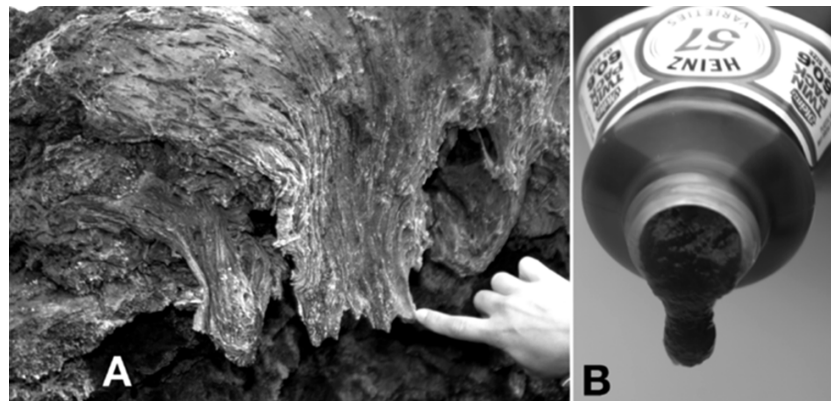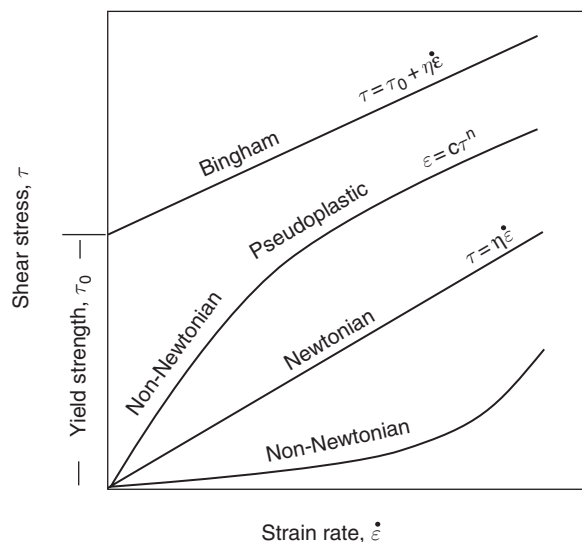
Fig. 2.5 Shear stress versus rate of shear strain for Newtonian and non-Newtonian liquids. The slope of these lines gives the viscosity of the liquid, which for a Newtonian liquid remains constant regardless of shear stress. The viscosity of non-Newtonian liquids varies with the applied shear stress.

Fig. 2.6 Laminar flow of liquid between two parallel plates resulting from the upper plate moving in the x direction with a velocity v_0 relative to the stationary lower plate when a shear stress, v, is applied. Note that the positive direction of z is upward (height).

paint to a vertical wall, the paint does not dribble (hopefully), because the weight of the paint is insufficient to overcome the paint's yield strength, which is commonly due to the presence of a fine suspension of mica crystals. However, if too thick a coat is applied, the yield strength is exceeded, and the paint begins to flow and dribble. A good paint that gives "no drips, no runs, and no errors" has a significant yield strength. Bingham behavior has profound effects on velocity profiles across bodies of flowing magma (Section 3.7) and on the ability of crystals to move buoyantly through magma (Section 14.3). Even when magmas do not contain crystals, Newtonian behavior may occur only at low strain rates. At higher strain rates, many magmas exhibit what is known as *shear thinning*; that is, they experience a decrease in viscosity with increasing shear rate (Dingwell, 1995). In this respect, they tend to show pseudoplastic behavior (Fig. 2.5).

Another type of non-Newtonian liquid is one that has low viscosity at low shear stresses (shallow slope in Fig. 2.5) but a high viscosity at high shear stresses. Silly Putty® is a common example of such a liquid. If you gently pull on a piece of Silly Putty, it stretches with almost no resistance, but if you suddenly pull forcefully, it resists and will probably fracture as if it were a solid that has an extremely high viscosity.

Viscosity is better appreciated by considering the following example. A shear stress τ (tau) is applied to the liquid between two parallel plates (Fig. 2.6) by moving the upper one steadily at a velocity v_0 relative to the lower one. Liquid in contact with the upper plate is dragged along at a velocity v_0, whereas that in contact with the lower plate remains stationary. The intervening liquid can be thought of as consisting of a series of thin parallel plates or lamellae, each of which slides over the one beneath. Such movement is referred to as *laminar flow*, in contrast to *turbulent flow*, where both the direction and rate of movement of the liquid experience wide
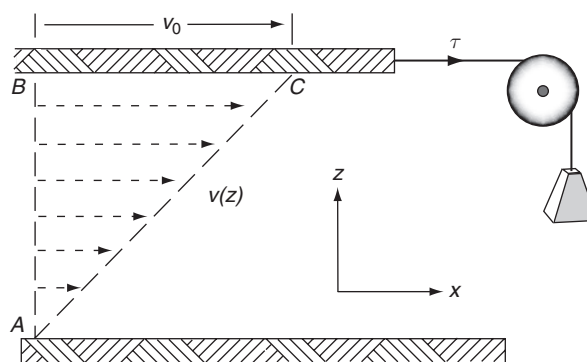
fluctuations. If the length of the arrows in the figure indicates the velocity in the x direction at various heights along the z axis, a marker line AB would move to the position AC after a unit interval of time. The strain rate must therefore be dv/dz, and the viscosity, denoted by η, is given by

$$\eta = \frac{\tau}{dv/dz} \quad \frac{N}{m^2} \frac{s}{m} m = Pa\,s \qquad (2.10)$$

In SI units, where shear stress is measured in newtons m^{-2}, the velocity in $m\,s^{-1}$, and distance in meters, the units of viscosity are Pa s. The cgs unit of viscosity is the *poise* (P), which is a dyne s cm^{-2} (1 P = 0.1 Pa s). In many fluid mechanics calculations, the viscosity is divided by the density of the fluid. This ratio is referred to as the *kinematic viscosity* and is defined as $v = \eta/\rho$ with units of $m^2\,s^{-1}$.

It is now possible to make a quantitative statement about the viscosity of magmas (Table 2.2). Measurements of viscosity have been made in the laboratory at both atmospheric and higher pressures on a variety of liquids formed by melting common igneous rocks. In addition, viscosities have been calculated from the actual measured rates of flow of lava issuing from active volcanoes. Common magmas cover a wide range of viscosities (Dingwell, 1995). At a temperature of 1200 °C and atmospheric pressure, basaltic magmas (gabbroic) have viscosities in the range 10 to 100 Pa s. This is about the same as tomato ketchup (Fig. 2.4), but the pulp in ketchup tends to make it behave as a Bingham liquid, which is why we often need to give an extra tap when pouring it to overcome the yield strength. The volatile-rich silica-poor magma kimberlite, which brings diamonds to the surface from great depth, has a viscosity of slightly less than 1 Pa s at temperatures of approximately 1000 °C. The magnesium rich, silica-poor lava komatiite which erupted only in the Archean had a viscosity of ~1 Pa s at its eruption temperature of ~1400 °C. Andesitic magmas (dioritic) at 1200 °C average about 10^3 Pa s, which is slightly greater than smooth peanut butter and about the same as Crisco® shortening. Most rhyolitic magmas (granitic) have viscosities above 10^5 Pa s and can be as high as 10^8 Pa s at 800 °C, which is a more normal magmatic temperature for rhyolite.

The viscosity of a magma is determined largely by its chemical composition, which controls its degree of polymerization. Just as silicate minerals show different degrees of polymerization, so do silicate melts. Most of the oxygen atoms in silica-rich melts are tetrahedrally (T) coordinated around silicon atoms. These silica-oxygen bonds are strong and give the melt a high viscosity. As the silica content decreases and the abundance of Fe, Mg, Ca, etc. increases so do the number of nonbridging oxygens (NBO). These are weaker bonds than the tetrahedral ones, and they therefore lower the viscosity. The degree of polymerization of a silicate melt is commonly given by the ratio of the average number of nonbridging oxygens to the number of tetrahedrally coordinated cations (NBO/T). Thus, in a melt of albite ($NaAlSi_3O_8$) composition, NBO/T = 0, whereas in a melt of diopside composition ($CaMgSi_2O_6$), NBO/T = 2 (draw a silica chain and check this ratio).

The viscosity of a silicate melt is controlled largely by the abundance of the polymerizing, tetrahedrally coordinated groups, SiO_2, $KAlO_2$, and $NaAlO_2$. The increase in viscosity from komatiite through basalt and andesite to rhyolite is attributable to this chemical variation, with komatiites containing approximately 40–45 wt% SiO_2, basalts ~50 wt% SiO_2, andesites ~60 wt%, and rhyolites ~73 wt%. This chemical control makes it possible to calculate reasonable viscosities for magmas of known composition, in much the same way that densities were calculated from partial molar volumes (Bottinga and Weill, 1972). The MELTS program also calculates the viscosity of melts based on their chemical composition (Problem 2.6).

Temperature has an enormous effect on viscosity, as anyone knows who has tried to start a car on a cold winter's day. The oil in the engine becomes extremely viscous at low temperatures. By heating the engine only slightly, for example by hanging a light bulb under the hood, the viscosity drops remarkably and starting becomes much easier. Viscosity is an exponential function of temperature. The viscosity of rhyolitic magma, for example, is shown in Table 2.2 as changing from 10^5 Pa s at 1200 °C to >10^8 Pa s at 800 °C (Problem 2.6c).

The range of viscosities of magmas becomes even greater when account is taken of the lower temperatures at which rhyolites, for example, crystallize compared with those at which basalt crystallizes. Rhyolitic magma is typically several hundred degrees cooler than basaltic magma and consequently has very much higher viscosity than does basalt. This single fact is largely responsible for the morphological differences between volcanoes and other igneous bodies formed from these different magmas.

The dramatic change in viscosity of magma with temperature generally obeys an Arrhenius relation of the form

$$\eta = \eta_0 e^{E/RT} \tag{2.11}$$

where η_0 is a constant, E the activation energy, R the gas constant, and T the absolute temperature. Such a relation, which describes the temperature dependency of many reaction rates, can be thought of as consisting of two parts; the preexponential factor η_0 expresses the frequency of a particular event involved in the reaction (in this case, perhaps the formation of a hole or the breaking of a bond in the liquid structure), and the exponential term gives the fraction of these events that have sufficient energy to permit the reaction to proceed.

Converting Eq. (2.11) to logarithmic form gives

$$\ln \eta = \ln \eta_0 + \frac{E}{RT}$$

which is the equation of a straight line. A plot of $\ln \eta$ versus $1/T$ for viscosities determined at several temperatures yields the activation energy from the slope of the line and the frequency factor from the intercept (see Problem 2.7). Once the values of η_0 and E are known, the viscosity can be calculated for any temperature.

Pressure has a very much smaller effect on magma viscosity than has temperature. The viscosity of a basaltic magma at 1400 °C, for example, decreases from 3.5 Pa s at atmospheric pressure to 1.5 Pa s at 2.0 GPa (Kushiro, 1980). Indirectly, however, pressure can modify viscosities more dramatically. Higher pressure permits more water to be dissolved in magmas, which depolymerizes the melt and lowers the viscosity (Shaw, 1965). The effect of water (independent of pressure) is demonstrated, for example, in experiments carried out on a melt of albitic composition at 1550 °C and a pressure of 2.5 GPa (Poe *et al.*, 2006). This melt, when dry, has a viscosity of 114 Pa s, but with 2.8 wt% H_2O, its viscosity decreases to 16.4 Pa s. Granitic melts containing ~6 wt% H_2O at a pressure of 0.8 GPa and 750 °C have a viscosity of ~10^5 Pa s (Petford, 2003). For more references to the literature on viscosity see Dingwell (2006).

Unless one is familiar with the numerical values for the viscosity of liquids, numbers such as 10^2 or 10^8 Pa s have little significance. They can, however, be made more meaningful by considering the effect that viscosity has on the flow rate of lava. Let us consider a lava flow (Fig. 2.7) of thickness h composed of an incompressible Newtonian magma with viscosity η and density ρ that flows with a steady rate down an unconfined plane surface with a slope of θ degrees. For convenience we shall choose a set of orthogonal axes, x, y, and z, such that x parallels the flow direction and z is normal to the plane surface. The lava flows in a laminar manner and for the moment we will neglect any effects due to cooling of the lava. Gravitational attraction causes the lava to flow, and if no other forces are present, the lava accelerates to greater and greater velocities. The viscous force, of course, prevents this from happening.

To analyze this situation we consider the forces acting on a very small volume of lava, $dx\,dy\,dz$. The gravitational force acting in the direction of flow is simply the mass times the acceleration, which is given by $\rho\,dx\,dy\,dz\,g\sin\theta$, where g is the acceleration of gravity. However, the shear stress in the liquid retards the flow. Because the shear stress varies with height in the flow, there is a difference in shear stress between the top and bottom of the volume, which we will designate $d\tau$. Because this stress (force/unit area) acts on an area $d\ \ d\ $, the

Fig. 2.7 Forces acting on a small element (dx dy dz) of lava within a flow of thickness h, descending a plane surface with a slope of θ degrees.
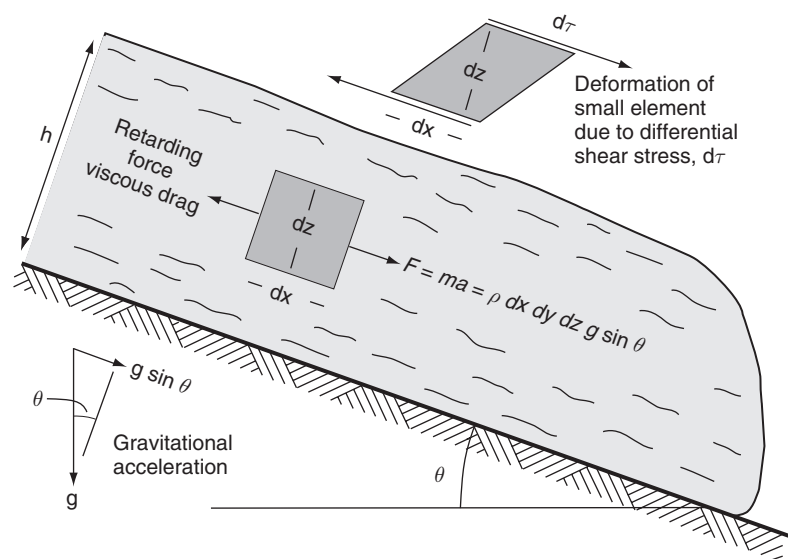
force retarding the flow is dτ dx dy. Because the flow rate of the lava is steady, there is no acceleration. Consequently, the sum of the viscous and gravitational forces acting in the plane of flow must be zero; that is,

$$d\tau\, dx\, dy + \rho\, dx\, dy\, dz\, g\, \sin\theta = 0$$

Dividing both sides of the equation by dx and dy and rearranging gives

$$d\tau = -\rho g\, \sin\theta\, dz$$

which, upon integration, becomes

$$\tau = -\rho g\, \sin\theta z + \text{constant}$$

We can determine the value of the constant of integration because we know that the shear stress is zero ($\tau = 0$) on the surface of the flow ($z = h$). The constant therefore equals $\rho g \sin\theta\, h$. Hence the shear stress at any point in the flow is given by

$$\tau = \rho g\, \sin\theta(h - z)$$

Because the lava is a Newtonian liquid in laminar flow, $\eta dv/dz$ can be substituted for τ (Eq. (2.10)), giving

$$\frac{dv}{dz} = \frac{\rho g\, \sin\theta}{\eta}(h - z) \tag{2.12}$$

This gives the velocity profile through a lava flow under the specified conditions. Integration of this equation gives

$$v = \frac{\rho g\, \sin\theta}{\eta}\left(hz - \frac{z^2}{2}\right) + \text{constant}$$

This integration constant is zero, because $v = 0$ where $z = 0$. Hence, the velocity at any height z is given by

$$v = \frac{\rho g\, \sin\theta}{\eta}\left(hz - \frac{z^2}{2}\right) \tag{2.13}$$

It is now possible, therefore, to use the viscosities of magmas to calculate flow rates for some common lavas. This is left for the reader to do in Problem 2.9 to 2.11.

When integrating Eq. (2.12), the viscosity was taken to be a constant. But, in reality, the magma would cool along the lower and upper surfaces of the flow, and this would cause a significant increase in the viscosity, which, in turn, would decrease flow velocities. To improve the calculated velocities, it would be necessary, before integrating Eq. (2.12), to express η as a function of height within the flow. This can be done, because a reasonable estimate of the temperature gradient within the flow can be made (see Chapter 5), and if the activation energy in Eq. (2.11) is known, the viscosity can be expressed as a function of z (see Problem 2.11).

2.5 PROBLEMS

2.1 Two lime-silica liquids at 1700 °C, one with 30 and the other with 60 mole % CaO, have densities of 2466 and 2665 kg m^{-3}, respectively. Determine the partial molar volumes of SiO_2 and CaO in these liquids at this temperature. (Note: Your answers may differ slightly from the values determined from Table 2.1, because the table is based on average values determined from many calculations such as those done in this problem.)

2.2 The molar volumes of SiO_2 and CaO at 1200 °C are 26.744×10^{-6} and 16.05×10^{-6} m^3 mol^{-1}, respectively, whereas at 1600 °C they are 26.755×10^{-6} and 17.15×10^{-6} m^3 mol^{-1}, respectively. Calculate the coefficients of expansion, α, for SiO_2 and CaO over this temperature range and give an explanation, in terms of the possible structures of the melts, for the large difference between these two values. (Reminder: $\int dV/V = \ln V + \text{constant}$.)

2.3 Using the partial molar volumes in Table 2.1, calculate the densities of typical rhyolitic (granitic) and basaltic (gabbroic) magmas at 1200 °C having the compositions

in Table 2.3. (Note that the compositions are expressed in weight percent of the oxides, and these must first be converted to mole fractions. The iron in these analyses is expressed as FeO for simplicity. Natural magmas contain both ferrous and ferric iron, the proportions depending on the degree of oxidation of the magma.)

Table 2.3 *For Problem 2.3*

| Oxide | Weight % oxides | |
	Rhyolite	Basalt
SiO_2	74.0	48.7
TiO_2	0.2	2.8
Al_2O_3	13.5	15.0
FeO	2.0	12.0
MgO	0.3	7.5
CaO	1.2	10.5
Na_2O	3.4	2.5
K_2O	5.4	1.0
Total	100.0	100.0

2.4 Using the MELTS program, calculate the density of the rhyolitic and basaltic magmas given in Problem 2.3 at 1200 °C and a pressure of 1 bar (~1 atmosphere). Set the oxygen fugacity to the quartz-fayalite-magnetite (Q-Fa-Mt) oxygen buffer (this is a common value for many magmas). Compare your results with those obtained in Problem 2.3 using Bottinga and Weill's partial molar volumes.

2.5 A sample of granite with density 2670 kg m^{-3} is fused and quenched to a glass that has a density of 2450 kg m^{-3} at 25 °C. The coefficient of expansion, α, for the glass is 2×10^{-5} °C^{-1}. Determine the density of the magma at 1025 °C, and compare this with the value obtained by simply taking 90% of the density of the rock.

2.6 (a) Using the MELTS program, calculate the viscosities of the rhyolitic and basaltic magmas given in Problem 2.3 at 1200 °C, a pressure of 1 bar (~1 atmosphere), and oxygen fugacity of the quartz-fayalite-magnetite (Q-Fa-Mt) oxygen buffer. Note that the MELTS program gives viscosity as log$_{10}$ in poise. You will have to convert to Pa s.

(b) Why are the viscosities of the two melts so different?

(c) Calculate the viscosity of the rhyolite for the same conditions except at a temperature of 800 °C.

(d) Assuming the variation in viscosity with temperature obtained in parts (a) and (c) obeys an Arrhenius relation, calculate the rhyolite's activation energy for viscous flow. Be sure to convert temperatures to kelvin.

(e) Although pressure has little effect on viscosity, higher pressures allow magmas to dissolve more water, which does lower the viscosity. At 1 GPa, granitic magma can dissolve up to ~10 wt% H_2O. Recalculate the granite analysis in part (a) to

include 10 wt% H_2O, and then calculate the viscosity at 1200 °C, 10 kb, under the Q-Fa-Mt oxygen buffer. By comparing your answer in part (a), comment on the effect of water on the viscosity of this magma.

2.7 The viscosity of a particular lava is found by experiment to vary with temperature in the following manner:

T (°C)	1100	1200	1300	1400
η (Pa s)	700	100	20	5

By plotting $\ln \eta$ versus $1/T$ and obtaining a least-squares best fit to the data, determine the value of the activation energy and the preexponential factor in the Arrhenius expression for the temperature dependency of viscosity. (Temperature must be expressed in kelvin.)

2.8 From the equation for the velocity distribution through a lava flow that is in steady-state laminar flow, derive an expression for the maximum velocity and show that this occurs at the upper surface of the flow. (Hint: Check your calculus book for maximum–minimum problems.)

2.9 Plot a graph of the velocity profile through a 2-m-thick basaltic lava flow with a viscosity of 300 Pa s and density 2750 kg m^{-3} descending a 2° slope. What is the shape of this profile?

2.10 Calculate the maximum velocity within the flow given in Problem 2.9. Do the same for a similar flow of rhyolite with a viscosity of 10^6 Pa s and density 2500 kg m^{-3}, and a lunar basalt from the Sea of Tranquility having a viscosity of 9 Pa s and a density of 3000 kg m^{-3}. Express your own maximum rate of walking (approximately 4 miles/hour) in the same units of velocity. Why would the flow velocities be too high? (Note that the acceleration of gravity on the Moon is only 1.62 m s^{-2}.)

2.11 Calculate the velocity profile through the same lava flow as given in Problem 2.9, but because of cooling toward the upper and lower surfaces, the viscosity increases away from the center of the flow. The viscosity at any height z, measured in meters, above the base of the flow is given by $\eta = \eta_c + f(z - h/2)^2$, where η_c, the viscosity at the center of the flow, is 300 Pa s, f is a constant with the value of 10^4 N s m^{-4}, and h is the thickness of the flow. Useful integrals are:

$$\int \frac{dz}{a + bz + cz^2} = \frac{2}{(4ac - b^2)^{1/2}} \tan^{-1} \frac{2cz + b}{(4ac - b^2)^{1/2}}$$
$$+ \text{constant} \quad \text{when} \quad 4ac - b^2 > c$$

$$\int \frac{z dz}{a + bz + cz^2} = \frac{1}{2c} \ln(a + bz + cz^2)$$
$$- \frac{b}{2c} \int \frac{dz}{a + bz + cz^2} + \text{constant}$$

3 Intrusion of magma

3.1 INTRODUCTION

Most field evidence indicates that igneous rocks have formed from upward-moving bodies of either magma, mixtures of magma and crystals, magma and gas bubbles, or even solid rock. Basaltic lava flows cover most of the ocean floor and large areas on continents. Rhyolitic lava flows and volcanic ash also cover large parts of continents. Clearly, magma of a wide range of composition is able to rise to the Earth's surface.

Exposures of deeply eroded parts of the crust reveal that basaltic magmas most commonly rise through fractures and, on cooling, form steeply dipping sheet-like intrusions known as *dikes*. Rhyolitic magmas, in contrast, tend to rise in large dome-like bodies known as *diapirs*, which, on solidifying beneath the surface, form granite batholiths. Still other magmas with high volatile contents such as kimberlite, which bring diamonds up from depths of approximately 200 km, rise through pipe-like bodies known as *diatremes* and erupt explosively onto the Earth's surface.

Regardless of the mode by which magma ascends, buoyancy is the main driving force. Magmas are simply less dense than most rocks and so tend to float toward the Earth's surface. When the upper mantle partially melts, a basaltic liquid is formed that is less dense than the refractory residue, so the liquid begins to rise. It does so slowly at first as an interstitial liquid moving through the pores of the deformable crystal mush (McKenzie, 1984). The flow then becomes channelized (Kelemen *et al*., 1995), and the pressure causing the magma to rise is sufficient to propagate fractures through the lithosphere. As the magma rises in these fractures, cracking at the fracture tip generates a characteristic seismic signal known as *harmonic tremor*, which can be used to track the advance of the magma (McNutt, 2005; Roman and Cashman, 2006; Burlini *et al*., 2007). Velocities of up to several meters per second have been recorded in this manner (Sigurdsson and Sparks, 1978; Aki and Koyanagi, 1981; Wright *et al*., 2006). Velocities have also been estimated from the width of dehydration rims on olivine crystals brought up from the mantle (Demouchy *et al*., 2006). Although olivine does not normally contain water, under high mantle pressures small amounts of hydrogen can enter the mineral and bond with oxygen to form hydroxyl. As magma rises toward the surface, the pressure decreases and the hydrogen starts diffusing out of the crystal. Knowing the rate of this diffusion allows the rate of magma ascent to be calculated from the shape of the dehydraion profile on the rim of the crystals (see Problem 5.17). Based on these profiles Demouchy *et al*. estimate that a basaltic magma in Pali-Aike, Chile, rose at a rate of $\sim 6\,\mathrm{m\ s^{-1}}$. Volatile-rich magma can rise even more rapidly. Based on the transport of large fragments of country rock (*xenoliths*) in kimberlites, velocities of up to $20\,\mathrm{m\ s^{-1}}$ have been calculated (Sparks *et al*., 2006). In volcanic explosions, such as the May 18, 1980, eruption of Mount St. Helens (Kieffer, 1981), magma velocities can even become supersonic ($>344\,\mathrm{m\ s^{-1}}$). In contrast, the rate at which granite diapirs rise is on the order of meters per year (Marsh and Kantha, 1978).

In considering the flow of magma, it is important to bear in mind that magmas, like other fluids, flow only in response to pressure gradients. Although these gradients are due largely to buoyancy, pressures can also result from volume increases on melting in the source region, liberation of gas from vapor-supersaturated magma, and from tectonic forces. We will consider each of these in turn.

3.2 BUOYANT RISE OF MAGMA

Within a body of magma surrounded by denser country rock, there is an excess pressure gradient (dP_{ex}/dz) due to the load of the surrounding rock, and it is this gradient that provides the buoyant force on magma. The density of the magma generates a pressure gradient in the magma itself, which we can refer to as the *magmastatic* pressure gradient ($dP_m/dz = \rho_m g$) to distinguish it from the surrounding *lithostatic* pressure gradient ($dP_r/dz = \rho_r g$), where ρ_m and ρ_r are the densities of magma and country rock respectively. Visualize two juxtaposed columns, one consisting of dense rock, and the other of less dense magma. At any given depth z, the pressure at the base of these two columns would be $P_r = \rho_r gz$ and $P_m = \rho_m gz$ respectively. Because $\rho_r > \rho_m$, the pressure at the base of the rock column would exceed that at the base of the magma column by an amount $(\rho_r - \rho_m)gz$. If the rock is able to flow, which we have argued is normally the case except very near the surface of the Earth (Section 1.3), the higher pressure at the base of the rock column relative to that in the adjoining magma column would cause the rock to flow into the magma and force it upward. If $\Delta\rho \equiv \rho_r - \rho_m$, then the excess pressure causing buoyant rise of the magma is $\Delta\rho gz$, and the excess pressure gradient in the magma due to the loading of the adjoining country rock is