15

Soil-Bearing Capacity for Shallow Foundations

The lowest part of a structure is generally referred to as the *foundation.* Its function is to transfer the load of the structure to the soil on which it is resting. A properly designed foundation transfers the load throughout the soil without overstressing the soil. Overstressing the soil can result in either excessive settlement or shear failure of the soil, both of which cause damage to the structure. Thus, geotechnical and structural engineers who design foundations must evaluate the bearing capacity of soils.

Depending on the structure and soil encountered, various types of foundations are used. Figure 15.1 shows the most common types of foundations. A *spread footing* is simply an enlargement of a load-bearing wall or column that makes it possible to spread the load of the structure over a larger area of the soil. In soil with low load-bearing capacity, the size of the spread footings required is impracticably large. In that case, it is more economical to construct the entire structure over a concrete pad. This is called a *mat foundation.*

Pile and *drilled shaft foundations* are used for heavier structures when great depth is required for supporting the load. Piles are structural members made of timber, concrete, or steel that transmit the load. of the superstructure to the lower layers of the soil. According to how they transmit their load into the subsoil, piles can be divided into two categories: friction piles and end-bearing piles. In the case of friction piles, the superstructure load is resisted by the shear stresses generated along the surface of the pile. In the end-bearing pile, the load carried by the pile is transmitted at its tip to a firm stratum.

In the case of drilled shafts, a shaft is drilled into the subsoil and is then filled with concrete. A metal casing may be used while the shaft is being drilled. The casing may be left in place or may be withdrawn during the placing of concrete. Generally, the diameter of a drilled shaft is much larger than that of a pile. The distinction between piles and drilled shafts becomes hazy at an approximate diameter of 1 m (3 ft), and the definitions and nomenclature are inaccurate.

Spread footings and mat foundations are generally referred to as *shallow foundations,* whereas pile and drilled shaft foundations are classified as *deep foundations.* In a more general sense, shallow foundations are foundations that have a depth-of-embedment-to-width ratio of approximately less than four. When the depth-of-embedment-to-width ratio of a foundation is greater than four, it may be classified as a deep foundation.

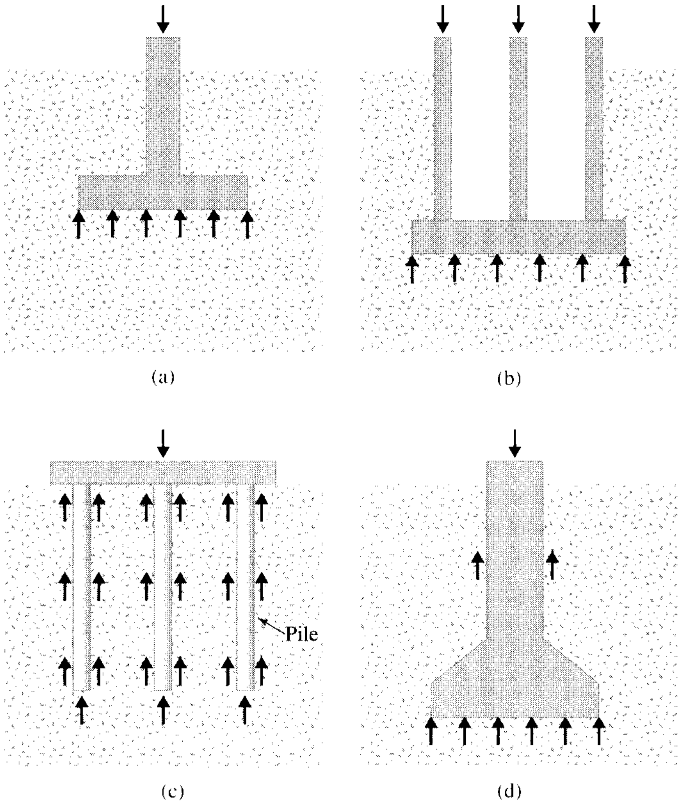


Figure 15.1 Common types of foundations: (a) spread footing; (b) mat foundation; (c) pile foundation; (d) drilled shaft foundation

In this chapter, we discuss the soil-bearing capacity for shallow foundations. As mentioned before, for a foundation to function properly, (1) the settlement of soil caused by the load must be within the tolerable limit, and (2) shear failure of the soil supporting the foundation must not occur. Compressibility of soil — consolidation and elasticity theory — was introduced in Chapter 10. This chapter introduces the load-carrying capacity of shallow foundations based on the criteria of shear failure in soil.

15.1 Ultimate Soil-Bearing Capacity for Shallow Foundations

To understand the concept of the ultimate soil-bearing capacity and the mode of shear failure in soil, let us consider the case of a long rectangular footing of width B located at the surface of a dense sand layer (or stiff soil) shown in Figure 15.2a. When a uniformly distributed load of q per unit area is applied to the footing, it settles. If the uniformly distributed load (q) is increased, the settlement of the footing gradually increases. When the value of $q = q_u$ is reached (Figure 15.2b), bearing capacity

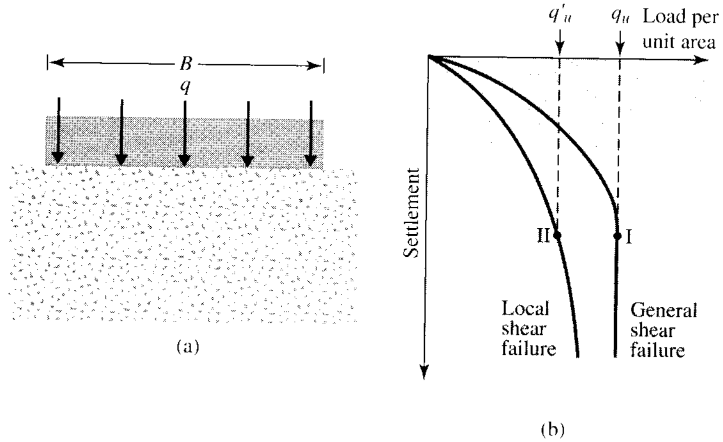


Figure 15.2 Ultimate soil-bearing capacity for shallow foundation: (a) model footing; (b) load settlement relationship

failure occurs; the footing undergoes a very large settlement without any further increase of q. The soil on one or both sides of the foundation bulges, and the slip surface extends to the ground surface. The load-settlement relationship is like curve I shown in Figure 15.2b. In this case, q_u is defined as the ultimate bearing capacity of soil.

The bearing capacity failure just described is called a *general shear failure* and can be explained with reference to Figure 15.3a. When the foundation settles under

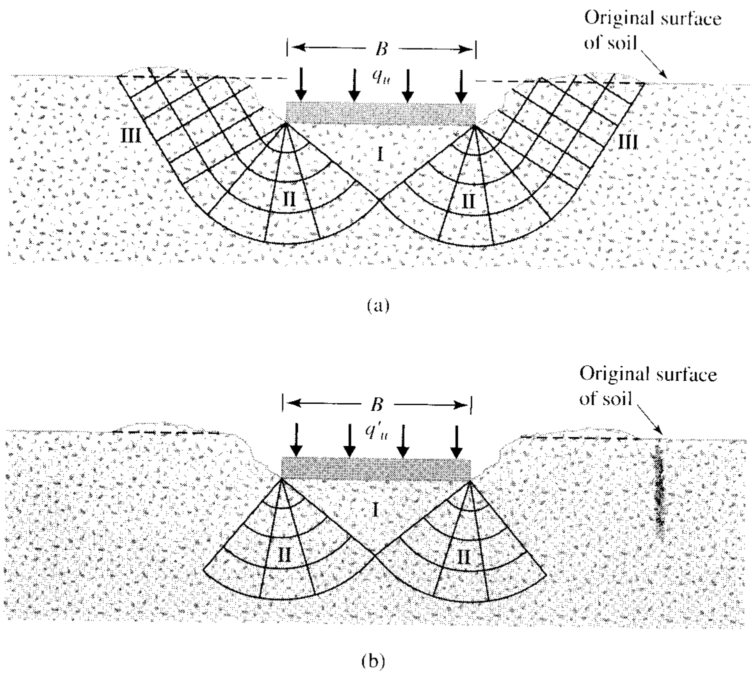


Figure 15.3 Modes of bearing capacity failure in soil: (a) general shear failure of soil; (b) local shear failure of soil

the application of a load, a triangular wedge-shaped zone of soil (marked I) is pushed down, and, in turn, it presses the zones marked II and III sideways and then upward. At the ultimate pressure, q_u, the soil passes into a state of plastic equilibrium and failure occurs by sliding.

If the footing test is conducted instead in a loose to medium dense sand, the load-settlement relationship is like curve II in Figure 15.2b. Beyond a certain value of $q = q'_u$, the load-settlement relationship becomes a steep inclined straight line. In this case, q'_u is defined as the ultimate bearing capacity of soil. This type of soil failure is referred to as *local shear failure* and is shown in Figure 15.3b. The triangular wedge-shaped zone (marked I) below the footing moves downward, but unlike general shear failure, the slip surfaces end somewhere inside the soil. Some signs of soil bulging are seen, however.

15.2 Terzaghi's Ultimate Bearing Capacity Equation

In 1921, Prandtl published the results of his study on the penetration of hard bodies, such as metal punches, into a softer material. Terzaghi (1943) extended the plastic failure theory of Prandtl to evaluate the bearing capacity of soils for shallow strip footings. For practical considerations, a long wall footing (length-to-width ratio more than about five) may be called a *strip footing.* According to Terzaghi, a foundation may be defined as a shallow foundation if the depth D_f is less than or equal to its width B (Figure 15.4). He also assumed that for ultimate soil-bearing capacity calculations, the weight of soil above the base of the footing may be replaced by a uniform surcharge, $q = \gamma D_f$.

The failure mechanism assumed by Terzaghi for determining the ultimate soil-bearing capacity (general shear failure) for a rough strip footing located at a depth D_f measured from the ground surface is shown in Figure 15.5a. The soil wedge ABJ (zone I) is an elastic zone. Both AJ and BJ make an angle ϕ' with the horizontal. Zones marked II (AJE and BJD) are the radial shear zones, and zones marked III are the Rankine passive zones. The rupture lines JD and JE are arcs of a logarithmic spiral, and DF and EG are straight lines. AE, BD, EG, and DF make angles of

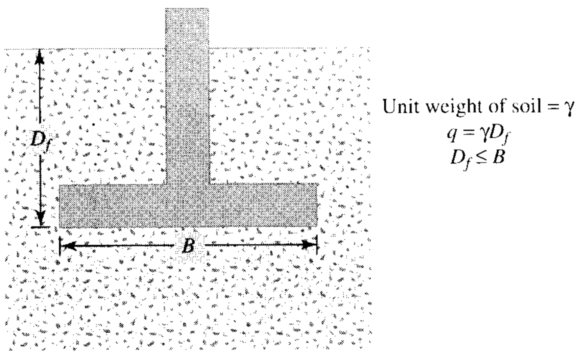
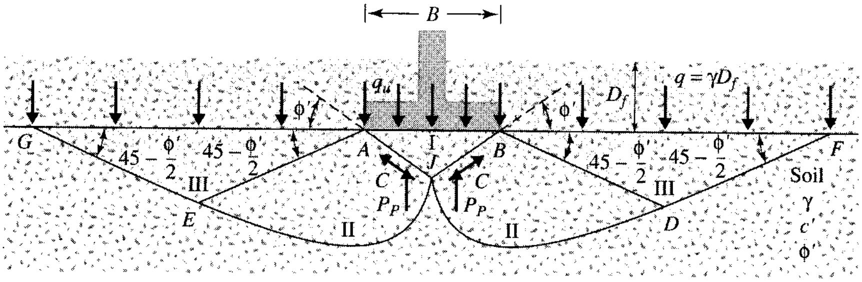
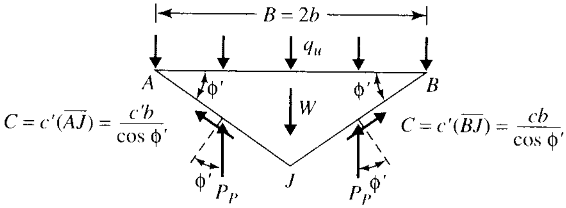


Figure 15.4 Shallow strip footing



(a)



(b)

Figure 15.5 Terzaghi's bearing capacity analysis

$45 - \phi'/2$ degrees with the horizontal. The equation of the arcs of the logarithmic spirals JD and JE may be given as

$$r = r_o e^{\theta \tan \phi'}$$

If the load per unit area, q_u, is applied to the footing and general shear failure occurs, the passive force P_p is acting on each of the faces of the soil wedge ABJ. This concept is easy to conceive of if we imagine that AJ and BJ are two walls that are pushing the soil wedges $AJEG$ and $BJDF$, respectively, to cause passive failure. P_p should be inclined at an angle δ (which is the angle of wall friction) to the perpendicular drawn to the wedge faces (that is, AJ and BJ). In this case, δ should be equal to the angle of friction of soil, ϕ'. Because AJ and BJ are inclined at an angle ϕ' to the horizontal, the direction of P_p should be vertical.

Now let us consider the free body diagram of the wedge ABJ as shown in Figure 15.5b. Considering the unit length of the footing, we have, for equilibrium,

$$(q_u)(2b)(1) = -W + 2C \sin \phi' + 2P_p \tag{15.1}$$

where $b = B/2$

W = weight of soil wedge $ABJ = \gamma b^2 \tan \phi'$

C = cohesive force acting along each face , AJ and BJ, that is equal to the unit cohesion times the length of each face $= c'b/(\cos \phi')$

Thus,

$$2bq_u = 2P_p + 2bc' \tan \phi' - \gamma b^2 \tan \phi' \tag{15.2}$$

or

$$q_u = \frac{P_p}{b} + c' \tan \phi' - \frac{\gamma b}{2} \tan \phi' \tag{15.3}$$

The passive pressure in Eq. (15.2) is the sum of the contribution of the weight of soil γ, cohesion c', and surcharge q. Figure 15.6 shows the distribution of passive pressure from each of these components on the wedge face BJ. Thus, we can write

$$P_p = \tfrac{1}{2}\gamma(b \tan \phi')^2 K_\gamma + c'(b \tan \phi')K_c + q(b \tan \phi')K_q \tag{15.4}$$

where K_γ, K_c, and K_q are earth pressure coefficients that are functions of the soil friction angle, ϕ'.

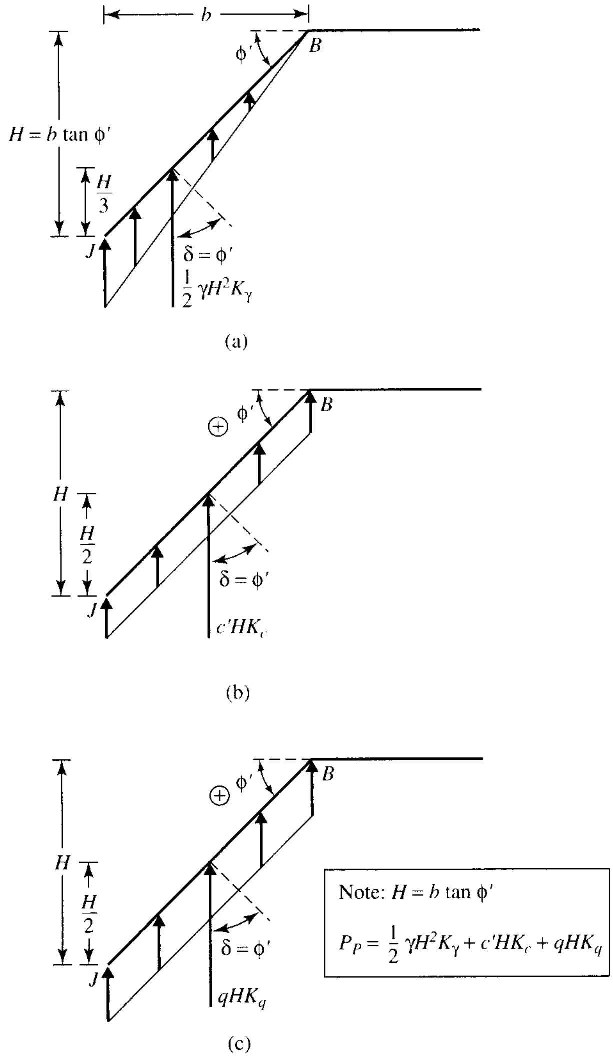


Figure 15.6 Passive force distribution on the wedge face BJ shown in Figure 15.5: (a) contribution of soil weight γ; (b) contribution of cohesion c'; (c) contribution of surcharge q.

Combining Eqs. (15.3) and (15.4), we obtain

$$q_u = c'N_c + qN_q + \frac{1}{2}\gamma B N_\gamma$$

where

$$N_c = \tan\phi'(K_c + 1) \tag{15.5}$$

$$N_q = K_q \tan\phi' \tag{15.6}$$

$$N_\gamma = \tfrac{1}{2}\tan\phi'(K_\gamma \tan\phi' - 1) \tag{15.7}$$

The terms N_c, N_q, and N_γ are, respectively, the contributions of cohesion, surcharge, and unit weight of soil to the ultimate load-bearing capacity. It is extremely tedious to evaluate K_c, K_q, and K_γ. For this reason, Terzaghi used an approximate method to determine the ultimate bearing capacity, q_u. The principles of this approximation follow:

1. If $c' = 0$ and surcharge $(q) = 0$ (that is, $D_f = 0$), then

$$q_u = q_\gamma = \tfrac{1}{2}\gamma B N_\gamma \tag{15.8}$$

2. If $\gamma = 0$ (that is, weightless soil) and $q = 0$, then

$$q_u = q_c = c'N_c \tag{15.9}$$

3. If $\gamma = 0$ (weightless soil) and $c' = 0$, then

$$q_u = q_q = qN_q \tag{15.10}$$

By the method of superimposition, when the effects of the unit weight of soil, cohesion, and surcharge are considered, we have

$$q_u = q_c + q_q + q_\gamma = c'N_c + qN_q + \tfrac{1}{2}\gamma B N_\gamma \tag{15.11}$$

Equation (15.11) is referred to as *Terzaghi's bearing capacity equation.* The terms N_c, N_q, and N_γ are called the *bearing capacity factors.* The values of these factors are given in Table 15.1.

For square and circular footings, Terzaghi suggested the following equations for ultimate soil-bearing capacity:

The square footing is

$$q_u = 1.3c'N_c + qN_q + 0.4\gamma B N_\gamma \tag{15.12}$$

The circular footing is

$$q_u = 1.3c'N_c + qN_q + 0.3\gamma B N_\gamma \tag{15.13}$$

where B = diameter of the footing.

Table 15.1 Terzaghi's Bearing Capacity Factors—N_c, N_q and N_γ—Eqs. (15.11), (15.12), and (15.13)

ϕ' (deg)	N_c	N_q	N_γ[a]	ϕ' (deg)	N_c	N_q	N_γ[a]
0	5.70	1.00	0.00	26	27.09	14.21	9.84
1	6.00	1.10	0.01	27	29.24	15.90	11.60
2	6.30	1.22	0.04	28	31.61	17.81	13.70
3	6.62	1.35	0.06	29	34.24	19.98	16.18
4	6.97	1.49	0.10	30	37.16	22.46	19.13
5	7.34	1.64	0.14	31	40.41	25.28	22.65
6	7.73	1.81	0.20	32	44.04	28.52	26.87
7	8.15	2.00	0.27	33	48.09	32.23	31.94
8	8.60	2.21	0.35	34	52.64	36.50	38.04
9	9.09	2.44	0.44	35	57.75	41.44	45.41
10	9.61	2.69	0.56	36	63.53	47.16	54.36
11	10.16	2.98	0.69	37	70.01	53.80	65.27
12	10.76	3.29	0.85	38	77.50	61.55	78.61
13	11.41	3.63	1.04	39	85.97	70.61	95.03
14	12.11	4.02	1.26	40	95.66	81.27	115.31
15	12.86	4.45	1.52	41	106.81	93.85	140.51
16	13.68	4.92	1.82	42	119.67	108.75	171.99
17	14.60	5.45	2.18	43	134.58	126.50	211.56
18	15.12	6.04	2.59	44	151.95	147.74	261.60
19	16.56	6.70	3.07	45	172.28	173.28	325.34
20	17.69	7.44	3.64	46	196.22	204.19	407.11
21	18.92	8.26	4.31	47	224.55	241.80	512.84
22	20.27	9.19	5.09	48	258.28	287.85	650.67
23	21.75	10.23	6.00	49	298.71	344.63	831.99
24	23.36	11.40	7.08	50	347.50	415.14	1072.80
25	25.13	12.72	8.34				

[a]From Kumbhojkar (1993)

Equation (15.11) was derived on the assumption that the bearing capacity failure of soil takes place by general shear failure. In the case of local shear failure, we may assume that

$$\bar{c}' = \tfrac{2}{3}c' \qquad (15.14)$$

and

$$\tan \bar{\phi}' = \tfrac{2}{3} \tan \phi' \qquad (15.15)$$

The ultimate bearing capacity of soil for a strip footing may be given by

$$q'_u = \bar{c}'N'_c + qN'_q + \tfrac{1}{2}\gamma BN'_\gamma \qquad (15.16)$$

Table 15.2 Terzaghi's Modified Bearing Capacity Factors—N_c', N_q', and N_γ'—Eqs. (15.16), (15.17), and (15.18)

ϕ' (deg)	N_c	N_q	N_γ	ϕ' (deg)	N_c	N_q	N_γ
0	5.70	1.00	0.00	26	15.53	6.05	2.59
1	5.90	1.07	0.005	27	16.30	6.54	2.88
2	6.10	1.14	0.02	28	17.13	7.07	3.29
3	6.30	1.22	0.04	29	18.03	7.66	3.76
4	6.51	1.30	0.055	30	18.99	8.31	4.39
5	6.74	1.39	0.074	31	20.03	9.03	4.83
6	6.97	1.49	0.10	32	21.16	9.82	5.51
7	7.22	1.59	0.128	33	22.39	10.69	6.32
8	7.47	1.70	0.16	34	23.72	11.67	7.22
9	7.74	1.82	0.20	35	25.18	12.75	8.35
10	8.02	1.94	0.24	36	26.77	13.97	9.41
11	8.32	2.08	0.30	37	28.51	15.32	10.90
12	8.63	2.22	0.35	38	30.43	16.85	12.75
13	8.96	2.38	0.42	39	32.53	18.56	14.71
14	9.31	2.55	0.48	40	34.87	20.50	17.22
15	9.67	2.73	0.57	41	37.45	22.70	19.75
16	10.06	2.92	0.67	42	40.33	25.21	22.50
17	10.47	3.13	0.76	43	43.54	28.06	26.25
18	10.90	3.36	0.88	44	47.13	31.34	30.40
19	11.36	3.61	1.03	45	51.17	35.11	36.00
20	11.85	3.88	1.12	46	55.73	39.48	41.70
21	12.37	4.17	1.35	47	60.91	44.54	49.30
22	12.92	4.48	1.55	48	66.80	50.46	59.25
23	13.51	4.82	1.74	49	73.55	57.41	71.45
24	14.14	5.20	1.97	50	81.31	65.60	85.75
25	14.80	5.60	2.25				

The modified bearing capacity factors N_c', N_q', and N_γ' are calculated by using the same general equation as that for N_c, N_q, and N_γ, but by substituting $\overline{\phi}' = \tan^{-1}(\frac{2}{3}\tan\phi')$ for ϕ'. The values of the bearing capacity factors for a local shear failure are given in Table 15.2. The ultimate soil-bearing capacity for square and circular footings for the local shear failure case may now be given as follows [similar to Eqs. (15.12) and (15.13)]:

The square footing is

$$q_u' = 1.3\overline{c}'N_c' + qN_q' + 0.4\gamma BN_\gamma' \tag{15.17}$$

The circular footing is

$$q_u' = 1.3\overline{c}'N_c' + qN_q' + 0.3\gamma BN_\gamma' \tag{15.18}$$

For undrained condition with $\phi = 0$ and $\tau_f = c_u$, the bearing capacity factors are $N_\gamma = N'_\gamma = 0$ and $N_q = N'_q = 1$. Also, $N_c = N'_c = 5.7$. In that case, Eqs. (15.11), (15.12), and (15.13) (which are the cases for general shear failure) take the forms

$$q_u = 5.7c_u + q \qquad \text{(strip footing)} \tag{15.18a}$$

and

$$q_u = (1.3)(5.7)c_u + q = 7.41c_u + q \qquad \text{(square and circular footing)} \tag{15.18b}$$

In a similar manner, Eqs. (15.16), (15.17), and (15.18), which are for the case of local shear failure, will take the forms

$$q'_u = \left(\frac{2}{3}c_u\right)(5.7) + q = 3.8c_u + q \qquad \text{(strip footing)} \tag{15.19a}$$

and

$$q'_u = (1.3)\left(\frac{2}{3}c_u\right)(5.7) + q = 4.94c_u + q \qquad \text{(square and circular footing)} \tag{15.19b}$$

15.3 General Bearing Capacity Equation

After the development of Terzaghi's bearing capacity equation, several investigators worked in this area and refined the solution (that is, Meyerhof, 1951, 1963; Lundgren and Mortensen, 1953; Balla, 1962). Different solutions show that the bearing capacity factors N_c and N_q do not change much. However, for a given value of ϕ', the values of N_γ obtained by different investigators vary widely. This difference is because of the variation of the assumption of the wedge shape of soil located directly below the footing, as explained in the following paragraph.

While deriving the bearing capacity equation for a strip footing, Terzaghi used the case of a rough footing and assumed that the sides *AJ* and *BJ* of the soil wedge *ABJ* (see Figure 15.5a) make an angle ϕ' with the horizontal. Later model tests (for example, DeBeer and Vesic, 1958) showed that Terzaghi's assumption of the general nature of the rupture surface in soil for bearing capacity failure is correct. However, tests have shown that the sides *AJ* and *BJ* of the soil wedge *ABJ* make angles of about $45 + \phi'/2$ degrees, instead of ϕ', with the horizontal. This type of failure mechanism is shown in Figure 15.7. It consists of a Rankine active zone *ABJ* (zone I), two radial shear zones (zones II), and two Rankine passive zones (zones III). The curves *JD* and *JE* are arcs of a logarithmic spiral.

On the basis of this type of failure mechanism, the ultimate bearing capacity of a strip footing may be evaluated by the approximate method of superimposition described in Section 15.2 as

$$q_u = q_c + q_q + q_\gamma \tag{15.20}$$

where q_c, q_q, and q_γ are the contributions of cohesion, surcharge, and unit weight of soil, respectively.

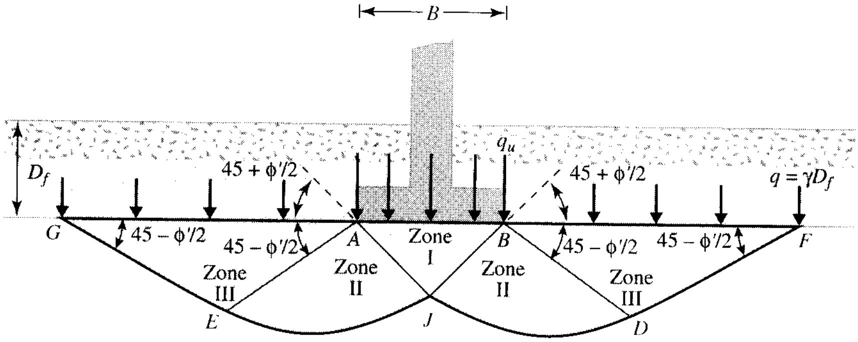


Figure 15.7 Soil-bearing capacity calculation — general shear failure

Reissner (1924) expressed q_q as

$$q_q = qN_q \tag{15.21}$$

where

$$N_q = e^{\pi \tan \phi'} \tan^2\left(45 + \frac{\phi'}{2}\right) \tag{15.22}$$

Prandtl (1921) showed that

$$q_c = c'N_c \tag{15.23}$$

where

$$N_c = (\underset{\substack{\uparrow \\ \text{Eq. (15.22)}}}{N_q} - 1)\cot \phi' \tag{15.24}$$

Meyerhof (1963) expressed q_γ as

$$q_\gamma = \tfrac{1}{2} B\gamma N_\gamma \tag{15.25}$$

where

$$N_\gamma = (\underset{\substack{\uparrow \\ \text{Eq. (15.22)}}}{N_q} - 1)\tan(1.4\phi') \tag{15.26}$$

Combining Eqs. (15.20), (15.21), (15.23), and (15.26), we obtain

$$q_u = c'N_c + qN_q + \tfrac{1}{2}\gamma BN_\gamma \tag{15.27}$$

This equation is in the same general form as that given by Terzaghi [Eq. (15.11)]; however, the values of the bearing capacity factors are not the same. The values of N_q, N_c, and N_γ, defined by Eqs. (15.22), (15.24), and (15.26), are given in Tables 15.3 and

Table 15.3 Bearing Capacity Factors N_c, and N_q [Eqs. (15.22) and (15.24)]

ϕ' (deg)	N_c	N_q	ϕ' (deg)	N_c	N_q
0	5.14	1.00	26	22.25	11.85
1	5.38	1.09	27	23.94	13.20
2	5.63	1.20	28	25.80	14.72
3	5.90	1.31	29	27.86	16.44
4	6.19	1.43	30	30.14	18.40
5	6.49	1.57	31	32.67	20.63
6	6.81	1.72	32	35.49	23.18
7	7.16	1.88	33	38.64	26.09
8	7.53	2.06	34	42.16	29.44
9	7.92	2.25	35	46.12	33.30
10	8.35	2.47	36	50.59	37.75
11	8.80	2.71	37	55.63	42.92
12	9.28	2.97	38	61.35	48.93
13	9.81	3.26	39	67.87	55.96
14	10.37	3.59	40	75.31	64.20
15	10.98	3.94	41	83.86	73.90
16	11.63	4.34	42	93.71	85.38
17	12.34	4.77	43	105.11	99.02
18	13.10	5.26	44	118.37	115.31
19	13.93	5.80	45	133.88	134.88
20	14.83	6.40	46	152.10	158.51
21	15.82	7.07	47	173.64	187.21
22	16.88	7.82	48	199.26	222.31
23	18.05	8.66	49	229.93	265.51
24	19.32	9.60	50	266.89	319.07
25	20.72	10.66			

15.4, but for all practical purposes, Terzaghi's bearing capacity factors will yield good results. Differences in bearing capacity factors are usually minor compared with the unknown soil parameters.

The soil-bearing capacity equation for a strip footing given by Eq. (15.27) can be modified for general use by incorporating the following factors:

1. *depth factor:* to account for the shearing resistance developed along the failure surface in soil above the base of the footing;
2. *shape factor:* to determine the bearing capacity of rectangular and circular footings; and
3. *inclination factor:* to determine the bearing capacity of a footing on which the direction of load application is inclined at a certain angle to the vertical.

Thus, the modified general ultimate bearing capacity equation can be written as

$$q_u = c'\lambda_{cs}\lambda_{cd}\lambda_{ci}N_c + q\lambda_{qs}\lambda_{qd}\lambda_{qi}N_q + \tfrac{1}{2}\lambda_{\gamma s}\lambda_{\gamma d}\lambda_{\gamma i}\gamma BN_\gamma \qquad (15.28)$$

where λ_{cs}, λ_{qs}, and $\lambda_{\gamma s}$ = shape factors
λ_{cd}, λ_{qd}, and $\lambda_{\gamma d}$ = depth factors
λ_{ci}, λ_{qi}, and $\lambda_{\gamma i}$ = inclination factors

Table 15.4 Bearing Capacity Factor N_γ [Eq. (15.26)]

ϕ' (deg)	N_γ	ϕ' (deg)	N_γ
0	0.000	27	9.463
1	0.002	28	11.190
2	0.010	29	13.236
3	0.023	30	15.668
4	0.042	31	18.564
5	0.070	32	22.022
6	0.106	33	26.166
7	0.152	34	31.145
8	0.209	35	37.152
9	0.280	36	44.426
10	0.367	37	53.270
11	0.471	38	64.073
12	0.596	39	77.332
13	0.744	40	93.690
14	0.921	41	113.985
15	1.129	42	139.316
16	1.375	43	171.141
17	1.664	44	211.406
18	2.003	45	262.739
19	2.403	46	328.728
20	2.871	47	414.322
21	3.421	48	526.444
22	4.066	49	674.908
23	4.824	50	873.843
24	5.716	51	1143.934
25	6.765	52	1516.051
26	8.002	53	2037.258

The approximate values of these shape, depth, and inclination factors recommended by Meyerhof are given in Table 15.5.

For undrained condition, if the footing is subjected to vertical loading (that is, $\alpha = 0°$), then

$$\phi = 0$$

$$c = c_u$$

$$N_\gamma = 0$$

$$N_q = 1$$

$$N_c = 1$$

$$\lambda_{ci} = \lambda_{qi} = \lambda_{\gamma i} = 1$$

So Eq. (15.28) transforms to

$$q_u = 5.14c_u\left[1 + 0.2\left(\frac{B}{L}\right)\right]\left[1 + 0.2\left(\frac{D_f}{B}\right)\right] + q \tag{15.29}$$

Table 15.5 Meyerhof's Shape, Depth, and Inclination Factors for a Rectangular Footing[a]

Shape factors

For $\phi = 0°$:

$$\lambda_{cs} = 1 + 0.2\left(\frac{B}{L}\right)$$

$$\lambda_{qs} = 1$$

$$\lambda_{\gamma s} = 1$$

For $\phi' \geq 10°$:

$$\lambda_{cs} = 1 + 0.2\left(\frac{B}{L}\right)\tan^2\left(45 + \frac{\phi'}{2}\right)$$

$$\lambda_{qs} = \lambda_{\gamma s} = 1 + 0.1\left(\frac{B}{L}\right)\tan^2\left(45 + \frac{\phi'}{2}\right)$$

Depth factors

For $\phi = 0°$:

$$\lambda_{cd} = 1 + 0.2\left(\frac{D_f}{B}\right)$$

$$\lambda_{qd} = \lambda_{\gamma d} = 1$$

For $\phi' \geq 10°$:

$$\lambda_{cd} = 1 + 0.2\left(\frac{D_f}{B}\right)\tan\left(45 + \frac{\phi'}{2}\right)$$

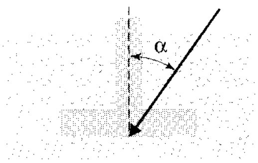
$$\lambda_{qd} = \lambda_{\gamma d} = 1 + 0.1\left(\frac{D_f}{B}\right)\tan\left(45 + \frac{\phi'}{2}\right)$$

Inclination factors

$$\lambda_{ci} = \left(1 - \frac{\alpha°}{90°}\right)^2$$

$$\lambda_{qi} = \left(1 - \frac{\alpha°}{90°}\right)^2$$

$$\lambda_{\gamma i} = \left(1 - \frac{\alpha°}{\phi'°}\right)^2$$



[a] B = width of footing; L = length of footing

15.4 Effect of Groundwater Table

In developing the bearing capacity equations given in the preceding sections we assumed that the groundwater table is located at a depth much greater than the width, B of the footing. However, if the groundwater table is close to the footing, some changes are required in the second and third terms of Eqs. (15.11) to (15.13), Eqs. (15.16) to (15.18), and Eq. 15.28. Three different conditions can arise regarding the location of the groundwater table with respect to the bottom of the foundation. They are shown in Figure 15.8. Each of these conditions is briefly described next.

- *Case I (Figure 15.8a):* If the groundwater table is located at a distance D above the bottom of the foundation, the magnitude of q in the second term of the bearing capacity equation should be calculated as

$$q = \gamma(D_f - D) + \gamma' D \tag{15.30}$$

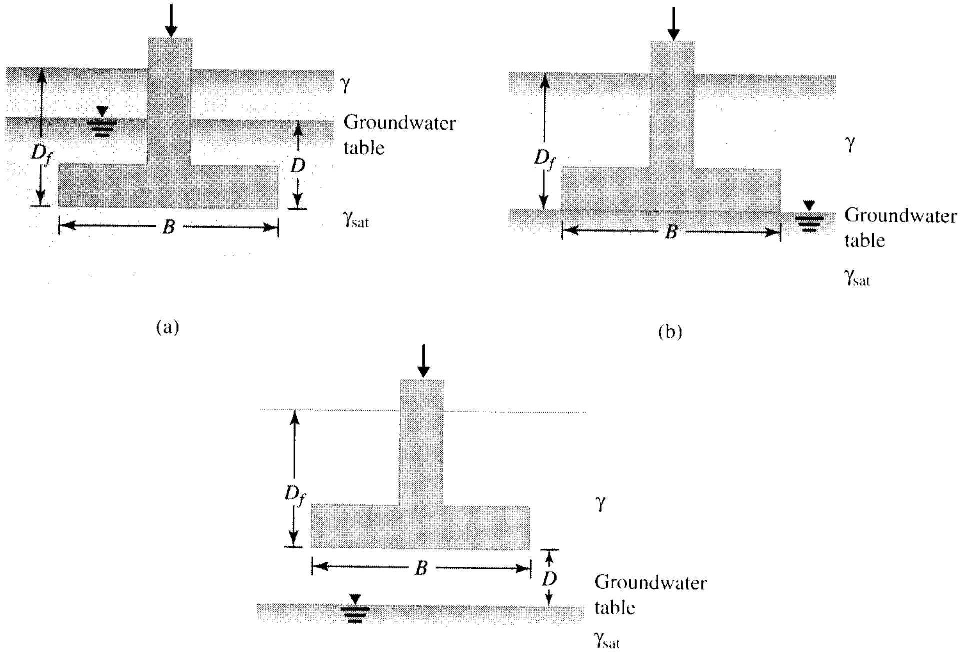


Figure 15.8 Effect of the location of groundwater table on the bearing capacity of shallow foundations: (a) Case I; (b) Case II; (c) Case III

where $\gamma' = \gamma_{sat} - \gamma_w$ = effective unit weight of soil. Also, the unit weight of soil, γ, that appears in the third term of the bearing capacity equations should be replaced by γ'.

- *Case II (Figure 15.8b):* If the groundwater table coincides with the bottom of the foundation, the magnitude of q is equal to γD_f. However, the unit weight, γ, in the third term of the bearing capacity equations should be replaced by γ'.
- *Case III (Figure 15.8c):* When the groundwater table is at a depth D below the bottom of the foundation, $q = \gamma D_f$. The magnitude of γ in the third term of the bearing capacity equations should be replaced by γ_{av}

$$\gamma_{av} = \frac{1}{B}[\gamma D + \gamma'(B - D)] \quad \text{(for } D \leq B\text{)} \tag{15.31}$$

$$\gamma_{av} = \gamma \quad \text{(for } D > B\text{)} \tag{15.32}$$

15.5 Factor of Safety

Generally, a factor of safety, F_s, of about 3 or more is applied to the ultimate soil-bearing capacity to arrive at the value of the allowable bearing capacity. An F_s of 3 or more is not considered too conservative. In nature, soils are neither homogeneous nor isotropic. Much uncertainty is involved in evaluating the basic shear strength parameters of soil.

There are two basic definitions of the allowable bearing capacity of shallow foundations. They are gross allowable bearing capacity, and net allowable bearing capacity.

The *gross allowable bearing capacity* can be calculated as

$$q_{\text{all}} = \frac{q_u}{F_s} \tag{15.33}$$

As defined by Eq. (15.33) q_{all} is the allowable load per unit area to which the soil under the foundation should be subjected to avoid any chance of bearing capacity failure. It includes the contribution (Figure 15.9) of (a) the dead and live loads above the ground surface, $W_{(D+L)}$; (b) the self-weight of the foundation, W_F; and (c) the weight of the soil located immediately above foundation, W_S. Thus,

$$q_{\text{all}} = \frac{q_u}{F_s} = \left[\frac{W_{(D+L)} + W_F + W_S}{A}\right]\frac{1}{F_S} \tag{15.34}$$

where A = area of the foundation.

The *net allowable bearing capacity* is the allowable load per unit area of the foundation in excess of the existing vertical effective stress at the level of the foundation. The vertical effective stress at the foundation level is equal to $q = \gamma D_f$. So the net ultimate load is

$$q_{u(\text{net})} = q_u - q \tag{15.35a}$$

Hence,

$$q_{\text{all(net)}} = \frac{q_{u(\text{net})}}{F_s} = \frac{q_u - q}{F_s} \tag{15.35b}$$

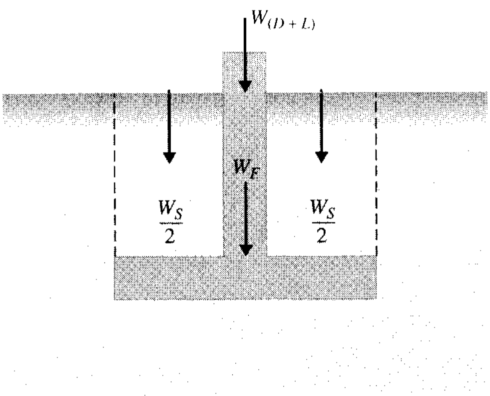


Figure 15.9 Contributions to q_{all}

If we assume that the weight of the soil and the weight of the concrete from which the foundation is made are approximately the same, then

$$q = \gamma D_f \approx \frac{W_S + W_F}{A}$$

Hence,

$$q_{\text{all(net)}} = \frac{W_{(D+L)}}{A} = \frac{q_u - q}{F_S} \tag{15.36}$$

Example 15.1

The plan of a 4-ft-square footing is shown in Figure 15.10. Determine the gross allowable load, Q_{all} ($Q_{\text{all}} = q_{\text{all}} \times$ area of the footing) that the footing can carry. A factor of safety of 3 is needed. Use Terzaghi's equation and assume general shear failure in soil.

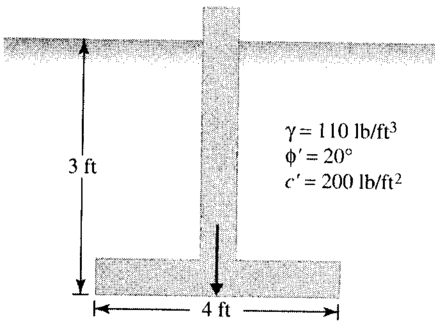


Figure 15.10

Solution

Assuming general shear failure of soil, we have

$$q_u = 1.3c'N_c + qN_q + 0.4\gamma BN_\gamma \qquad \text{[Eq. (15.12)]}$$

From Table 15.1, for $\phi' = 20°$, $N_c = 17.69$, $N_q = 7.44$, and $N_\gamma = 3.64$,

$$q = \gamma D_f = 110 \times 3 = 330 \text{ lb/ft}^2$$

So

$$q_u = (1.3)(200)(17.69) + (330)(7.44) + (0.4)(110)(4)(3.64)$$
$$= 4599 + 2455 + 641 = 7695 \text{ lb/ft}^2$$

$$q_{\text{all}} = \frac{q_u}{F_s} = \frac{7695}{3} = 2565 \text{ lb/ft}^2$$

Hence,

$$Q_{\text{all}} = 2565 \times B^2 = 2565 \times 16 = \mathbf{41{,}040\ lb}$$

■

Example 15.2

Redo Example Problem 15.1 assuming local shear failure in soil. Use Eq. 15.17.

Solution

From Eq. (15.17),

$$q'_u = 1.3\bar{c}'N'_c + qN'_q + 0.4\gamma BN'_\gamma$$

and

$$\bar{c}' = \frac{2}{3}(200) = 133.3 \text{ lb/ft}^2$$

From Table 15.2, $N'_c = 11.85$, $N'_q = 3.88$, and $N'_\gamma = 1.12$. So,

$$q'_u = (1.3)(133.3)(11.85) + (110 \times 3)3.88 + 0.4(110)(4)(1.12)$$
$$= 2054 + 1280 + 197 = 3531 \text{ lb/ft}^2$$

and

$$q_{all} = \frac{q'_u}{3} = \frac{3531}{3} = 1177 \text{ lb/ft}^2$$

Hence,

$$Q_{all} = 1177 \times B^2 = 1177 \times 16 = \mathbf{18{,}832\ lb}$$

■

Example 15.3

A square footing is shown in Figure 15.11. The footing will carry a gross mass of 30,000 kg. Using a factor of safety of 3, determine the size of the footing — that is, the size of B. Use Eq. (15.12).

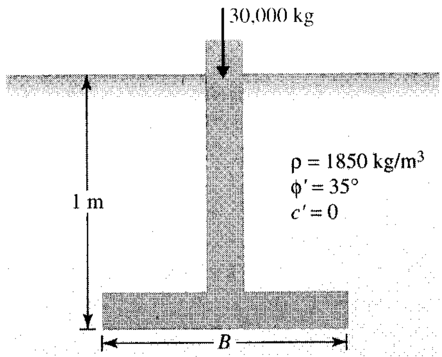


Figure 15.11

Solution

It is given that soil density = 1850 kg/m^3. So

$$\gamma = \frac{1850 \times 9.81}{1000} = 18.15 \text{ kN/m}^3$$

Total gross load to be supported by the footing is

$$\frac{(30{,}000)9.81}{1000} = 294.3 \text{ kN} = Q_{\text{all}}$$

From Eq. (15.12)

$$q_u = 1.3c'N_c + qN_q + 0.4\gamma BN_\gamma$$

With a factor of safety of 3

$$q_{\text{all}} = \frac{q_u}{3} = \frac{1}{3}(1.3c'N_c + qN_q + 0.4\gamma BN_\gamma) \quad \text{(a)}$$

Also,

$$q_{\text{all}} = \frac{Q_{\text{all}}}{B^2} = \frac{294.3}{B^2} \quad \text{(b)}$$

From Eqs. (a) and (b),

$$\frac{294.3}{B^2} = \frac{1}{3}(1.3c'N_c + qN_q + 0.4\gamma BN_\gamma) \quad \text{(c)}$$

From Table 15.1, for $\phi' = 35°$, $N_c = 57.75$, $N_q = 41.44$, and $N_\gamma = 45.41$. Substituting these values into Eq. (c) yields

$$\frac{294.3}{B^2} = \frac{1}{3}[(1.3)(0)(57.75) + (18.15 \times 1)(41.44) + 0.4(18.15)(B)(45.41)]$$

or

$$\frac{294.3}{B^2} = 250.7 + 109.9$$

The preceding equation may now be solved by trial and error, and from that we get

$$B \simeq \mathbf{0.95\ m}$$

■

Example 15.4

Refer to Example 15.1. Determine the net allowable load $Q_{\text{all(net)}}$ with an $F_s = 3$ against the net ultimate bearing capacity.

Solution

From Example 15.1

$$q_u = 7695 \text{ lb/ft}^2$$

$$q_{u(\text{net})} = q_u - q = 7695 - 330 = 7365 \text{ lb/ft}^2$$

$$q_{\text{all(net)}} = \frac{q_{u(\text{net})}}{F_s} = \frac{7365}{3} = 2455 \text{ lb/ft}^2$$

So

$$Q_{\text{all(net)}} = (q_{\text{all(net)}})(B^2) = (2455)(4^2) = \mathbf{39{,}280\ lb}$$ ■

Example 15.5

A square footing is shown in Figure 15.12. Determine the safe gross load (factor of safety of 3) that the footing can carry. Use Eq. (15.28).

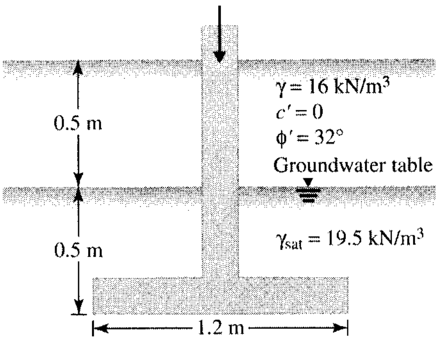


Figure 15.12

Solution

From Eq. (15.28),

$$q_u = c'\lambda_{cs}\lambda_{cd}N_c + q\lambda_{qs}\lambda_{qd}N_q + \tfrac{1}{2}\gamma'\lambda_{\gamma s}\lambda_{\gamma d}BN_\gamma$$

(*Note*: λ_{ci}, λ_{qi}, and $\lambda_{\gamma i}$ are all equal to 1 because the load is vertical.)

Because $c' = 0$,

$$q_u = q\lambda_{qs}\lambda_{qd}N_q + \tfrac{1}{2}\gamma'\lambda_{\gamma s}\lambda_{\gamma d}BN_\gamma$$

From Tables 15.3 and 15.4, for $\phi' = 32°$, $N_q = 23.18$ and $N_\gamma = 22.02$. From Table 15.5,

$$\lambda_{qs} = \lambda_{\gamma s} = 1 + 0.1\left(\frac{B}{L}\right)\tan^2\left(45 + \frac{\phi'}{2}\right)$$

$$= 1 + 0.1\left(\frac{1.2}{1.2}\right)\tan^2\left(45 + \frac{32}{2}\right) = 1.325$$

$$\lambda_{qd} = \lambda_{\gamma d} = 1 + 0.1\left(\frac{D_f}{B}\right)\tan\left(45 + \frac{\phi'}{2}\right)$$

$$= 1 + 0.1\left(\frac{1}{1.2}\right)\tan\left(45 + \frac{32}{2}\right) = 1.15$$

The groundwater table is located above the bottom of the foundation, so, from Eq. (15.30),

$$q = (0.5)(16) + (0.5)(19.5 - 9.81) = 12.845 \text{ kN/m}^2$$

Thus,

$$q_u = (12.845)(1.325)(1.15)(23.18) + (\tfrac{1}{2})(19.5 - 9.81)(1.325)(1.15)(1.2)(22.02)$$

$$= 453.7 + 195.1 = 648.8 \text{ kN/m}^2$$

$$q_{\text{all}} = \frac{q_u}{3} = \frac{648.8}{3} = 216.3 \text{ kN/m}^2$$

Hence, the gross load is as follows:

$$Q = q_{\text{all}}(B^2) = 216.3(1.2)^2 = \mathbf{311.5\ kN}$$

■

15.6 Ultimate Load for Shallow Foundations under Eccentric Load

One-Way Eccentricity

To calculate the bearing capacity of shallow foundations with eccentric loading, Meyerhof (1953) introduced the concept of *effective area*. This concept can be explained with reference to Figure 15.13, in which a footing of length L and width B is subjected to an eccentric load, Q_u. If Q_u is the ultimate load on the footing, it may be approximated as follows:

1. Referring to Figures 15.13b and 15.13c, calculate the effective dimensions of the foundation. If the eccentricity (e) is in the x direction (Figure 15.13b), the *effective dimensions* are

$$X = B - 2e$$

and

$$Y = L$$

However, if the eccentricity is in the y direction (Figure 15.13c), the effective dimensions are

$$Y = L - 2e$$

and

$$X = B$$

2. The lower of the two effective dimensions calculated in step 1 is the *effective width* (B') and the other is the *effective length* (L'). Thus,

$$B' = X \text{ or } Y\text{, whichever is smaller}$$

$$L' = X \text{ or } Y\text{, whichever is larger}$$

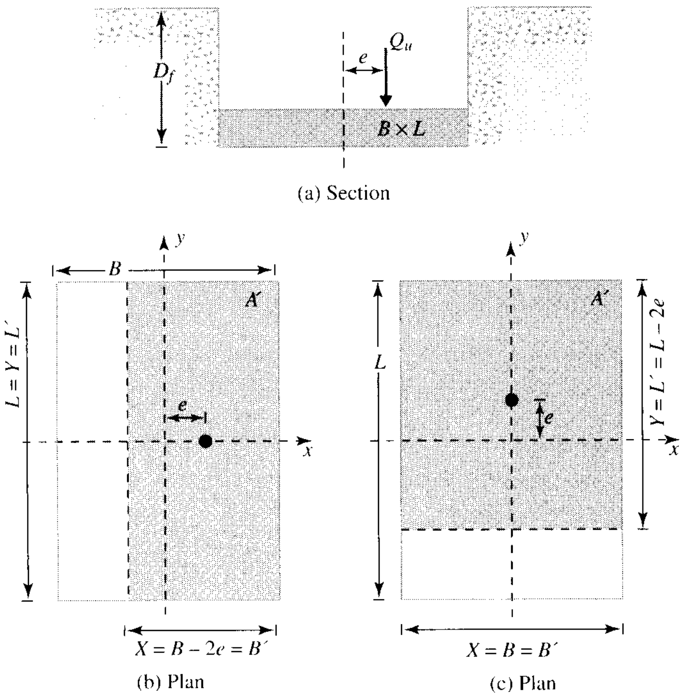


Figure 15.13 Ultimate load for shallow foundation under eccentric load

3. So the effective area is equal to B' times L'. Now, using the effective width, we can rewrite Eq. (15.28) as

$$q_u = c'\lambda_{cs}\lambda_{cd}N_c + q\lambda_{qs}\lambda_{qd}N_q + \tfrac{1}{2}\lambda_{\gamma s}\lambda_{\gamma d}\gamma B'N_\gamma \tag{15.37}$$

   Note that the preceding equation is obtained by substituting B' for B in Eq. (15.28). While computing the shape and depth factors, one should use B' for B and L' for L.

4. Once the value of q_u is calculated from Eq. (15.37), we can obtain the total gross ultimate load as follows:

$$Q_u = q_u(B'L') = q_uA' \tag{15.38}$$

   where A' = effective area.

Two-Way Eccentricity

When foundations are subjected to loads with two-way eccentricity, as shown in Figure 15.14, the effective area is determined such that the centroid coincides with the load. The procedure for finding the effective dimensions, B' and L', are beyond the scope of this text and readers may refer to Das (1999). Once B' and L' are determined, Eqs. (15.37) and (15.38) may be used to determine the ultimate load.

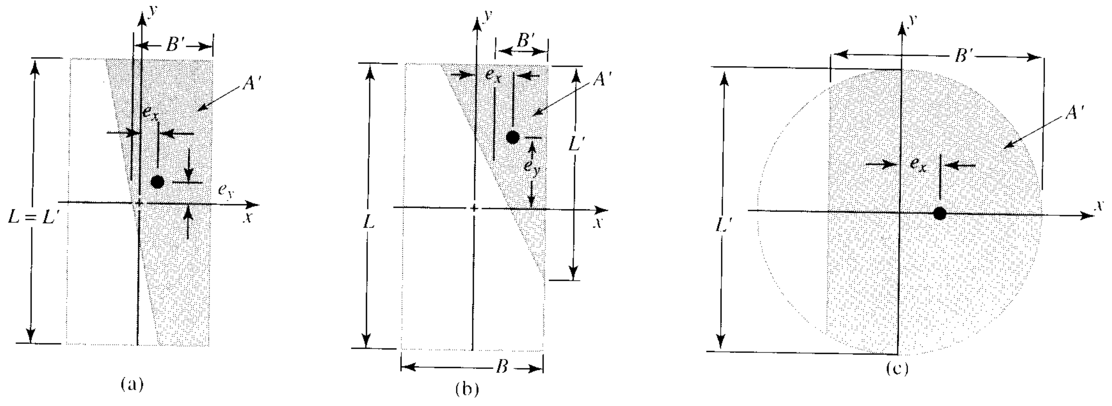


Figure 15.14 Foundation subjected to two-way eccentricity

Example 15.6

A rectangular footing 1.5 m × 1 m is shown in Figure 15.15. Determine the magnitude of the gross ultimate load applied eccentrically for bearing capacity failure in soil.

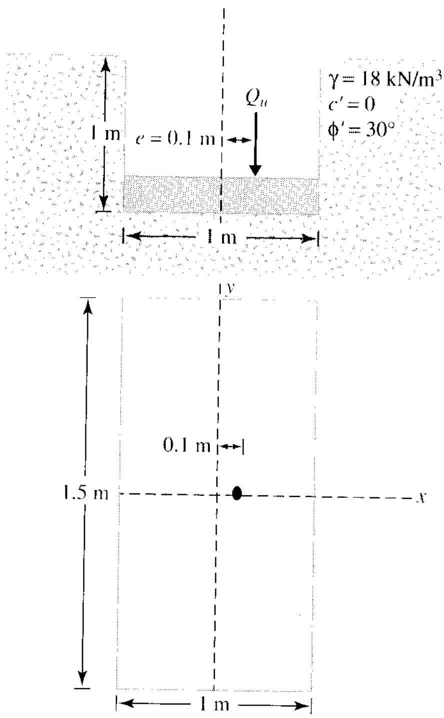


Figure 15.15

Solution

From Figure 15.13b and 15.15,

$$X = B - 2e = 1 - 2e = 1 - (2)(0.1) = 0.8 \text{ m}$$

$$Y = L = 1.5 \text{ m}$$

So the effective width $(B') = 0.8$ m and the effective length $(L') = 1.5$ m. From Eq. (15.37),

$$q_u = q\lambda_{qs}\lambda_{qd}N_q + \tfrac{1}{2}\lambda_{\gamma s}\lambda_{\gamma d}\gamma B' N_\gamma$$

From Tables 15.3 and 15.4, for $\phi' = 30°$, $N_q = 18.4$ and $N_\gamma = 15.668$. From Table 15.5,

$$\lambda_{qs} = \lambda_{\gamma s} = 1 + 0.1\left(\frac{B'}{L'}\right)\tan^2\left(45 + \frac{\phi'}{2}\right)$$

$$= 1 + 0.1\left(\frac{0.8}{1.5}\right)\tan^2\left(45 + \frac{30}{2}\right) = 1.16$$

$$\lambda_{qd} = \lambda_{\gamma d} = 1 + 0.1\left(\frac{D_f}{B'}\right)\tan\left(45 + \frac{\phi'}{2}\right)$$

$$= 1 + 0.1\left(\frac{1}{0.8}\right)\tan\left(45 + \frac{30}{2}\right) = 1.217$$

So

$$q_u = (1 \times 18)(1.16)(1.217)(18.4)$$
$$+ (\tfrac{1}{2})(1.16)(1.217)(18)(0.8)(15.668) \approx 627 \text{ kN/m}^2$$

Hence, from Eq. (15.38),

$$Q_u = q_u(B'L') = (627)(0.8)(1.5) \approx \mathbf{752\ kN}$$ ■

15.7 Bearing Capacity of Sand Based on Settlement

Obtaining undisturbed specimens of cohesionless sand during a soil exploration program is usually difficult. For this reason, the results of standard penetration tests (SPTs) performed during subsurface exploration are commonly used to predict the allowable soil-bearing capacity of foundations on sand. (The procedure for conducting SPTs is discussed in detail in Chapter 17.)

Meyerhof (1956) proposed a correlation for the *net allowable bearing pressure* for foundations with the corrected standard penetration resistance, N_{cor}. The net allowable pressure was defined in Eq. (15.36).

(For definition of the corrected standard penetration resistance, please see Section 17.5.)

According to Meyerhof's theory, for 25 mm (1 in.) of estimated maximum settlement,

$$q_{\text{all(net)}}(\text{kN/m}^2) = 11.98N_{\text{cor}} \qquad (\text{for } B \leq 1.22 \text{ m}) \tag{15.39}$$

$$q_{\text{all(net)}}(\text{kN/m}^2) = 7.99N_{\text{cor}}\left(\frac{3.28B + 1}{3.28B}\right)^2 \qquad (\text{for } B > 1.22 \text{ m}) \tag{15.40}$$

where N_{cor} = corrected standard penetration number

Note that in Eqs. (15.39) and (15.40) B is in meters.

In English units,

$$q_{\text{all(net)}}(\text{kip/ft}^2) = \frac{N_{\text{cor}}}{4} \qquad (\text{for } B \leq 4 \text{ ft}) \tag{15.41}$$

and

$$q_{\text{all(net)}}(\text{kip/ft}^2) = \frac{N_{\text{cor}}}{6}\left(\frac{B + 1}{B}\right)^2 \qquad (\text{for } B > 4 \text{ ft}) \tag{15.42}$$

Since Meyerhof proposed his original correlation, researchers have observed that its results are rather conservative. Later, Meyerhof (1965) suggested that the net allowable bearing pressure should be increased by about 50%. Bowles (1977) proposed that the modified form of the bearing pressure equations be expressed as

$$q_{\text{all(net)}}(\text{kN/m}^2) = 19.16N_{\text{cor}}F_d\left(\frac{S_e}{25}\right) \qquad (\text{for } B \leq 1.22 \text{ m}) \tag{15.43}$$

and

$$q_{\text{all(net)}}(\text{kN/m}^2) = 11.98N_{\text{cor}}\left(\frac{3.28B + 1}{3.28B}\right)^2 F_d\left(\frac{S_e}{25}\right) \qquad (\text{for } B > 1.22 \text{ m}) \tag{15.44}$$

where F_d = depth factor = $1 + 0.33(D_f/B) \leq 1.33$ (15.45)

S_e = tolerable elastic settlement, in mm

Again, the unit of B is meters.

In English units,

$$q_{\text{all(net)}}(\text{kip/ft}^2) = \frac{N_{\text{cor}}}{2.5}F_dS_e \qquad (\text{for } B \leq 4 \text{ ft}) \tag{15.46}$$

and

$$q_{\text{all(net)}}(\text{kip/ft}^2) = \frac{N_{\text{cor}}}{4}\left(\frac{B + 1}{B}\right)^2 F_dS_e \qquad (\text{for } B > 4 \text{ ft}) \tag{15.47}$$

where F_d is given by Eq. (15.45)

S_e = tolerable elastic settlement, in in.

The empirical relations just presented may raise some questions. For example, which value of the standard penetration number should be used, and what is the effect of the water table on the net allowable bearing capacity? The design value of N_{cor} should be determined by taking into account the N_{cor} values for a depth of $2B$ to $3B$, measured from the bottom of the foundation. Many engineers are also of the opinion that the N_{cor} value should be reduced somewhat if the water table is close to the foundation. However, the author believes that this reduction is not required because the penetration resistance reflects the location of the water table.

15.8 Plate Load Test

In some cases, conducting field load tests to determine the soil-bearing capacity of foundations is desirable. The standard method for a field load test is given by the American Society for Testing and Materials (ASTM) under Designation D-1194 (ASTM, 1997). Circular steel bearing plates 152 to 760 mm (6 to 30 in.) in diameter and 305 mm × 305 mm (1 ft × 1 ft) square plates are used for this type of test.

A diagram of the load test is shown in Figure 15.16. To conduct the test, one must have a pit of depth D_f excavated. The width of the test pit should be at least four times the width of the bearing plate to be used for the test. The bearing plate is placed on the soil at the bottom of the pit, and an incremental load on the bearing plate is applied. After the application of an incremental load, enough time is allowed for settlement to occur. When the settlement of the bearing plate becomes negligible, another incremental load is applied. In this manner, a load-settlement plot can be obtained, as shown in Figure 15.17.

From the results of field load tests, the ultimate soil-bearing capacity of actual footings can be approximated as follows:

For clays,

$$q_{u(\text{footing})} = q_{u(\text{plate})} \tag{15.48}$$

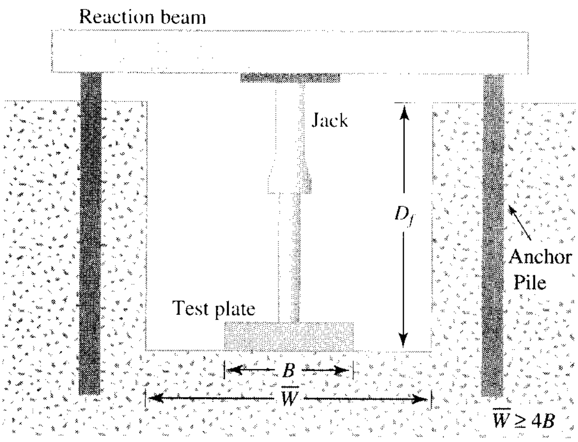


Figure 15.16 Diagram of plate load test

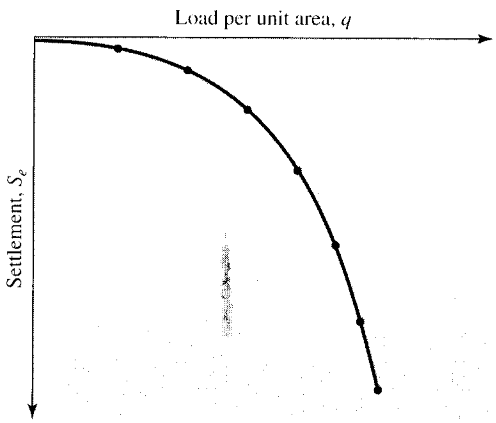


Figure 15.17 Typical load-settlement curve obtained from plate load test

For sandy soils,

$$q_{u(\text{footing})} = q_{u(\text{plate})}\frac{B_{(\text{footing})}}{B_{(\text{plate})}} \tag{15.49}$$

For a given intensity of load q, the settlement of the actual footing can also be approximated from the following equations:

In clay,

$$S_{e(\text{footing})} = S_{e(\text{plate})}\frac{B_{(\text{footing})}}{B_{(\text{plate})}} \tag{15.50}$$

In sandy soil,

$$S_{e(\text{footing})} = S_{e(\text{plate})}\left[\frac{2B_{(\text{footing})}}{B_{(\text{footing})} + B_{(\text{plate})}}\right]^2 \tag{15.51}$$

Housel (1929) also proposed a method for obtaining the soil-bearing capacity of a footing that rests on a cohesive soil for a given settlement S_e. According to this procedure, the total load carried by a footing of area A and perimeter P can be given by

$$Q = Aq + Ps \tag{15.52}$$

where q = compression stress below the footing
s = unit shear stress at the perimeter

Note that q and s are the two unknowns that must be determined from the results of the field load tests conducted on two different-size plates. If Q_1 and Q_2 are the loads required to produce a settlement S_e in plates 1 and 2, respectively, then

$$Q_1 = A_1q + P_1s \tag{15.53}$$

and

$$Q_2 = A_2q + P_2s \tag{15.54}$$

Solution of Eqs. (15.53) and (15.54) yields the values of q and s. Housel's method is not widely used in practice.

Example 15.7

The ultimate bearing capacity of a 700 mm diameter plate as determined from field load tests is 280 kN/m². Estimate the ultimate bearing capacity of a circular footing with a diameter of 1.5 m. The soil is sandy.

Solution

From Eq. (15.49),

$$q_{u(\text{footing})} = q_{u(\text{plate})}\frac{B_{(\text{footing})}}{B_{(\text{plate})}} = 280\left(\frac{1.5}{0.7}\right)$$

$$= \mathbf{680\ kN/m^2}$$

■

Example 15.8

Following are the results of two plate load tests in a cohesive soil:

Plate size (ft)	Settlement (in.)	Total load, Q (lb)
1.5 × 1.5	0.5	15,750
2.5 × 2.5	0.5	33,750

If a square footing 5.75 ft × 5.75 ft is to be constructed and the allowable settlement is 0.5 in., what is the magnitude of the total load that it can carry?

Solution

From Eq. (15.52),

$$Q = Aq + Ps$$

So

$$15{,}750 = (1.5)^2 q + (4 \times 1.5)s \tag{a}$$

$$33{,}750 = (2.5)^2 q + (4 \times 2.5)s \tag{b}$$

From Eqs. (a) and (b),

$$q = 3{,}000 \text{ lb/ft}^2 \quad \text{and} \quad s = 1{,}500 \text{ lb/ft}$$

$$Q = Aq + Ps = (5.75)^2(3{,}000) + (4 \times 5.75)(1{,}500)$$

$$= 133{,}687.5 \text{ lb} \approx \mathbf{133.69 \ kip}$$

■

15.9 Ultimate Bearing Capacity on Layered Soil

The ultimate and allowable bearing capacities of shallow foundations on weaker (loose) sands and soft clays can be increased by placing a layer of compact (dense) sand over it. This is essentially a bearing capacity problem on a layered soil, which is the subject of discussion in this section. It is divided into two parts — the first discusses the bearing capacity on layered sand (dense over loose) followed by an evaluation of the bearing capacity on a stronger sand layer underlain by a weaker saturated clay layer.

Foundations on Layered Sand—Dense over Loose

A simple theory for determining the ultimate bearing capacity of a foundation that rests on a layer of dense sand underlain by loose sand has been proposed by Meyerhof and Hanna (1978). The basic principle of this theory can be explained with the aid of Figure 15.18, which is for a strip foundation. When the top dense sand layer is

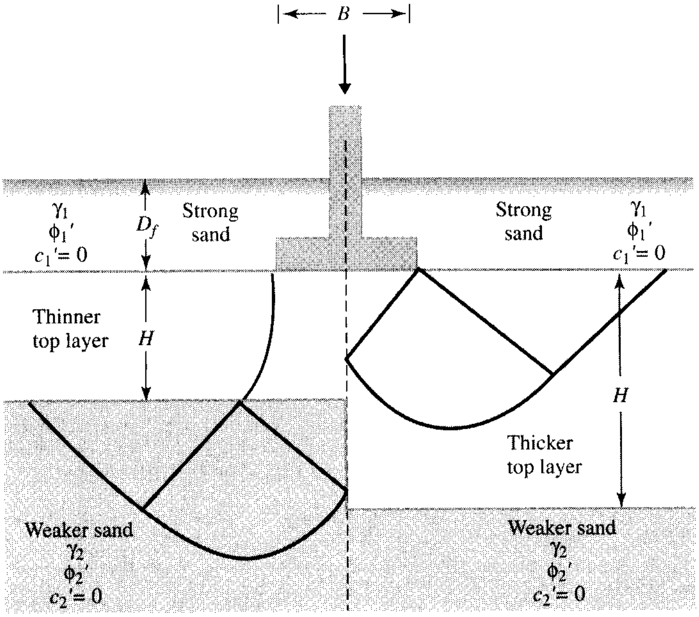


Figure 15.18 Bearing capacity in layered sand — strong sand underlain by weak sand

relatively thick, as shown by the right-hand side of Figure 15.18, the failure surface in soil under the foundation will be fully located inside the dense sand. For this case,

$$q_u = q_{u(t)} = \gamma_1 D_f N_{q(1)} + \frac{1}{2}\gamma_1 B N_{\gamma(1)}$$

(for strip foundations) (15.55)

$$q_u = q_{u(t)} = \gamma_1 D_f N_{q(1)} + 0.3\gamma_1 B N_{\gamma(1)}$$

(for circular or square foundations) (15.56)

and

$$q_u = q_{u(t)} = \gamma_1 D_f N_{q(1)} + \frac{1}{2}\left[1 - 0.4\left(\frac{B}{L}\right)\right]\gamma_1 B N_{\gamma(1)} \qquad (15.57)$$

(for rectangular foundations)

where γ_1 = unit weight of top layer (dense sand in this case)
$N_{q(1)}$ and $N_{\gamma(1)}$ = bearing capacity factors with reference to the soil friction angle, ϕ_1' (Tables 15.3 and 15.4)

Note that Eqs. (15.55), (15.56), and (15.57) are similar to Eq. (15.28). However, the depth factors have not been incorporated; they can be assumed to be somewhat conservative.

If the thickness of the dense sand layer under the foundation H is relatively thin, the failure in soil would take place by *punching* in the dense sand layer followed by a general shear failure in the bottom (or weaker) sand layer, as shown in the left-hand side of Figure 15.18. For such a case, the ultimate bearing capacity for the foundation can be given as

$$q_u = q_{u(b)} + \gamma_1 H^2\left(1 + \frac{2D_f}{H}\right)K_s \frac{\tan\phi_1'}{B} - \gamma_1 H \le \underset{\substack{\uparrow \\ [\text{Eq. (15.55)}]}}{q_{u(t)}}$$

(for strip foundations) (15.58)

$$q_u = q_{u(b)} + 2\gamma_1 H^2\left(1 + \frac{2D_f}{H}\right)\left(\frac{K_s \tan\phi_1'}{B}\right)\lambda_s' - \gamma_1 H \le \underset{\substack{\uparrow \\ [\text{Eq. (15.56)}]}}{q_{u(t)}}$$

(for square or circular foundation) (15.59)

and

$$q_u = q_{u(b)} + \left(1 + \frac{B}{L}\right)\gamma_1 H^2\left(1 + \frac{2D_f}{H}\right)\left(\frac{K_s \tan\phi_1'}{B}\right)\lambda_s' - \gamma_1 H \le \underset{\substack{\uparrow \\ [\text{Eq. (15.57)}]}}{q_{u(t)}}$$

(for rectangular foundations)

(15.60)

where K_s = punching shear coefficient
λ_s' = shape factor
$q_{u(b)}$ = ultimate bearing capacity of the bottom soil layer

The value of the shape factor λ_s' can be taken to be approximately 1. The punching shear coefficient is

$$K_s = f(\gamma_1, \gamma_2, N_{\gamma(1)}, N_{\gamma(2)}) \tag{15.61}$$

where γ_2 = unit weight of the lower layer of sand
$N_{\gamma(2)}$ = bearing capacity factor for the soil friction angle, ϕ_2'

The variation of K_s is shown in Figure 15.19. The term $q_{u(b)}$ in Eqs. (15.58), (15.59), and (15.60) is given by the relationships

$$q_{u(b)} = \gamma_1(D_f + H)N_{q(2)} + \frac{1}{2}\gamma_2 BN_{\gamma(2)}$$

(for strip foundations) (15.62)

$$q_{u(b)} = \gamma_1(D_f + H)N_{q(2)} + 0.3\gamma_2 BN_{\gamma(2)}$$

(for circular or square foundations) (15.63)

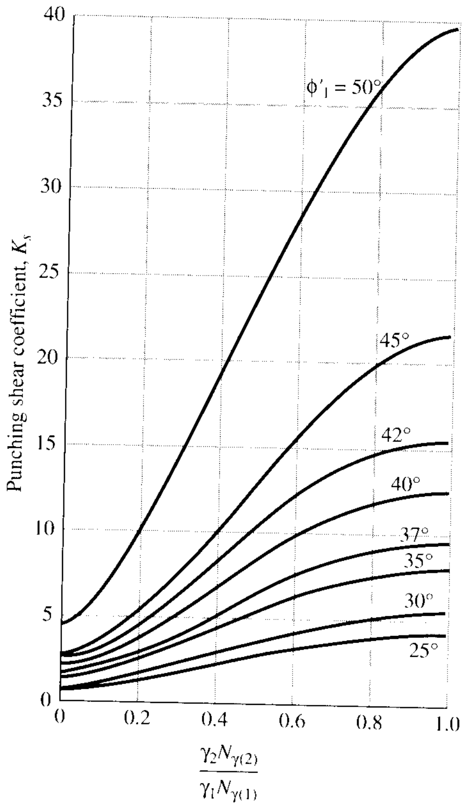


Figure 15.19
Variation of K_s with $(\gamma_2 N_{\gamma(2)})/(\gamma_1 N_{\gamma(1)})$

and

$$q_{u(b)} = \gamma_1(D_f + H)N_{q(2)} + \frac{1}{2}\left[1 - 0.4\left(\frac{B}{L}\right)\right]\gamma_2 B N_{\gamma(2)}$$

(for rectangular foundations) (15.64)

Foundations on Dense or Compacted Sand Overlying Soft Clay

If the thickness of the sand layer under the foundation is relatively small, the failure surface may extend into the soft clay layer. This is shown in the left half of Figure 15.20. However, if the sand layer under the foundation is large, the failure surface will lie entirely in the sand layer, as shown in the right half of Figure 15.20. According to Meyerhof and Hanna (1978), in this case the ultimate bearing capacity of a strip foundation may be given by

$$q_u = c_u N_c + \gamma H^2\left(1 + \frac{2D_f}{H}\right)K_s \frac{\tan\phi'}{B} + \gamma D_f \qquad (15.65)$$

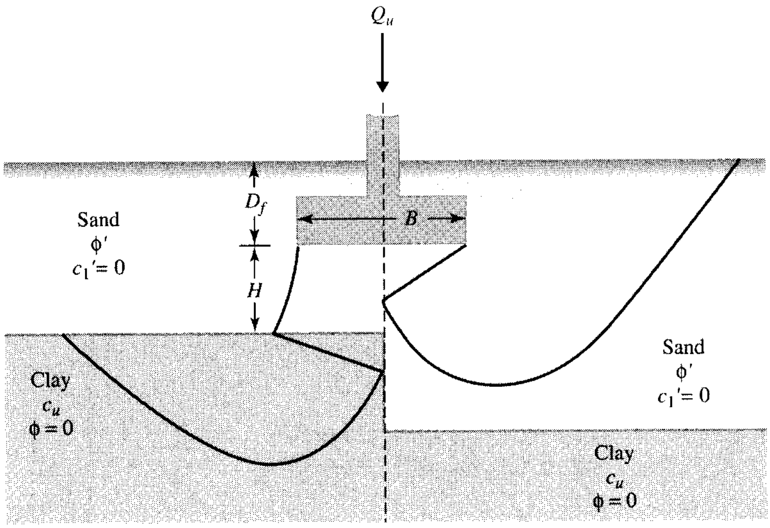


Figure 15.20 Foundation on compacted sand layer overlying soft clay

with a maximum of

$$q_u = \frac{1}{2}\gamma B N_\gamma + \gamma D_f N_q \tag{15.66}$$

where ϕ' = angle of friction of top sand layer
γ = unit weight of sand
K_s = punching shear resistance coefficient

N_γ and N_q correspond to the angle of friction, ϕ', for sand (Tables 15.3 and 15.4). *Note*: for $\phi = 0$, $N_c = 5.14$, as determined from Table 15.3.

For rectangular foundations,

$$q_u = \left(1 + 0.2\frac{B}{L}\right)c_u N_c + \left(1 + \frac{B}{L}\right)\gamma H^2\left(1 + \frac{2D_f}{H}\right)K_s\frac{\tan\phi'}{B} + \gamma D_f \tag{15.67}$$

with a maximum of

$$q_u = \frac{1}{2}\left(1 - 0.4\frac{B}{L}\right)\gamma B N_\gamma + \gamma D_f N_q \tag{15.68}$$

The variation of the punching shear resistance factor, K_s, is given in Figure 15.21. Equations (15.66) and (15.68) are estimates of the values of q_u for strip and rectangular foundations, respectively, in the upper sand layer. This condition corresponds to that shown in the right half of Figure 15.20.

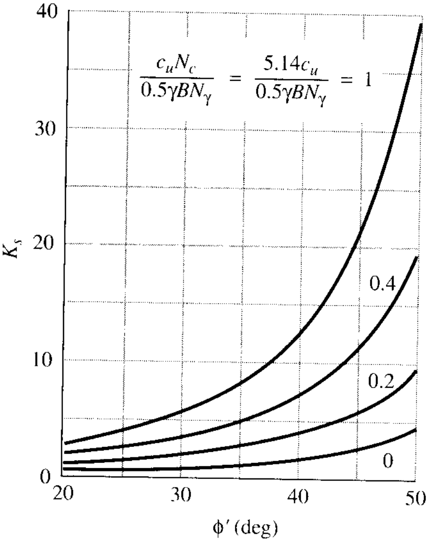


Figure 15.21
Variation of K_s with ϕ' (according to Meyerhof and Hanna)

Example 15.9

Figure 15.22 shows a rectangular foundation with $B = 4$ ft and $L = 6$ ft. Using a factor of safety of 3, determine the net allowable load the foundation can carry.

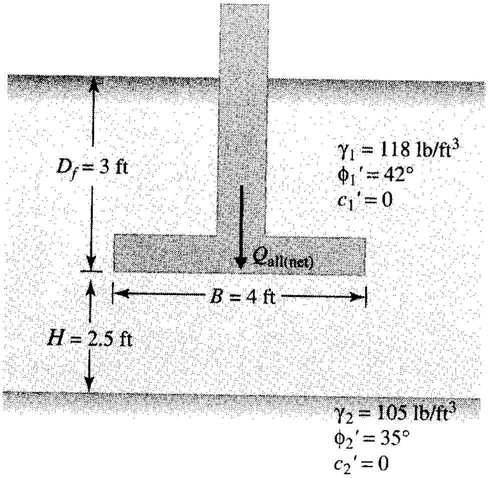


Figure 15.22

Solution

The top layer of sand is dense since it has $\phi_1' = 42°$, which is greater than $\phi_2' = 35°$. Also, $\gamma_1 > \gamma_2$. So Eq. (15.60) should be used to calculate q_u:

$$q_u = q_{u(b)} + \left(1 + \frac{B}{L}\right)\gamma_1 H^2 \left(1 + \frac{2D_f}{H}\right)\left(\frac{K_s \tan \phi_1'}{B}\right)\lambda_s' - \gamma_1 H$$

It is given that $\gamma_1 = 118\ \text{lb/ft}^3$, $\gamma_2 = 105\ \text{lb/ft}^3$, $\phi'_1 = 42°$, and $\phi'_2 = 35°$. Also, from Table 15.4, $N_{\gamma(1)} = 139.32$ and $N_{\gamma(2)} = 37.12$. So

$$\frac{\gamma_2 N_{\gamma(2)}}{\gamma_1 N_{\gamma(1)}} = \frac{(105)(37.12)}{(118)(139.32)} = 0.237$$

From Figure 15.19, for $\phi'_1 = 42°$, the value of K_s is 6. Thus, from Eq. (15.60),

$$q_u = q_{u(b)} + \left(1 + \frac{4}{6}\right)(118)(2.5)^2\left[1 + \frac{(2)(3)}{2.5}\right]\left[\frac{(6)(\tan 42°)}{4}\right](1) - (118)(2.5)$$

$$= q_{u(b)} + 5349 \qquad \text{(a)}$$

From Eq. (15.64),

$$q_{u(b)} = \gamma_1(D_f + H)N_{q(2)} + \frac{1}{2}\left[1 - 0.4\left(\frac{B}{L}\right)\right]\gamma_2 B N_{\gamma(2)}$$

From Tables 15.3 and 15.4, for $\phi'_2 = 35°$, the values of $N_{q(2)} = 33.3$ and $N_{\gamma(2)} = 37.15$. Hence,

$$q_{u(b)} = (118)(3 + 2.5)(33.3) + \frac{1}{2}\left[1 - 0.4\left(\frac{4}{6}\right)\right](105)(4)(37.15) = 27{,}333\ \text{lb/ft}^2 \qquad \text{(b)}$$

From Eqs. (a) and (b),

$$q_u = 27{,}333 + 5349 = 32{,}682\ \text{lb/ft}^2 \qquad \text{(c)}$$

We also need to check Eq. (15.57):

$$q_u = \gamma_1 D_f N_{q(1)} + \frac{1}{2}\left[1 - 0.4\left(\frac{B}{L}\right)\right]\gamma_1 B N_{\gamma(1)}$$

From Tables 15.3 and 15.4, for $\phi'_1 = 42°$, $N_{\gamma(1)} = 139.32$ and $N_{q(1)} = 85.38$, so

$$q_u = (118)(3)(85.38) + \left(\frac{1}{2}\right)\left[1 - 0.4\left(\frac{4}{6}\right)\right](118)(4)(139.32)$$

$$= 54{,}336\ \text{lb/ft}^2 \qquad \text{(d)}$$

Comparing Eqs. (c) and (d), $q_u = 32{,}682\ \text{lb/ft}^2$, we have

$$q_{u(\text{net})} = q_u - \gamma_1 D_f = 32{,}682 - (3)(118) = 32{,}328\ \text{lb/ft}^2$$

$$Q_{\text{all}} = \frac{q_{u(\text{net})}BL}{F_s} = \frac{(32{,}328)(4)(6)}{(1000)(3)} \approx \mathbf{258.6\ kips}$$ ■

Example 15.10

Refer to Figure 15.20. For sand

$$\gamma = 117 \text{ lb/ft}^3$$
$$\phi' = 40°$$

and for clay

$$c_u = 400 \text{ lb/ft}^2$$

For the foundation

$$B = 3 \text{ ft}$$
$$L = 4.5 \text{ ft}$$
$$D_f = 3 \text{ ft}$$
$$H = 4 \text{ ft}$$

Determine the gross ultimate bearing capacity of the foundation.

Solution

The foundation is rectangular, so Eqs. (15.67) and (15.68) will apply. For $\phi' = 40°$, from Table 15.4, $N_\gamma = 93.69$ and

$$\frac{c_u N_c}{0.5\gamma B N_\gamma} = \frac{(400)(5.14)}{(0.5)(117)(3)(93.69)} = 0.125$$

From Figure 15.21, for $c_u N_c / 0.5\gamma B N_\gamma = 0.125$ and $\phi' = 40°$, the value of $K_s \approx 2.5$. Equation (15.67) gives

$$q_u = \left[1 + (0.2)\left(\frac{B}{L}\right)\right] c_u N_c + \left(1 + \frac{B}{L}\right)\gamma H^2\left(1 + \frac{2D_f}{H}\right)K_s \frac{\tan\phi'}{B} + \gamma D_f$$

$$= \left[1 + (0.2)\left(\frac{3}{4.5}\right)\right](400)(5.14) + \left(1 + \frac{3}{4.5}\right)(117)(4)^2$$

$$\times \left[1 + \frac{(2)(3)}{4}\right](2.5)\frac{\tan 40}{3} + (117)(3)$$

$$= 2330 + 5454 + 351 = 8135 \text{ lb/ft}^2$$

Again, from Eq. (15.68),

$$q_u = \frac{1}{2}\left[1 - (0.4)\left(\frac{B}{L}\right)\right]\gamma B N_\gamma + \gamma D_f N_q$$

For $\phi = 40°$, $N_q = 64.20$ (Table 15.3) and

$$q_u = (0.5)\left[1 - (0.4)\left(\frac{3}{4.5}\right)\right](117)(3)(93.69) + (117)(3)(64.20)$$

$$= 12{,}058 + 22{,}534 = 34{,}592 \text{ lb/ft}^2$$

Hence,

$$q_u = \mathbf{8135\ lb/ft^2}$$ ■

15.10 Summary and General Comments

In this chapter, theories for estimating the ultimate and allowable bearing capacities of shallow foundations were presented. Procedures for field load tests and estimation of the allowable bearing capacity of granular soil based on limited settlement criteria were briefly discussed.

Several building codes now used in the United States and elsewhere provide presumptive bearing capacities for various types of soil. It is extremely important to realize that they are *approximate values only.* The bearing capacity of foundations depends on several factors:

1. Subsoil stratification
2. Shear strength parameters of the subsoil
3. Location of the ground water table
4. Environmental factors
5. Building size and weight
6. Depth of excavation
7. Type of structure.

Hence, it is important that the allowable bearing capacity at a given site be determined based on the findings of soil exploration at that site, past experience of foundation construction, and fundamentals of the geotechnical engineering theories for bearing capacity.

The allowable bearing capacity relationships based on settlement considerations such as those given in Section 15.7 do not take into account the settlement caused by consolidation of the clay layers. Excessive settlement usually causes the building to crack, which may ultimately lead to structural failure. Uniform settlement of a structure does not produce cracking; on the other hand, differential settlement may produce cracks and damage to a building.

Problems

15.1 For the continuous footing shown in Figure 15.23, determine the gross allowable bearing capacity. Use Terzaghi's bearing capacity factors and a factor of safety of 4. Assume general bearing capacity failure.

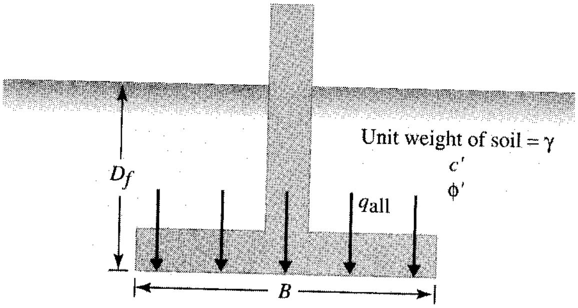


Figure 15.23

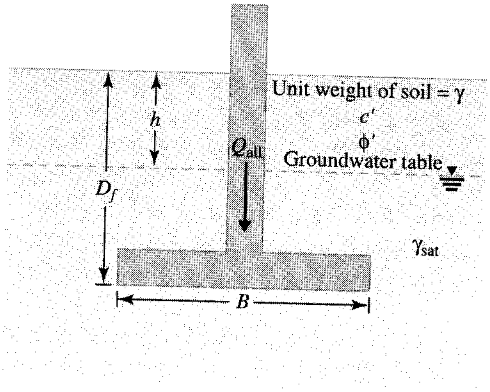


Figure 15.24

a. $\gamma = 120\ \text{lb/ft}^3$, $c' = 0$, $\phi' = 40°$, $D_f = 3$ ft, $B = 3.5$ ft
b. $\gamma = 115\ \text{lb/ft}^3$, $c' = 600\ \text{lb/ft}^3$, $\phi' = 25°$, $D_f = 3.5$ ft, $B = 4$ ft
c. $\gamma = 17.5\ \text{kN/m}^3$, $c' = 14\ \text{kN/m}^2$, $\phi' = 20°$, $D_f = 1.0$ m, $B = 1.2$ m
d. $\gamma = 118\ \text{lb/ft}^3$, $c' = 450\ \text{lb/ft}^2$, $\phi' = 28°$, $D_f = 4$ ft, $B = 4$ ft
e. $\gamma = 17.7\ \text{kN/m}^3$, $c = 48\ \text{kN/m}^2$, $\phi = 0°$, $D_f = 0.6$ m, $B = 0.8$ m

15.2 Repeat Problem 15.1 assuming local shear failure.

15.3 Redo Problem 15.1 using the Prandtl, Reissner, and Meyerhof bearing capacity factors given in Tables 15.3 and 15.4 and Eq. (15.28).

15.4 A square footing has the following values:
Gross allowable load = 42,260 lb
Factor of safety = 3
$D_f = 3$ ft
Soil properties: $\gamma = 110\ \text{lb/ft}^3$
$\phi' = 20°$
$c' = 200\ \text{lb/ft}^3$
Use Eq. (15.12) to determine the size of the footing.

15.5 Repeat Problem 15.4 for the following data:
Gross allowable load = 1870 kN
Factor of safety = 3
$D_f = 1$ m
$\gamma = 17\ \text{kN/m}^3$
$\phi' = 35°$
$c' = 0$

15.6 A square footing is shown in Figure 15.24. For the following cases, determine the gross allowable load, Q_{all}, that the footing can carry. Use Terzaghi's equation for general shear failure ($F_s = 3$).
a. $\gamma = 105\ \text{lb/ft}^3$, $\gamma_{\text{sat}} = 118\ \text{lb/ft}^3$, $c' = 0$, $\phi' = 35°$, $B = 5$ ft, $D_f = 4$ ft, $h = 2$ ft
b. $\rho = 1800\ \text{kg/m}^3$, $\rho_{\text{sat}} = 1980\ \text{kg/m}^3$, $c' = 23.94\ \text{kN/m}^2$, $\phi' = 25°$, $B = 1.8$ m, $D_f = 1.2$ m, $h = 2$ m

15.7 A square footing is shown in Figure 15.24. Use Eq. (15.28) for general shear failure and a factor of safety of 3. Determine the safe gross allowable load. Use the following values:

$\gamma = 100 \text{ lb/ft}^3$
$\gamma_{sat} = 115 \text{ lb/ft}^3$
$c' = 0$
$\phi' = 30°$
$B = 4 \text{ ft}$
$D_f = 3.5 \text{ ft}$
$h = 2 \text{ ft}$

15.8 Solve Problem 15.7 with the following values:

$\gamma = 15.72 \text{ kN/m}^3$
$\gamma_{sat} = 18.55 \text{ kN/m}^3$
$c' = 0$
$\phi' = 35°$
$B = 1.53 \text{ m}$
$D_f = 1.22 \text{ m}$
$h = 0.61 \text{ m}$

15.9 Solve Problem 15.7 with the following data:

$\gamma = 115 \text{ lb/ft}^3$
$\gamma_{sat} = 122.4 \text{ lb/ft}^3$
$c' = 100 \text{ lb/ft}^2$
$\phi' = 30°$
$B = 4 \text{ ft}$
$D_f = 3 \text{ ft}$
$h = 4 \text{ ft}$

15.10 The square footing shown in Figure 15.25 is subjected to an eccentric load. For the following cases, determine the gross allowable load that the footing could carry (use $F_s = 3$):

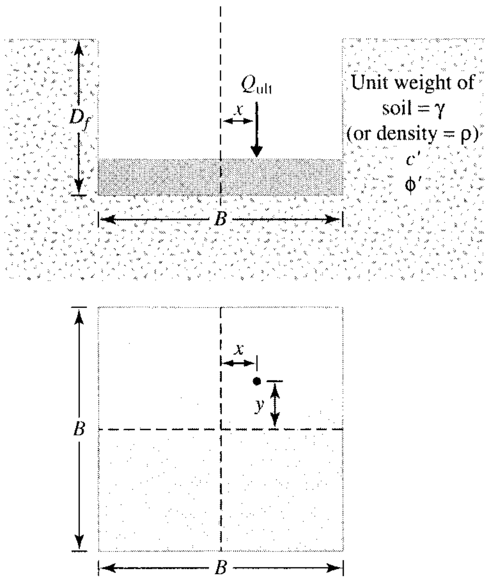


Figure 15.25

a. $\gamma = 16 \text{ kN/m}^3$, $c' = 0$, $\phi' = 30°$, $B = 1.5$ m, $D_f = 1$ m, $x = 0.15$ m, $y = 0$
b. $\rho = 2000 \text{ kg/m}^3$, $c' = 0$, $\phi' = 42°$, $B = 2.5$ m, $D_f = 1.5$ m, $x = 0.2$ m, $y = 0$
c. $\rho = 1950 \text{ kg/m}^3$, $c' = 0$, $\phi' = 40°$, $B = 3$ m, $D_f = 1.4$ m, $x = 0.3$ m, $y = 0$

15.11 For a square footing supported by a sand, given that $B = 2$ m, $D_f = 1.5$ m, corrected standard penetration number $N_{cor} = 9$, allowable settlement $S_e =$ 20 mm, estimate the net allowable bearing capacity.

15.12 A plate load test was conducted in a sandy soil in which the size of the bearing plate was 1 ft × 1 ft. The ultimate load per unit area (q_u) for the test was found to be 4200 lb/ft². Estimate the total allowable load (Q_{all}) for a footing of size 5.5 ft × 5.5 ft. Use a factor of safety of 4.

15.13 A plate load test (bearing plate of 762 mm diameter) was conducted in clay. The ultimate load per unit area, q_u, for the test was found to be 200 kN/m². What should be the total allowable load, Q_{all}, for a column footing 1.75 m in diameter? Use a factor of safety of 3.

15.14 The results of two field load tests in a clay soil are given in the following table:

Plate diameter (mm)	Settlement (mm)	Total load (kN)
204.8	15	49.5
457.2	15	133.1

Based on these results, determine the size of a square footing that will carry a total load of 300 kN with a maximum settlement of 15 mm.

15.15 Figure 15.26 shows a footing on layered sand. Determine the net allowable load it can carry, given the following conditions:

Square footing: $B = 5$ ft
Factor of safety required = 4
$D_f = 3.5$ ft
$H = 2$ ft
$\gamma_1 = 118 \text{ lb/ft}^3$
$\gamma_2 = 105 \text{ lb/ft}^3$
$\phi'_1 = 40°$
$\phi'_2 = 30°$

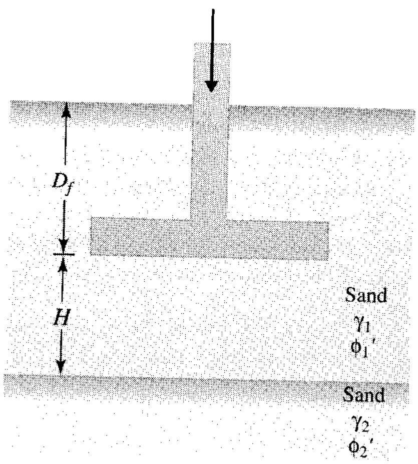


Figure 15.26

15.16 Redo Problem 15.15 with the following values:

Rectangular footing: $B = 1$ m; $L = 1.5$ m
Factor of safety required = 3
$D_f = 1$ m
$H = 0.6$ m
$\gamma_1 = 17.5$ kN/m^3
$\gamma_2 = 15$ kN/m^3
$\phi'_1 = 42°$
$\phi'_2 = 34°$

15.17 Refer to Figure 15.20. The foundation is 1 m × 2 m in plan. $D_f = 1$ m and $H = 1.5$ m. For the sand layer, $\phi' = 35°$, $c' = 0$, $\gamma = 17.8$ kN/m^3; and, for the clay layer, $\phi = 0°$, $c_u = 60$ kN/m^2, $\gamma = 18.2$ kN/m^3. Determine the gross allowable load that the foundation could carry. Use $F_s = 4$.

15.18 Redo Problem 15.17 with the following data:

Foundation: $B \times L = 3$ ft × 6 ft
$D_f = 2.5$ ft
$H = 3$ ft

Sand: $\phi' = 40°$
$c' = 0$
$\gamma = 115$ lb/ft^3

Clay: $\phi = 0°$
$c_u = 750$ lb/ft^2
$\gamma = 118$ lb/ft^3

References

AMERICAN SOCIETY FOR TESTING AND MATERIALS (1997). *Annual Book of Standards,* ASTM, Vol. 04.08, West Conshohocken, Pa.

BOWLES, J. E. (1977). *Foundation Analysis and Design,* 2nd ed., McGraw-Hill, New York.

DAS, B. M. (1999), *Principles of Foundation Engineering,* 4th ed., Plus Publishing Co., Boston.

DEBEER, E. E., and VESIC, A. S. (1958). "Etude Experimentale de la Capacité Portante du Sable Sous des Fondations Directes Etablies en Surface," *Ann. Trav. Publics Belg.,* Vol. 59, No. 3.

HOUSEL, W. S. (1929). "A Practical Method for the Selection of Foundations Based on Fundamental Research in Soil Mechanics," University of Michigan Research Station, *Bulletin No. 13,* Ann Arbor.

KUMBHOJKAR, A. S. (1993). "Numerical Evaluation of Terzaghi's N_γ," *Journal of Geotechnical Engineering,* ASCE, Vol. 119, No. GT3, 598–607.

MEYERHOF, G. G. (1953). "The Bearing Capacity of Foundations Under Eccentric and Inclined Loads," *Proceedings,* 3rd International Conference on Soil Mechanics and Foundation Engineering, Vol. 1, 440–445.

MEYERHOF, G. G. (1956). "Penetration Tests and Bearing Capacity of Cohesionless Soils, *Journal of the Soil Mechanics and Foundations Division,* American Society of Civil Engineers, Vol. 82, No. SM1, pp. 1–19.

MEYERHOF, G. G. (1963). "Some Recent Research on the Bearing Capacity of Foundations," *Canadian Geotechnical Journal,* Vol. 1, 16–26.

MEYERHOF, G. G. (1965). "Shallow Foundations," *Journal of the Soil Mechanics and Foundations Division,* ASCE, Vol. 91, No. SM2, pp. 21–31.

MEYERHOF, G. G., and HANNA, A. M. (1978). "Ultimate Bearing Capacity of Foundations on Layered Soil under Inclined Load," *Canadian Geotechnical Journal,* Vol. 15, No. 4, 565–572.

PRANDTL, L. (1921). "Über die Eindringungsfestigkeit (Harte) plasticher Baustoffe und die Festigkeit von Schneiden," *Zeitschrift für Angewandte Mathematik und Mechanik,* Basel, Switzerland, Vol. 1, No. 1, 15–20.

REISSNER, H. (1924). "Zum Erddruckproblem," *Proceedings,* 1st International Congress of Applied Mechanics, 295–311.

TERZAGHI, K. (1943). *Theoretical Soil Mechanics,* Wiley, New York.