2

MECHANICAL PRINCIPLES

Materials of Structural Geology

Structural geology is concerned primarily with solids, but also with liquids and to some extent with gases.

In a *gas* the atoms are in rapid motion, move independently of one another, and have no orderly arrangement. The forces of mutual attraction are less than the forces of movement. Gases have high mobility.

In a *liquid* the atomic forces are strong enough to keep the atoms together, but there is either no orderly arrangement or only a limited orderly arrangement.

In *crystalline solids* the atoms have an orderly arrangement. Common salt, for example, is composed of sodium and chlorine atoms in a ratio of one to one, and forming a cubic lattice (Fig. 2-1); the external shape of a salt crystal is a cube or a related form. A rock such as basalt is composed primarily of two minerals, plagioclase and augite. The sodium and alumin

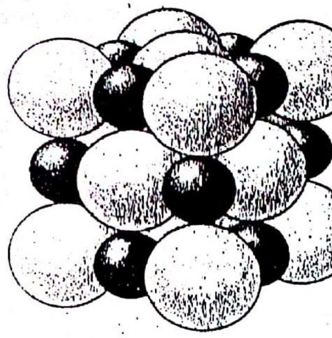

Fig. 2-1. Atomic structure of common salt. The large spheres represent chlorine atoms; the small spheres represent sodium atoms. (From R. W. G. Wyckoff's *Structure of crystals*, 2d ed., The Chemical Catalog Co., 1931.)

as well as some of the calcium and silicon atoms, combine to form crystals of plagioclase, characterized by its own lattice. Similarly, the atoms of iron and magnesium, as well as some of the atoms of calcium and silicon, combine to form crystals of augite, also characterized by its own lattice.

In *noncrystalline solids* there is no such systematic arrangement of the atoms into crystals. The *vitreous solids* or *glasses* are the product of liquids that cooled so rapidly that the atoms could not organize into crystals. Window glass is a vitreous solid. A basaltic glass is composed of the same elements as basalt, but is composed of glass rather than crystals of plagioclase and augite. *Amorphous solids* are a special group that lack crystal structure, but are not the product of rapidly cooled liquids.

The outer shell of the earth consists predominantly of solids, but gases and liquids are also present; their importance varies with time and space. Groundwater and active volcanoes attest to the importance of liquids at the present time, and the igneous rocks of intrusive bodies indicate the abundance of liquids in the past. Gases, present in the outer shell of the earth, are strikingly manifested in regions where petroleum is found; vast quantities of gas are sometimes expelled by active volcanoes. Never, however, does the gas occupy great underground chambers. The natural gas associated with petroleum occupies small pore spaces and fractures in solid rock, and the gas of volcanoes effervesces from magma.

In this section of the book we are concerned primarily with solids. Gases and liquids are important only if their presence in pore spaces modifies the behavior of the solids.

The outer shell of the earth consists of sedimentary, igneous, and metamorphic rocks. The structural geologist, however, is interested primarily in

the mechanical properties of the rocks with which he deals rather than in their origin.[1,2] Is the rock well consolidated or not? A poorly cemented sandstone will be weaker than a well-cemented one, and quartzite will have greater strength than lava full of gas bubbles. Is the rock massive or not? Thin-bedded strata are weaker than thick-bedded formations. A thick, massive limestone will be stronger than a series of thin lava flows; although in laboratory tests of individual specimens, the lava may be the stronger of the two. A thick, massive sandstone can be stronger than a highly fractured granite. Is the composition such that the fractures may be readily healed? Specimens of quartzite may be stronger than a limestone. But fractures in quartzite heal less readily than those in limestone.

FORCE AND ACCELERATION

Force

Force is an explicitly definable vector quantity that changes or tends to produce a change in the motion of a body. The locomotive of a train exerts the force that moves the cars. Force is defined by its magnitude and direction, hence it may be expressed by an arrow, the length of which is proportional to the magnitude of the force, and the direction of which indicates the direction in which the force is acting.

An *unbalanced force* is one that causes a change in the motion of a body. The *acceleration* is the rate of change of velocity. If a train starts from rest and acquires a velocity of 20 miles per hour at the end of 10 minutes, the acceleration is two miles per hour per minute. A body dropped from a high building is subjected to an unbalanced force because of the gravitational pull of the earth, and the body accelerates at the rate of approximately 32 feet per second per second.

Balanced forces exist where no change in motion occurs. If a train is moving at a constant velocity, the frictional resistance of the tracks and the air equals the force exerted by the locomotive. If a man pushes against a wall that he cannot move, the wall is exerting a force equal and opposite to that exerted by the man.

Most problems confronting the structural geologist may be analyzed by assuming balanced forces because the velocity of rock bodies is so small that acceleration is negligible. Along faults, however, the motion causing earthquakes may be so rapid that acceleration is important.

UNITS OF MEASUREMENT

Two different systems of measurement are commonly used in English-speaking countries, one, the so-called absolute or C.G.S. system, the other the English or Engineer's system.

In the C.G.S. system the principal units are length (centimeters), mass (grams), time (seconds), and force (dynes).

$$F = ma, \qquad (1)$$

where F is force, m is mass, and a is acceleration.

A dyne is the unbalanced force that will give a mass of one gram an acceleration of one centimeter per second per second. The force of gravity at sea level is approximately 980 dynes per square centimeter, producing an acceleration of 980 centimeters per second per second.

In the English system the units are length (feet or inches), mass (pounds), time (seconds), and force (poundal). A *poundal* is the unbalanced force that will give a mass of one pound an acceleration of one foot per second per second. The force of gravity at sea level is approximately 32.2 poundals per square foot, producing an acceleration of 32.2 feet per second per second.

In practice, force is often expressed in terms of grams or grams per square centimeter (or kilograms, that is, thousands of grams, per square centimeter) or in pounds or pounds per square inch.

Other units are also used, such as atmospheres and bars. Table 2-1 shows how the various units may be converted into one another.

Table 2-1. Conversion Table for Units of Mass and Force (Clark, 1967)

Pounds per Square Inch	*Atmosphere*	*Bars*	*Kilograms per Square Centimeter*	*Dynes per Square Centimeter**
1	0.0680458	0.0689474	0.0703070	68,947
14.6960	1	1.01325	1.03323	1,013,250
14.5038	0.986924	1	1.01972	1,000,000
14.2234	0.967842	0.980665	1	980,665

*Assuming "normal" acceleration of gravity of 980.665 cm/sec.²

COMPOSITION AND RESOLUTION OF FORCES

Force may be represented by a *vector*, that is, a line oriented in the direction in which the force is operating and proportional in length to the intensity of the force. Two or more forces may act in different directions at a point, as in Fig. 2-2, where OX (8 pounds) and OY (12 pounds) act at O. The same result would be produced by the force OZ (14¼ pounds) acting in the direction indicated; OZ is the resultant of OX and OY. A *resultant* is the single force that produces the same result as two or more forces, and it may be represented by the diagonal of a parallelogram constructed on two arrows that represent the two forces. The *equilibrant* is the force necessary to balance two or more forces. In Fig. 2-2, OW is the force necessary to balance OX and OY; it is equal to the resultant of the two forces, but acts in the opposite direction. The process of finding the resultant of two or more forces is called the *composition of forces*.

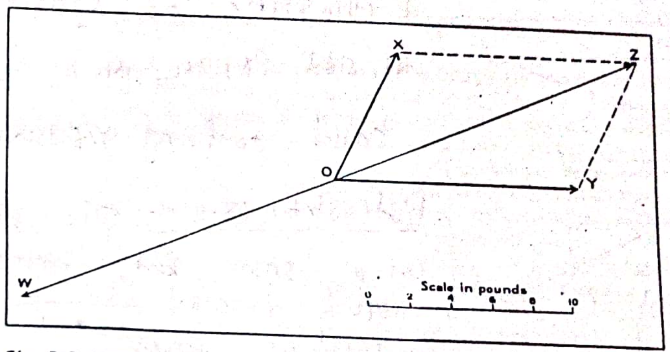

Fig. 2-2. Composition of forces.

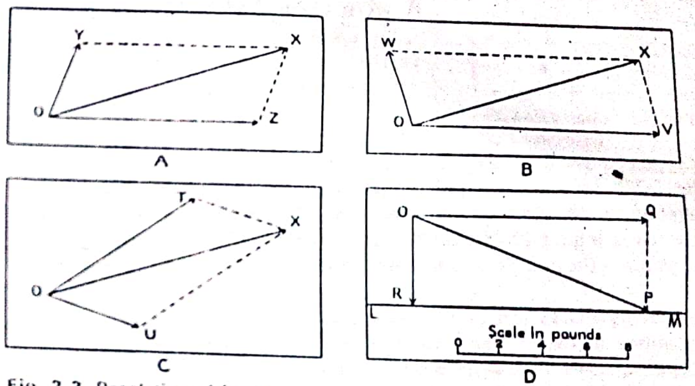

Fig. 2-3. Resolution of forces.

Conversely, the effect of a single force may be considered in terms of two or more forces that would produce the same result. Thus, in Fig. 2-3A, OY and OZ would produce the same result as OX; in Fig. 2-3B, OW and OV would produce the same result as OX; in Fig. 2-3C, OT and OU would produce the same result as OX. A single force may thus be resolved into two *components*, acting in defined directions, by constructing a parallelogram, the diagonal of which represents the given force, and the sides of which have the directions of the components. The process of finding the components of a single force is called the *resolution of forces*.

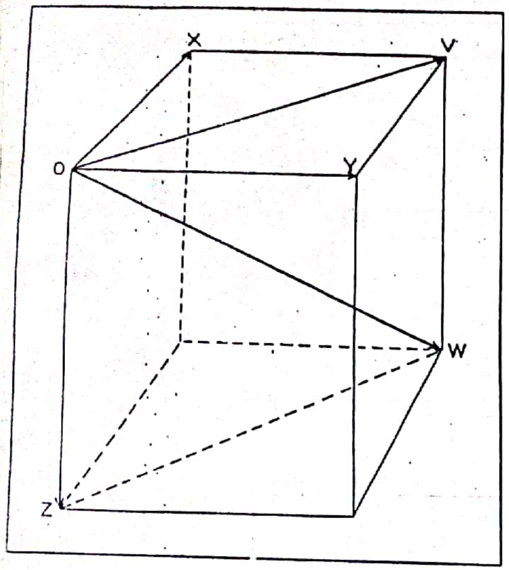

Fig. 2-4. Resolution of forces in three dimensions.

Lithostatic or Confining pressure.
All sided pressure on a solid is called confining pressure,
Hydrostatic pressure The pressure on a small body immersed in a liquid is called ———.
→ Lithostatic pressure cause increase in density and decrease in volume.

In Fig. 2-3D the force OP (12 lb) impinges on the line LM, and it is necessary to find the value of the component parallel to LM. This component OQ has a value of about 11 lb, as can be determined from the scale in the figure. OR, which is the component perpendicular to LM, has a value of about 5 lb.

The preceding discussion of the composition and resolution of forces has been confined to two dimensions, but geology is concerned with three dimensions. In Fig. 2-4 an inclined force OW lies in the vertical plane $OZVW$. This force may be resolved into two components, one of which, OZ, is vertical; the other, OV, lies in the horizontal plane $OXVY$. The OV may in turn be resolved into OX and OY, which lie in the horizontal plane and at right angles to each other. Moreover, any force, regardless of its value and its angle of inclination, may be similarly resolved into three components parallel to the X, Y, and Z axes of Fig. 2-4.

LITHOSTATIC OR CONFINING PRESSURE

The pressure on a small body immersed in a liquid is described as *hydrostatic pressure*. For example, at a depth of a mile in the ocean, the pressure is equal to the weight of a column of salt water one mile high. The pressure is 337,900 pounds per square foot, or 2346 pounds per square inch. Every square inch of the surface of a small sphere at this depth would be under a

pressure of 2346 pounds per square inch. Such an undirected, all-sided pr sure is called hydrostatic pressure.

Rocks in the lithosphere, because of the weight of whatever rocks above them, are subjected to a similar but not identical kind of pressu The weight of a column of rock one mile high will be several times tha an equally high column of water, because rocks have a higher specific grav The weight of a column of granite one mile high and one inch square wo be 6178 pounds. A small imaginary sphere at a depth of one mile in the gra would be subjected to an all-sided pressure that would simulate hydrost pressure. This type of pressure may be called *lithostatic pressure*, but in exp mental work this equal, all-sided pressure on solids is called the *confi pressure*.

Obviously, the lithostatic pressure increases with depth in the earth reaches tremendous values in the interior. It is equal to the weight of overlying column of rocks, but near the surface this is only approxima true.

An increase in confining or lithostatic pressure causes a decrease in volume of rocks but an increase in the density. A decrease in conf pressure causes an increase in volume but a decrease in density.

DIFFERENTIAL FORCES

In many instances the forces acting on a body are not equal on all A body is said to be under *tension* when it is subjected to external force tend to pull it apart. Tension may be represented, as in Fig. 2-5A, b arrows that are on the same straight line and are directed away from other; the arrows represent the forces, whereas the rectangle represen body or part of a body upon which the forces act. The rectangle m omitted.

A body is said to be under *compression* when it is subjected to ex forces that tend to compress it. Compression may be represented, as 2-5B, by two arrows that are on the same straight line and are directed each other; the arrows represent the forces, whereas the rectangle rep the body or part of the body acted upon. The rectangle may be o

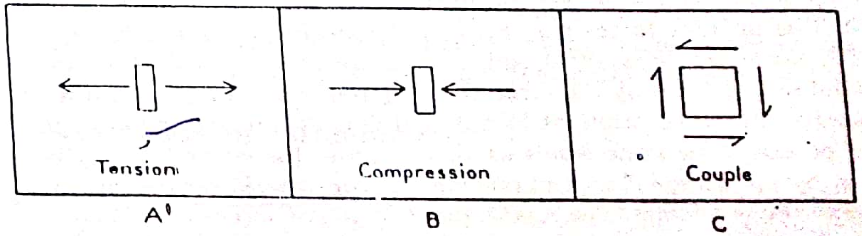

Fig. 2-5. Arrows representing tension, compression, and a couple.

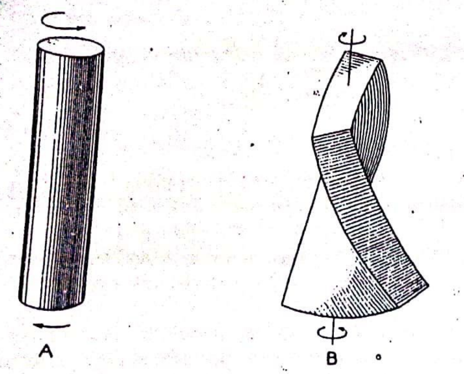

Fig. 2-6. Torsion. A rod (A) or plate (B) is subjected to torsion when the ends are twisted in opposite directions.

A *couple* consists of two equal forces that act in opposite directions in the same plane, but not along the same line. In Fig. 2-5C a couple is represented by the upper and lower arrows, which are not on the same straight line and which are directed away from each other. To prevent rotation and preserve equilibrium a second couple is necessary, as shown by the vertical arrows. The rectangle, which represents the body or part of the body acted upon, may be omitted.

Torsion results from twisting. If the two ends of a rod are turned in opposite directions, the rod is subjected to torsion (Fig. 2-6A). A plate undergoes torsion, as in Fig. 2-6B, if two diagonally opposite corners are subjected to forces acting in one direction while the other two corners are subjected to forces acting in the opposite direction.

Stress

CONCEPT OF STRESS

Imagine a vertical column of material. Along any imaginary horizontal plane within this column, the material above the plane, because of its weight, pushes downward on the material below the plane. Similarly, the part of the column below the plane pushes upward with an equal force on the material above the plane. The mutual action and reaction along a surface constitutes a stress.

Moreover, along any imaginary plane within the column there are

similar actions and reactions. The imaginary plane may be horizontal, vertical, or inclined at any angle. The force, due to the weight of that part of the column that lies above the plane, acts in a vertical direction. This force would be directed normally to a horizontal plane. Along an inclined plane, however, the vertically directed force would be resolved into a normal component and a tangential component. If Fig. 2-3D were turned so that *OP* were vertical, *LM* would represent the inclined plane, *OR* the normal component, and *OQ* the tangential component.

The normal component is a *compressive stress* if it tends to push together the material on opposite sides of the plane. The normal component is a *tensile stress* if it tends to pull apart the material on opposite sides of the plane. The tangential component is generally called a *shearing stress* or *shear*.

In this book, in accordance with common geological practice, a compressive stress will be considered positive, and a tensile stress will be considered negative. In engineering and physics the opposite convention is often followed.

The *stress-difference* at any point in a body is the algebraic difference between the greatest stress and the least stress at that point. This concept is more fully developed in Chap. 7.

Physicists measure stress as the force per unit area; it is stated in pounds per square inch, tons per square foot, kilograms per square centimeter, or similar convenient units. Engineers prefer to use *unit stress* for the force per unit area.

It is essential to distinguish between the external force that is applied to a body and the resulting internal actions and reactions that constitute the stress.

CALCULATION OF STRESS

There is no direct way to measure the stresses in a body, but they may be calculated if the external forces are known. If a body is compressed or stretched, the stress is referred to a plane perpendicular to the direction in which the external forces are acting. Thus, if a vertical square column 10 inches on a side supports a load of 5000 pounds, every horizontal plane in the column is subjected to a compressive force of 5000 pounds if we neglect the weight of the column itself. Each square inch of these horizontal planes supports a load of 50 pounds per square inch. The *compressive stress* is said to be 50 pounds per square inch. If a vertical rod with a cross-sectional area of 10 square inches carries a weight of 5000 pounds at its lower end, every horizontal plane in the rod is subjected to a pull of 500 pounds per square inch. The *tensile stress* is said to be 500 pounds per square inch.

Various techniques have been devised for deducing the stresses in open cuts, bore holes, tunnels, and other underground openings. These devices are attached to the wall of the opening or placed in a bore hole. They actually measure the strain, from which the stresses are deduced. One method employs photoelastic techniques.[3] Others use signals derived from electrical, mechanical, or hydraulic components.[4,5] Such measurements are important in planning large underground openings.

Strain

DEFINITION

Strain is the deformation caused by stress; strain may be *dilation*, which is a change in volume, or *distortion*, which is a change in form, or both.

When there is a change in the confining pressure, an isotropic body—that is, a body whose mechanical properties are uniform in all directions—will change in volume, but not in shape. With increasing confining pressure, the volume of the body decreases and the dilation is negative. With decreasing confining pressure, the volume of the body increases and the dilation is positive.

Under directed forces distortion occurs. For example, a steel rod 10 inches long with a cross section of one square inch is subjected to tension. A pull of 20,000 pounds stretches the rod 0.007 inch. The stress is 20,000 pounds per square inch and the strain is 0.0007 inch per inch.

THREE STAGES OF DEFORMATION

If a body is subjected to directed forces lasting over a short period of time—minutes or hours—it usually passes through three stages of deformation, although in brittle substances the intermediate stage may be omitted. At first, the deformation is *elastic;* that is, if the stress is withdrawn, the body returns to its original shape and size. There is always a limiting stress, called the *elastic limit;* if this is exceeded, the body does not return to its original shape. Below the elastic limit, the deformation obeys *Hooke's law* which states that strain is proportional to stress.

If the stress exceeds the elastic limit, the deformation is *plastic;* that is, the specimen only partially returns to its original shape even if the stress is removed. Steel rods under tension, for example, begin to get thinner or "neck" in the middle, and, even after the stress is released, the constriction remains in the rod.

When there is a continued increase in the stress, one or more fractures develop, and the specimen eventually fails by *rupture*. The arrangement and form of the fractures depend upon several factors which are fully discussed in Chapter 7.

Brittle substances are those that rupture before any significant plastic deformation takes place. *Ductile substances* are those that undergo a large plastic deformation before rupture. After the elastic limit has been exceeded,

ductile substances undergo a long interval of plastic deformation, and in some instances they may never rupture.

ELASTIC DEFORMATION

At room temperature and pressures, and under stresses applied for a short period of time, most rocks are brittle. They behave elastically until they fail by rupture. For such rocks the elastic limit or yield point is the stress at rupture. Ideally an elastic substance will return to the original shape after the deforming stress has been removed, although there may be a slight delay as unloading occurs.

If a solid cylinder of rock is subjected to stress parallel to its long axis, it will lengthen under tension and shorten under compression. The ratio of the stress to the deformation is a measure of the property of the rock to resist deformation.

$$E = \frac{\sigma}{\epsilon} \qquad (2)$$

where E is *Young's modulus* (also called *modulus of elasticity*), σ is stress, and ϵ is strain.

$$\epsilon = \frac{\Delta l}{l_o} \qquad (3)$$

where Δl is change in length, l_o is original length. Thus, if a cylinder 10 centimeters long is stretched 0.001 centimeters under tension by a stress of 10^8 dynes per square centimeter, Young's modulus is 10^{12} dynes per square centimeter. This is an average value for rocks; in the English system it is 1.45×10^7 lb/in.^{2}.

Under tension the diameter of a cylinder subjected to tension parallel to the axis becomes smaller; under compression parallel to the axis the diameter becomes greater. Poisson's ratio is the ratio of transverse strain to axial strain.

$$\upsilon = \frac{\Delta d/d_o}{\Delta l/l_o} \qquad (4)$$

where υ is Poisson's ratio, Δd is change in diameter, d_o is original diameter, and Δl and l_o are as above.

If in the preceding example the diameter decreased by 0.00025 centimeters, Poisson's ratio is 0.25; this is a good average for rocks.

Rigidity measures the resistance to change in shape (Fig. 2-7).

$$G = \frac{\tau}{\gamma} \qquad (5)$$

where G is rigidity modulus, τ is shear stress and γ is the shear strain.

$$\gamma = \frac{ab}{ac} \qquad (6)$$

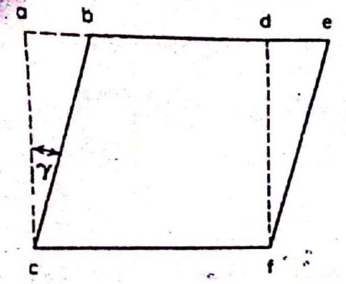

Fig. 2-7. Rigidity. Square *acdf* is deformed into parallelogram *bcef*.

If ac were 10 centimeters, ab were 0.001 centimeters, and τ were 10^7 dynes per square centimeter, the rigidity modulus would be 10^{11} dynes/cm². In rocks the rigidity averages 2 to 5×10^{11} dynes per square centimeter or 2.9 to 7.25×10^6 lb/in.².

G may also be expressed in terms of Young's modulus and Poisson's ratio.

$$G = \frac{E}{2(1 + \upsilon)} \tag{7}$$

The bulk modulus or incompressibility is

$$K = \frac{\Delta h}{\Delta V / V_o} \tag{8}$$

where K is the bulk modulus, Δh is change in hydrostatic pressure, ΔV is change in volume, and V_o is original volume. Δh may refer to lithostatic pressure instead of hydrostatic pressure. Bulk modulus may be expressed in dynes per square centimeter or some equivalent unit.

A rock at a depth of 10 kilometers (6.214 miles) is under a lithostatic pressure of 2.7×10^6 g/cm². If it is raised to a depth of 4 km, Δh becomes 1.62×10^6 g/cm². If the original volume were 1000 cubic centimeters and the new volume is 999.000 cubic centimeters, $\Delta V / V_o$ is 10^{-3}. The bulk modulus is 1.62×10^9 g/cm² or 1.59×10^{10} dynes/cm².

The compressibility is the reciprocal of the bulk modulus

$$\beta = \frac{1}{K} \tag{9}$$

where β is compressibility.

Elastic deformation is primarily of importance in analyzing tidal deformation of the solid earth and in investigating the transmission of seismic waves through the earth. It is of even more direct significance to structural geology in studying elastic rebound associated with earthquakes, in the fracturing of rocks to produce joints and faults, and in certain aspects of fold-

ing. Although most structural features observed by structural geologists are the result of deformation beyond the elastic limit, the same parameters of length, mass, time, and force are involved.

PLASTIC DEFORMATION

Although most rocks at room temperatures and pressures fail by rupture before attaining a stage of plastic deformation, most rocks, at sufficiently high temperatures and confining pressures deform plastically even in experiments lasting for a short time. This plastic deformation is not recoverable or is only partially recoverable. That is, if the stress is removed, the material does not return to its original shape.

In much experimental work the deformation of rocks results from internal adjustments within the mineral grains, notably gliding and dislocations (Chap. 20). Hence metallurgists tend to confine the concept of plasticity to such deformation. But in nature other factors are also important, notably the rotation of grains and recrystallization. But the internal changes that take place in a rock undergoing plastic deformation are discussed more fully in Chap. 20.

Stress-Strain Diagrams

The relation existing between stress and strain is commonly expressed in graphs known as stress-strain diagrams (Fig. 2-8). The stress is plotted on the ordinate (vertical axis), whereas the strain is plotted on the abscissa (horizontal axis). In Fig. 2-8 the material is under compression and the compressive stress parallel to the axis of the cylinder is in pounds per square inch. With increasing stress the specimen becomes shorter and the strain is plotted in terms of the percentage of the shortening of the specimen.

Curve A is the stress-strain diagram of a brittle substance. It deforms elastically up to a stress of 20,000 lb/in.² and has shortened one-half of one percent; it then fails by rupture.

Curve B is an ideal plastic substance. It behaves elastically at first. At a stress of about 24,000 lb/in.² it reaches the *proportional elastic limit*, which is the point at which the curve departs from the straight line. The shortening is slightly less than one percent. Thereafter the specimen deforms continuously without any added stress.

Curve C represents a more normal type of plastic behavior. At a stres of about 28,000 lb/in.² and a strain of somewhat over 1 percent, the specime reaches the proportional elastic limit and thereafter deforms plastically. Bu for every increment of strain an increase in stress is necessary. This is th result of what is called *work hardening*; that is, the specimen becomes pr gressively more difficult to deform.

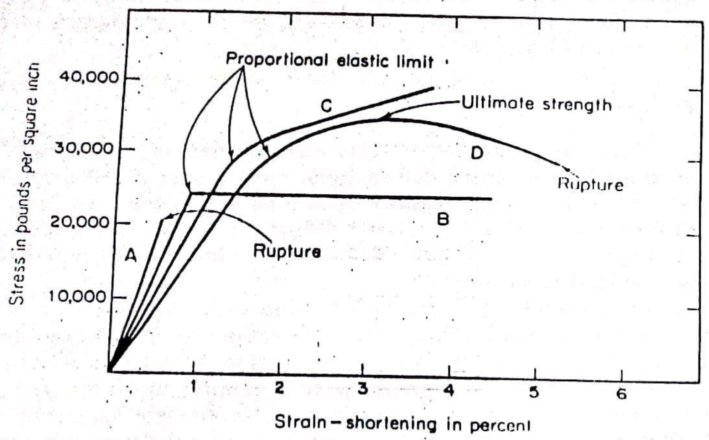

Fig. 2-8. Stress-strain diagrams.

Curve *D* represents a very common type of plastic deformation. The specimen deforms elastically up to a stress of about 28,000 lb/in.² and a shortening of somewhat less than 2 percent. At first an increase in stress is necessary for continued deformation. But when the shortening is somewhat over 3 percent, progressively less stress is necessary to continue the deformation. This high point on the curve is the *ultimate strength*. However, the ultimate strength of a rock is a function of many variables, such as confining pressure and temperature.

The term "strength" is a rather meaningless term unless all the environmental conditions are specified. The value of the breaking strength—normally applied to brittle materials under temperature and pressures at the surface of the earth—is of use in engineering projects. Under compression, diabase, basalt, felsite, and quartzite are strong, with an average breaking strength of about 1800–2600 kg/cm². Marlstone, limestone, marble, and sandstone are weak, with an average breaking strength 700–1000 kg/cm². Under tension, the breaking strengths are far less, averaging only 5 percent to 10 percent of the breaking strength under compression.

More precise values are not given intentionally, because the figures may be misleading. The shape of the test specimen may control the results. Moreover, the strength of the same kind of rock may vary greatly, depending on the locality from which it comes. Finally, the published data usually represent "intact" rocks, that is, small specimens that are not marred by flaws. Larger units, such as those significant to regional tectonics, are generally characterized by joints and other planes of weakness.

Factors Controlling Behavior of Materials

CONFINING PRESSURE

The mechanical behavior of rocks is controlled not only by their inherent properties—mineralogy, grain size, porosity, fractures, etc.—but also by factors that are of little or no concern in planning man-made structures at the surface of the earth. These factors are confining pressure, temperature, time, and solutions. Tremendous ingenuity has been required to design the experimental apparatus that has been utilized in obtaining the data presented in the following pages.

Cylinders of rock are prepared for the experiments; usually the length is several times the diameter. In some experiments the cylinders are small, the length not exceeding one or a few inches. By using small cylinders it is possible to employ both high confining pressures and high temperatures. When high temperatures are not a factor, much larger specimens, several feet long, may be used. The confining pressure—in this case a hydrostatic pressure—is obtained by a fluid under pressure. The compressive (or tensile) stress is applied by a plunger (or gripping device) at the end of the cylinders. Often the specimens are "jacketed" by aluminum foil or some similar material to prevent fluid from escaping into the specimen and thus weakening it.

In this experimental work various systems of units have been used. Some experiments use the C.G.S. system exclusively. Others use the English system. Others use a combination of the two. Table 2-1 is a conversion table to show how the various systems are related.

Figure 2-9 illustrates the behavior of Solenhofen Limestone under such conditions.[6] The compressive stress on the ends of the cylinder is given on the ordinate in kilograms per square centimeter. The percentage of shortening of the cylinder is given on the abscissa. Seven separate experiments are shown at confining pressures of 1, 300, 700, 1000, 2000, 3000, and 4000 kilograms per square centimeter. Separate curves are given for the behavior at each of these confining pressures. Below a compressive stress of 3700 kg/cm² the curves run together and appear as one. One experiment was run in air so that the confining pressure was equal to 1 kg/cm², that is, 14.7 lb/in.², or 1 atmosphere. This specimen behaved elastically up to a compressive stress of 2800 kg/cm², when it failed by rupture. The specimens tested under confining pressures of 300 and 700 kg/cm² deformed elastically, went through a short stage of plastic deformation—that portion of the lines that is bending—and then failed by rupture. The specimens tested under confining pressures of 1000 or more kg/cm² began to deform plastically at a compressive stress of about 4000 kg/cm² and continued to deform plastically. The specimen tested under a confining pressure of 2000 kg/cm² had shortened 30 percent

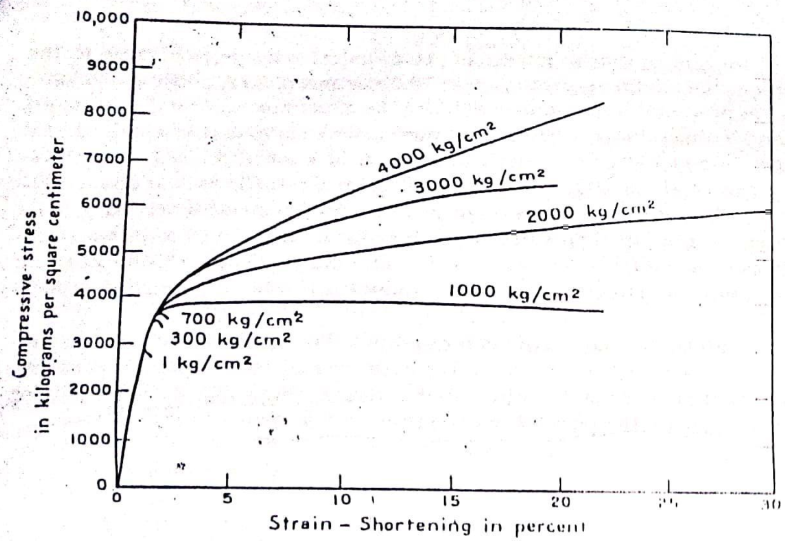

Fig. 2-9. Effect of confining pressure on behavior of Solenhofen Limestone under compression. (After E. Robertson.[6])

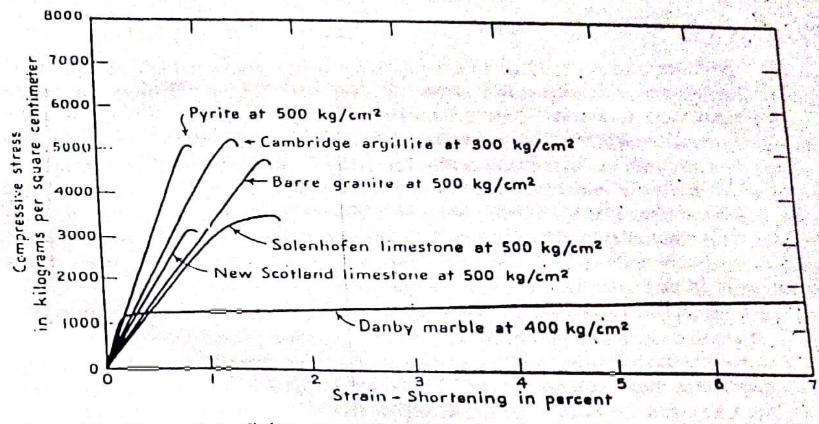

Fig. 2-10. Effect of confining pressure on behavior of various rocks and pyrite under compression. (After E. Robertson, "An experimental study of flow and fracture in rocks," doctoral thesis, Harvard University, 1952.)

when the test was terminated. The curves representing the tests at a confining pressure of 1000, 2000, 3000, and 4000 kg/cm² end, not because of failure by rupture, but because the tests were not carried any further.

It is also readily apparent that the strength increases with the confining pressure. Whereas the specimen tested at a confining pressure of 1 kg/cm² fails by rupture at a compressive stress of 2800 kg/cm² and the specimen tested at a confining pressure of 1000 kg/cm² cannot support a compressive stress greater than 3900 kg/cm², the specimen tested at a confining pressure of 4000 kg/cm² can support a compressive stress in excess of 8000 kg/cm².

Such experiments indicate that rocks exhibiting very little plastic deformation near the surface of the earth may be very plastic under high confining pressure. Thus under a confining pressure of 1000 kg/cm² or greater, Solenhofen limestone will deform plastically. This means that at a depth of 2.5 miles Solenhofen limestone will deform plastically if sufficient compressive stress is applied; as will be shown later, this figure may be still less because of other factors.

Different rocks, of course, behave differently. Figure 2-10 shows the stress-strain diagram for several rocks and one mineral.[6] The results are not strictly comparable because, as the figure shows, the confining pressure was not the same in all experiments, ranging from 300 to 500 kg/cm². Pyrite, Cambridge

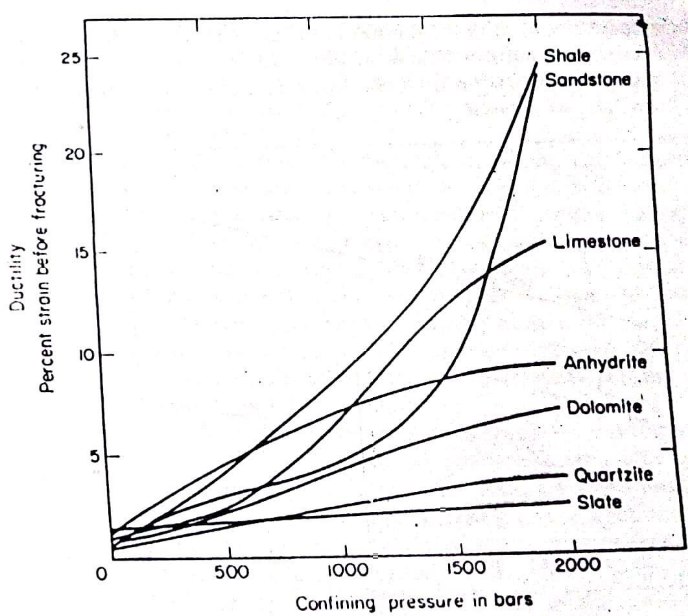

Fig. 2-11. Ductility of several common rocks under differing confining pressures. (After Donath.[7] Reprinted by permission, *American Scientist*, Journal of The Society of Sigma Xi.)

Argillite, and Barre Granite are relatively brittle rocks, which behave elastically up to a compressive stress of over 4500 kg/cm². Above the elastic limit there is a small zone of plastic deformation, and then rupture takes place. New Scotland limestone was elastic up to a compressive stress of about 3000 kg/cm², deformed plastically for a short interval, and then ruptured at 3200 kg/cm². Solenhofen Limestone shows a still larger range of plastic deformation. Danby Marble is much weaker. It deforms elastically up to a compressive stress of 1000 kg/cm² and then deforms plastically. Although the curve scale ends at 7 percent, the original data show that the specimen shortened 14 percent before the test was ended.

Figure 2-11 illustrates the effect of confining pressure on the breaking strength of several different rocks.[7] At atmospheric pressure—confining pressure of one bar—the rocks deform only a few percent before fracturing. Under a confining pressure of 1000 bars the sandstone and shale deform more than 5 percent before rupturing. Under a confining pressure of 2000 bars the limestone deforms nearly 15 percent and shale and sandstone over 20 percent before rupturing.

TEMPERATURE

Changes in temperature modify the strength of rocks.[8] Hot steel, for example, undergoes plastic deformation much more readily than does cold steel. Figure 2-12 shows two tests run on Yule Marble. Conditions were identical

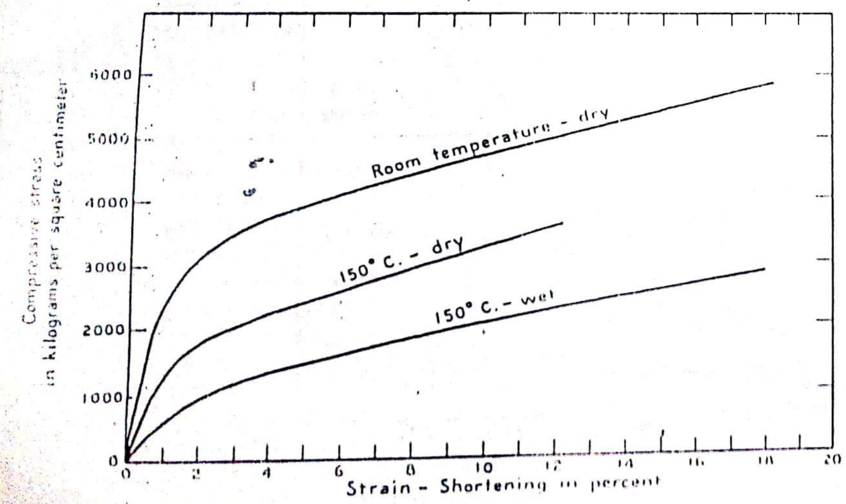

Fig. 2-12. Effect of temperature and solutions on deformation of marble. Yule Marble, confining pressure of 10,000 atmospheres, cylindrical specimen cut perpendicular to foliation. (After D. T. Griggs, et al.[9])

except for temperature; the axes of the cylinders were perpendicular to the foliation, the confining pressure was 10,000 atmospheres, and the deformation was produced by compressive stress. The uppermost curve is that obtained at room temperatures, whereas the intermediate curve is that obtained at a temperature of 150°C. At room temperature the elastic limit is at a compressive stress of about 2000 kg/cm², at 150°C the elastic limit is at about 1000 kg/cm². Moreover, to produce a given strain far less stress is necessary when the specimen is hot than when it is cold. For example, to produce a strain of 10 percent at 150°C the compressive stress is 3000 kg/cm², but at room temperature the stress necessary to produce a similar deformation is 4500 kg/cm².

It is apparent that plastic deformation is far less common near the surface of the earth, where the confining pressure and the temperature are low, than it is at greater depths, where higher temperatures and greater confining pressure increase the possibility of plastic deformation.

TIME

Geological processes have great lengths of time in which to operate. Although geologic time is impossible to duplicate experimentally, it is possible from experiments to make some deductions concerning the influence of time. An analysis of the effects of time is concerned with such subjects as creep, strain-rate, and viscosity.

Creep refers to the slow continuous deformation with the passage of time." The stresses may be above or below the elastic limit, but we are especially interested in creep caused by stresses below the elastic limit.

Solenhofen Limestone under atmospheric pressure and at room temperature has a strength of 2560 kg/cm². In a long-time experiment, Solenhofen Limestone subjected to a compressive stress of 1400 kg/cm²—half the value of the strength—deforms rapidly at first, then more slowly (Fig. 2-13). At the end of one day, it has been shortened about 0.006 percent; after 10 days about 0.011 percent; after 100 days about 0.016 percent; and after 400 days a little more than 0.019 percent.

The general form of a creep curve is shown in Fig. 2-14. The ordinate is the total strain and the abscissa is time. The intercept A on the ordinate represents the instantaneous strain when the load is added. The first part of the curve, B, represents primary creep, when the strain decreases with time. The main part of the curve, C, represents secondary or steady-state creep. Finally, in tertiary creep, D, the curve sharply rises just before rupture.

This curve may be expressed as an equation

$$S = A + B \log t + Ct + D \tag{10}$$

where S is total strain, t is time, A, B, and C are constants, and D is strain during tertiary creep.

Strain rate is the amount of strain (pp. 18 and 21) divided by the time.

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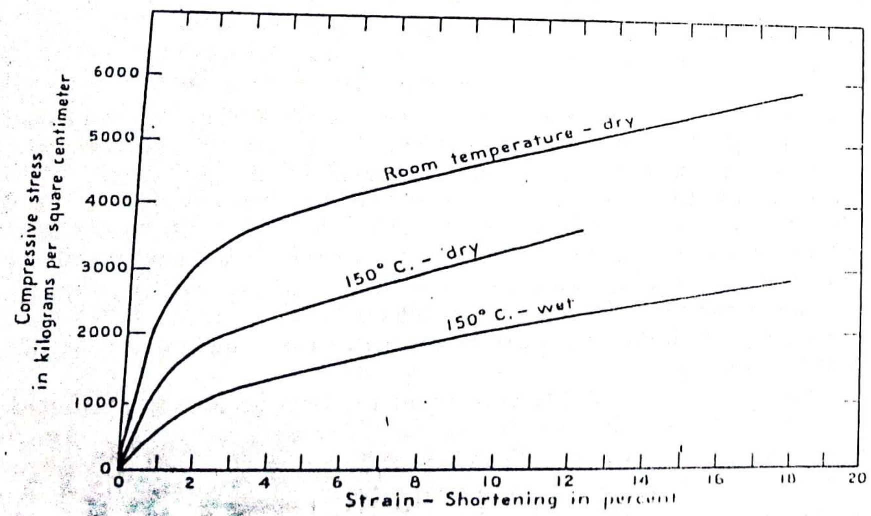

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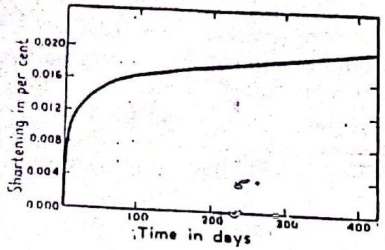

Fig. 2-13. Creep curve for Solenhofen Limestone under a stress of 1400 kg/cm². (After D. T. Griggs.[9])

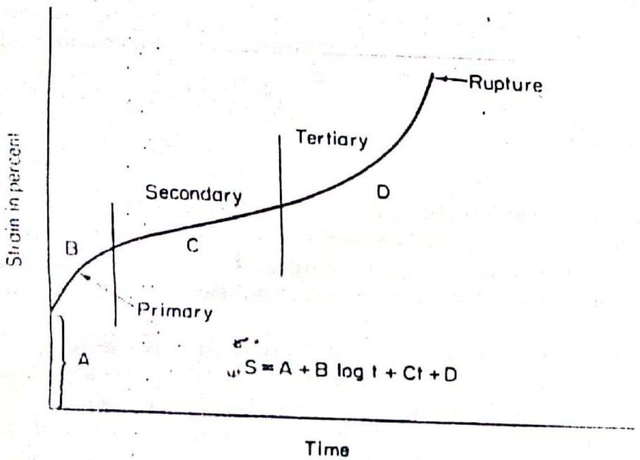

Fig. 2-14. Ideal creep curve. (A) Instantaneous deformation. (B) Primary creep. (C) Secondary creep. (D) Tertiary creep. S is the total strain; t is time.

$$\dot{\epsilon} = \frac{\epsilon}{t} \qquad (11)$$

where $\dot{\epsilon}$ is strain rate, ϵ is strain, and t is time, usually given in seconds. In Fig. 2-13 the instantaneous deformation and the primary and secondary creep are shown well. For that part of the curve in Figure 2-13 in which the shortening exceeds 0.016, the strain rate is

$$\dot{\epsilon} = \frac{3.2 \times 10^{-5}}{325 \times 24 \times 60 \times 60} = \frac{3.2 \times 10^{-5}}{2.81 \times 10^{7}} = 1.139 \times 10^{-12}\ \text{sec} \qquad (12)$$

Experimental apparatus has been devised so that the strain rate can be kept constant.[10] Figures 2-15 and 2-16 show two sets of graphs for Yule Marble

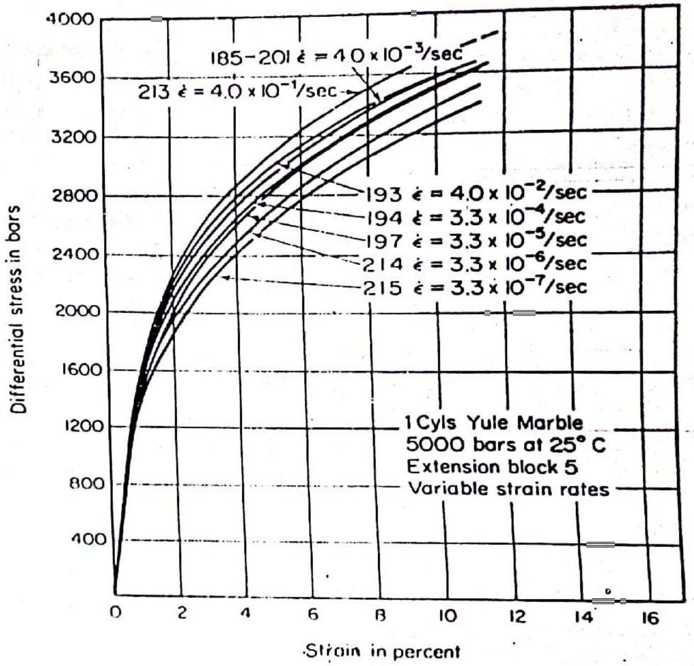

Fig. 2-15. Effect of strain rate on stress-strain curve at 25°C, confining pressure of 5000 bars. (Hugh C. Heard.[10]) Permission University of Chicago Press.

subjected to tension under a confining pressure of 5000 bars. The cylinders were cut perpendicular to the foliation. Figure 2-15 gives the results for experiments conducted at 25°C, Fig. 2-16 for experiments at a temperature of 500°C. In Fig. 2-16 eight experiments were run, at strain rates of 4.0×10^{-1}/sec, 4.0×10^{-2}/sec, etc., up to a minimum strain rate of 3.3×10^{-8}/sec. Curve 153, $\dot{\epsilon} = 3.3 \times 10^{-7}$/sec shows that the strain is very small until the differential stress reaches 500 bars, when the strain increases relatively rapidly, attaining a value of 10 percent at a differential stress of about 725 bars. The time to reach this strain can be calculated from equation (11) as 3.03×10^{4} sec or $8\frac{1}{2}$ hr.

From Figs. 2-15 and 2-16 two important principles may be established. (1) The slower the strain rate, the less the differential stress to attain a given strain. In Fig. 2-16 a differential stress of 1820 bars is necessary to attain 10 percent lengthening if the strain rate is 4.0×10^{-1}/sec, but only 450 bars are necessary if the strain rate is 3.3×10^{-8} sec.
(2) The higher the temperature, the less the required differential stress for a given strain; compare Fig. 2-15 (25°C) and Fig. 2-16 (500°C).

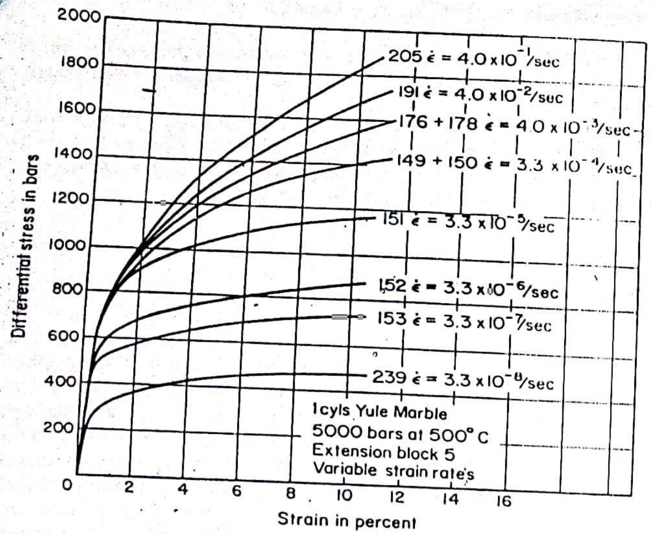

Fig. 2-16. *Effect of strain rate on stress-strain curves at 500°C, confining pressure of 5000 bars. (Hugh C. Heard.[10]) Permission University of Chicago Press.*

Normally we think of *viscosity* in relation to liquids and more specifically the ease with which they flow. Water is much less viscous than syrup, and syrup much less viscous than tar. But the concept of viscosity, or, more precisely, apparent viscosity, may be applied to solids.

In a perfectly viscous substance, characterized by Newtonian viscosity, viscosity is defined as the ratio of shearing stress to the rate of shear. Referring to Fig. 2-7, if shear stress is applied the rapidity with which the square *adcf* is deformed into the parallelogram *becf* is a function of the viscosity. The rate of shear is measured by the change in angle γ per unit of time. If the stress is measured in dynes/cm² and the rate of shear in seconds, the viscosity is given in dynes-sec/cm²; this unit is called the *poise*.

$$\dot{\gamma} = \frac{\gamma}{t} \tag{13}$$

and

$$\eta = \frac{\tau}{\dot{\gamma}} \tag{14}$$

where $\dot{\gamma}$ is rate of shear strain, γ is shear strain, t is time, η is viscosity, and τ is shearing stress.

Calculations show that the viscosity of Yule Marble, used in the experiments referred to in Figs. 2-15 and 2-16, ranges from 10^{23} poises at 25°C to 10^{18} poises at 500°C. Additional values of viscosity are given in Table 2-2.

Table 2-2. Some Values of Viscosity in Poises

Water at 100°C and one atmosphere	0.00284
Water at 30°C and one atmosphere	0.00801
Water at 0°C and one atmosphere	0.01792
Corn syrup, room temperature and pressure	7×10^2
Roofing tar, ready to apply	3×10^7
Lava, Mt. Vesuvius, 1400°C	2.66×10^2
Lava, Mt. Vesuvius, 1100°C	2.83×10^4
Rock salt, near surface	10^{17}
Rocks in general	10^{17} to 10^{22}
Mantle of earth	10^{23}

Creep is the combined effect of an elastic strain and a permanent strain. It is viscoelastic deformation rather than pure viscous deformation or elastic deformation. The specimen recovers from that portion of the deformation that is caused by the elastic strain, whereas that portion of the deformation that is the result of permanent strain is unrecoverable.

The *fundamental strength* of any material is defined as the stress which that material is able to withstand, regardless of time, under given physical conditions—temperature, pressure, solutions—without rupturing or deforming continuously. The fundamental strength, which is always less than the breaking strength and the ultimate strength, is much more significant to the geologist. Unfortunately, at the present time we have few data on the value of the fundamental strength of rocks.

The mechanics of plastic deformation and creep—that is, the changes that take place in the rock during these processes—is discussed in Chap. 20.

SOLUTIONS

Much rock deformation takes place while solutions capable of reacting chemically with the rock are present in the pore spaces. This is notably true of metamorphic rocks, in which extensive or complete recrystallization occurs. The solutions dissolve old minerals and precipitate new ones. Under such conditions the mechanical properties of rock are greatly modified.

Creep experiments have been performed[11] on alabaster (a variety of gypsum) with solutions present (Fig. 2-17). In all cases the compressive stress was 205 kg/cm² (less than half the normal elastic limit of 480 kg/cm²), and the temperature 24°C. The lowest curve represents the deformation of a dry specimen. Within a few days the specimen had shortened about 0.03 percent,

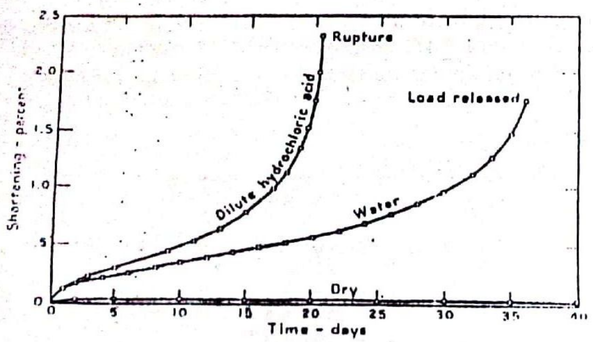

Fig. 2-17. Effect of solutions on deformation of alabaster. (After D. T. Griggs.[9])

but there was no further detectable deformation even after 40 days. A specimen deformed under such conditions that water had access to the alabaster (intermediate curve) had shortened 1 percent at the end of 30 days and 1.75 percent by the end of 36 days, when the load was released. A specimen deformed with access of dilute hydrochloric acid had deformed more than 2 percent before rupturing at the end of 20 days. Whereas the strength of the dry alabaster under room temperature and a confining pressure of 1 atmosphere is 480 kg/cm², and the ultimate strength is 520 kg/cm², the fundamental strength under similar conditions, but with the specimen free to react with water, is estimated to be only 92 kg/cm². In this particular case, therefore, the fundamental strength is less than 20 percent of the strength and the ultimate strength.

The lowest curve in Fig. 2-12 shows the effect of water on the strength of Yule Marble. At a temperature of 150°C the elastic limit and strength of the wet specimen is much less than the strength of the dry specimen at the same temperature.

PORE PRESSURE

In recent years it has been suggested that high fluid pressure in the pore spaces of rocks partially balances the lithostatic pressure. The reality of such high pressures has been demonstrated in some oil fields. Imagine an open bore hole 1 km deep filled with water. The hydrostatic pressure at the bottom of the hole is 10^5 g/cm². If the rock is sufficiently permeable for water to penetrate into all the pore spaces, this pressure in part balances the lithostatic pressure.

Let λ be the ratio of pore pressure to lithostatic pressure. In the case just cited $\lambda = 1/2.3 = 0.435$. In some deep wells λ is much greater, and may approach 1 in value.

This pore pressure weakens the rock. Normally the strength of a rock increases at depth because of the increase in confining pressure. But with increasing pore pressure the effective confining pressure decreases. Moreover, with increasing pore pressure the rocks are less coherent.

ANISOTROPY AND INHOMOGENEITY

Most of the tests described in the preceding sections were made on isotropic materials, that is, rocks whose mechanical properties were uniform in all directions. Rocks that show bedding, banding, or foliation are not isotropic. The strength of such rocks would depend upon the orientation of the applied forces to the planar structures of the rock. This point is well illustrated in Fig. 2-18. The rock was Yule Marble, confining pressure was 10,000 kg/cm², and the tests were run at room temperature. All the specimens show great plastic deformation. The solid lines represent experiments under compression; in this case the stress is compressive and the strain is shortening parallel to the axis of the cylinder. Under compression the cylinder perpendicular to the foliation is stronger than the cylinder parallel to the foliation. The broken lines represent tests under tension; in this case the stress is tensile and the strain is lengthening parallel to the cylinders. Under tension the cylinder

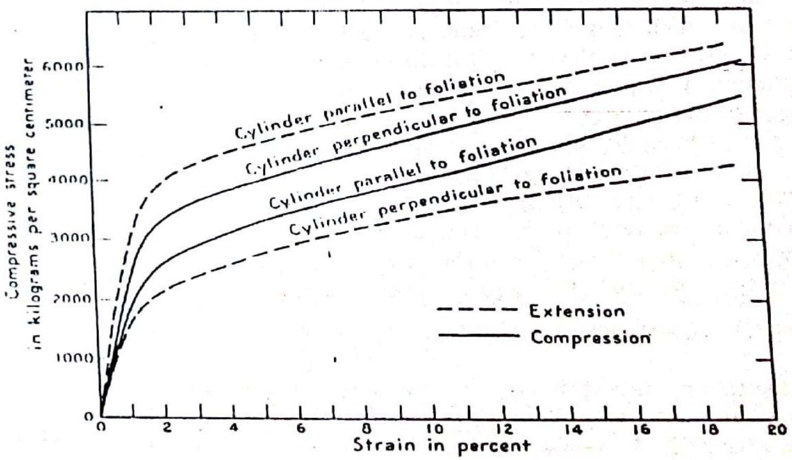

Fig. 2-18. Effect of anisotropy on deformation of marble. Yule Marble at confining pressure of 10,000 kg/cm² and room temperature. (After D. T. Griggs, et al.[9])

parallel to the foliation is much stronger than the cylinder perpendicular to the foliation.

SUMMARY

It is clear that the mechanical properties of rocks are profoundly modified by confining pressure, temperature, the time factor, and the presence of reacting solutions. The combined effect of these factors is so great that it is impossible in the present state of our knowledge to treat rock deformation in a quantitative way. Increase in confining pressure increases the elastic limit and the ultimate strength. Increase in the temperature weakens the rocks. After long continued stress the rocks become much weaker. The fundamental strength is of more interest to the structural geologist than is the strength or ultimate strength. Reacting solutions lower the strength, the ultimate strength, and the fundamental strength of rocks.

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