7

Rock Testing

7·1. Introduction

When a rock is being used for engineering structures, there may be two possibilities of using it.

(*a*) Rock on/in which construction will be made ;

(*b*) Rock with which construction will be made.

Construction of dams, tunnels, underground power houses etc. come under the first category. Dam is supported on the rock mass, tunnels and underground power houses are made making cavities or openings through the rock mass. Similar case is there when mine shafts are made into the rock mass.

Examples of construction, pertaining to the second category are seen where rock mass in the form of aggregates are used for construction of structures such as masonry dams, quay walls, jetties etc. In such cases. properties of rock aggregates such as crushing strength, impact values, soundness etc. are determined in a laboratory and these are discussed in text books of concrete and roads.

In rock mechanics, we are mainly concerned with the strength properties of the rock mass on which construction is to take place. When the load will be applied due to the construction of a structure it is to be seen as to how a rock mass will behave ; how much stress it can take and how much deformation in the rock mass will be there. Engineers are mainly concerned with these informations, because after knowing these parameters only, they will be able to design the structure. They will be able to fix the dimensions of different components of the structure.

Therefore, the mechanical or engineering properties of a rock mass which are to be investigated for designing foundations, hydraulic structures, underground openings etc. are—

(*i*) Strength ; and

(*ii*) Elastic constants.

With strength properties, we can ascertain whether a particular rock mass is suitable for a particular stress system or not. The strength properties, which are investigated for rock mass, are—

(*a*) Uniaxial compressive strength,

(*b*) Uniaxial tensile strength,

(*c*) Flexural strength,

(*d*) Shear strength,

(*e*) Triaxial strength.

Elastic constants relate different types of stresses with the corresponding strains. Sometimes, stress measurement in the rock mass is difficult especially in case of 'in-situ' conditions. But the measurement of deformations can be done easily. In such cases, necessity of elastic constants arise. Therefore, a knowledge of measurement of elastic constants is also necessary for an engineer dealing with problems of rock mechanics. The important elastic constants are—

(*i*) Modulus of elasticity,

(*ii*) Poisson's ratio,

(*iii*) Modulus of rigidity,

(*iv*) Bulk modulus,

(*v*) Lame's constants.

7·2. Sampling

The behaviour of rock mass in-situ conditions are not governed by the materials of compositions but by the defects existing in the rock mass. Because on application of a load, the weakest plane will be the joints, fissures, faults etc. along which the failure will start before propagation of failure in the body of the rock material. That is why, great importance has been given to the methods of "In-situ testing" in rock mechanics. But, always 'in-situ' tests may not be possible. Sometimes, finance may be limited or heavy equipment may not be available. In such cases the engineer has to ascertain the behaviour of the rock mass with testing rock samples in the laboratory. Hence, great importance has been given to "Sampling" in rock mechanics. Rock mass in general is non-homogeneous and the properties of the samples taken from one portion of the rock mass may be different from those taken from another portion. Therefore, samples collected must truly represent the rock mass, the properties of which are to be determined. To ensure proper sampling, following points should be kept in mind.

(*i*) Lithological studies—so that regions may be identified which differ in their mineral compositions, nature of cementing material and texture ;

(*ii*) Existence of bedding planes and their identification ;

(*iii*) Presence of planes of weaknesses such as cleavage planes, joints, cracks etc. ;

(*iv*) Presence of faults, dykes, folds if any, because rock properties in these regions vary much.

7·2·1. Sample Preparation

Samples for laboratory testing are collected in the form of large blocks from the field. Out of these samples, test specimens are prepared for a particular type of testing. While taking samples from field following care should be taken.

(*i*) Samples of hard rocks are taken by drilling closely spaced overlapping holes or chiselling, if drilling equipment is not available ;

(*ii*) Samples from soft rocks should be taken with care. They may be cut by a rock saw ;

(*iii*) Sometimes hard rock samples are taken after blasting. Blasting, of course, should be avoided. If at all it is necessary, care should be taken that high intensity of blasting should not be used. High blasting may produce cracks in the rock mass due to which test result may be misleading ;

(*iv*) Samples from greater depths may be obtained in the form of cores by diamond drilling ;

(*v*) If irregular specimens are to be tested then sampling by blasting should be avoided because in case of blasting weaker portion of the rock may be broken into very small pieces and while making specimen they may be left over. Therefore, in such cases, care should be taken that blasting intensity should not be too high. To overcome the difficulty, sometimes a bigger block is broken manually and specimens are taken.

(*vi*) The collected samples should be marked on the map to indicate their original position and orientation in the parent rock mass.

7·3. Specimen

When the rock samples are brought from field to the laboratory, the test specimen are required to be prepared for a particular test. The shape and size of specimen depend upon the type of the test to be conducted. The specimen may be of regular or irregular shape. Irregular shapes are used for specific tests but most of the specimen for mechanical properties are of regular shape.

Regular shaped specimen are

(*a*) Cylindrical.

(*b*) Prismatic,

(*c*) Cubical.

Most of the specimens for tests to determine mechanical properties are cylindrical. Since, they are obtained by drilling the bore holes at different depths, they are used with minimum specimen preparation formalities. Typical diameter range used for cylindrical samples is 25 mm to 50 mm. Diameter of the specimen generally coincides with the diameter of core samples from the bore holes. Length to diameter ratios for different types of tests are as follows.

Compressive strength test	: 2 to 3
Bending test	: 3 to 7
Brazilian test (For tensile strength)	: 0·5 to 1·0

To avoid the stress concentration at the time of testing it should be seen that both the ends of the specimen should be perfectly plane, parallel to each other and normal to the axis of the specimen. Stress concentration causes the sample to fail at a smaller load.

To avoid surface irregularity the specimen should be cut by disc saw and finished by a surface grinder.

In addition to cylindrical shapes, specimens are of prismatic or cubical shapes also. From the irregular rock samples, first plates are cut out. Next from plates bars and finally prism shaped specimens are prepared as per requirement. At the time of cutting water is continuously poured to dissipate the heat generated which may cause cracks in the specimen due to rise of temperature.

The result obtained from testing a single specimen may not represent the property of the parent rock mass yet the result will be nearer to the absolute value, if a large number of specimens are tested. But unlimited number of specimens cannot be tested to get an absolute value. To limit the testing work, it is necessary to ascertain the minimum number of specimen to be tested without sacrificing the reliability of the test result.

7·3·1. The minimum number of specimen to be tested depends upon

(*a*) Variability of the test results,

(*b*) Desired accuracy.

The variability depends upon non-homogeneity of a rock mass and the size of specimen. The smaller the size of the specimen the larger will be the variance.

Desired accuracy depends upon the importance of a structure and the stage of investigation.

The following relation from statistical theory gives the number of specimen n to be tested. (Vutukuri et al 1974)

$$p=\frac{\left(100+\frac{kV}{\sqrt{n}}\right)}{\left(100-\frac{kV}{\sqrt{n}}\right)} \qquad \ldots(7{\cdot}1)$$

where V=coefficient of variation

p= a number

$$V=\frac{s}{\bar{x}}\times 100\%$$

where $\bar{x}$=the mean value

$$=\frac{\Sigma x_1}{N}$$

s=standard deviation

$$=\sqrt{\left\{\frac{\Sigma_i (x_i-\bar{x})^2}{N-1}\right\}}$$

The value of k depends upon the desired confidence interval and the number of specimen tested in N preliminary experiments. The value p depends upon the permissible deviation. For permissible deviation of $\pm$ 20% the mean value for p is 1·5.

For strength (compressive and tensile) determination, literatures report that for marble 2 to 3, shale 5, sandstone 5 to 10, granite, andesite and sandy tuff 10 or more number of specimen are required to be tested.

7·4. Uniaxial Compressive Strength Test (Brady + Brown).

In case of underground mines, rock columns support the roof. Similarly, for other situations also, rock mass has to support loads. For stability of columns and other supporting structures, it is necessary to know the compressive strength of the rock material.

The specimens to be tested are either cylindrical or cubical in shape. But cylindrical samples are more common. The cylindrical samples are cut to size by a diamond saw and surface irregularities if any, are smoothened by a surface polishing machine. The length of the specimen preferable should be 2·5 times the diameter. Otherwise L/D ratio may be 2 to 3. The ends of the specimen should be parallel to each other and normal to the axis of the specimen.

The Committee of International Society for Rock Mechanics (I.S.R.M.) on Laboratory Tests (1972) has recommended the following tolerance on dimensions of cylindrical specimen for compressive strength test.

(*a*) The ends of the specimen shall be flat to 0·02 mm.

(*b*) The ends of specimen shall be perpendicular to the axis of the specimen within 0·001 radian (3·5 minutes).

(c) The sides of the specimen shall be smooth and free of abrupt irregularities and straight to within 0·3 mm over the full length of the specimen.

Other type of specimen for test is cubical which has L/D ratio 1.

The specimen is kept between two loading platens of a compression testing machine. The applied load is read in the proving ring as shown in Fig. 7·1. After putting the specimen it is seen that the specimen is in full contact with the loading platens. If there is some gap, the surfaces should be polished. If full contact is not there, the application of load will be on a lesser area and the sample will fail earlier. Sometimes, the crushed piece may fly off the sample due to which some accident may occur. To prevent this the specimen is enclosed in some flexible wire or plastic mesh.

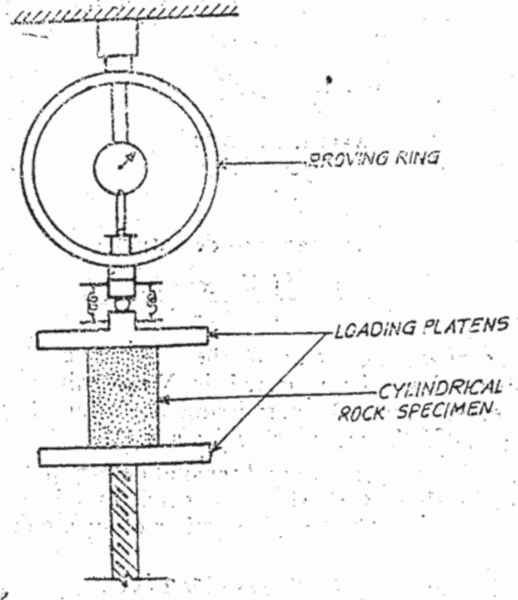


Fig. 7·1. Setup for uniaxial compressive strength test.

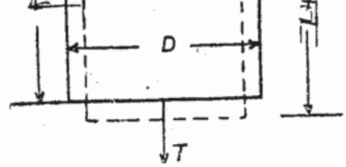


Fig. 7·1 (a)

The load should be applied at the rate of 0·5 to 1·0 MP_a/sec ($\approx$5 to 10 kgf/cm^2) so that failure occurs in 5 to 10 minutes of time. Rate of loading is an important factor for testing because strength depends on the rate of loading also.

Knowing the failure load and cross sectional area of a sample, compressive strength of the specimen can be known which will be equal to failure load divided by cross-sectional area of the sample.

If the specimens of different lengths are there, then the values can be adjusted for D/L ratio of 1 by the formula suggested by ASTM.

$$\sigma_{cu} = \frac{\sigma_{ult}}{[0{\cdot}778 + 0{\cdot}222\,(D/L)]} \quad ...(7{\cdot}2)$$

where σ_{cu} = Computed ultimate compressive strength of an equivalent cubical or cylindrical specimen whose D/L ratio is 1. This value is also termed as adjusted compressive strength.

σ_{ult} = Ultimate compressive strength of the test specimen whose length L is greater than its diameter D.

D = Dia. of a cylindrical sample and the side length in case of a cubical sample.

L = Length or height of the sample.

In addition to the rate of loading and specimen size, other factors on which compressive strength of rock depends are shape, surface quality of loading platens, rock specimen surface, porosity and moisture content of the rock. Compressive strength of a rock decreases with an increase in its porosity. Water in rock pores reduces the magnitude of internal friction of rock resulting in a decrease of its strength. Hence increase in moisture content of the rock reduces its strength. The ratio of dry to wet strength of rock is known as its softening factor. For most of the cases this value is found to be nearer to 3, which means that a wet sample has its strength one third of a dry sample.

7·5. Tensile Strength Tests

Tensile strength may be defined as the maximum stresses developed in a specimen in a tension test performed to rupture it. In rock mechanics a knowledge of tensile strength of rock is important in designing roofs and domes of underground openings. A rock slab or beam subjected to bending also experiences a tensile stress. Although rock is weak in tension, an average tensile strength of rock has been found to be one tenth of its compressive strength.

Although the principle of measurement of tensile strength is to

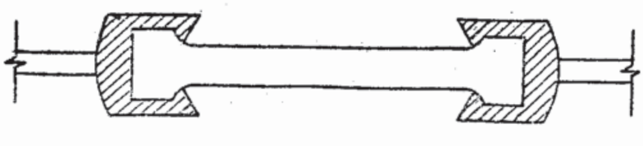


Fig. 7·2. Shape of a specimen for tensile test.

apply an axial tensile force to a rock specimen the method is not successful due to difficulties in making specimen of rock samples for the test. For a grip at the two ends, the shape of the specimen has to be as shown in Fig. 7·2. With rock, it is very difficult to make specimen of such a shape. An alternative is to grip the cylindrical sample at two ends with some fixing material, and then to apply the tensile force at the two ends. Due to the gripping difficulties, the tensile strength of rock is estimated by following two indirect methods.

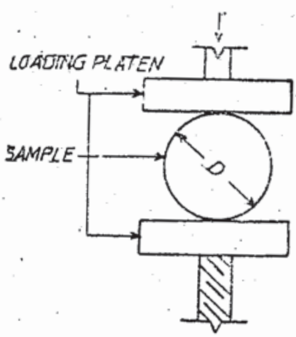
(*a*) Brazilian test,

(*b*) Flexural strength or Bending test.

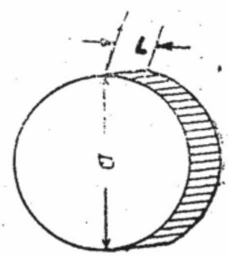
7·5·1. Brazilian Test

Thin discs are cut out of the cylindrical cores by a diamond saw in order to have L/D ratio as 0·5. The periphery of the specimen should be smooth.

This specimen is placed between the loading platens of the compression testing machine as shown in Fig. 7·3. A compressive load is applied to the specimen slowly till failure takes place. The rate of loading is normally 200 N/sec (220 kgf/sec) so that failure takes place in about 5 minutes time. The test may be stress controlled or strain controlled. If a loading frame is used which works on an electric motor the test will be strain controlled. Because speed of machine, thus rate of lifting of loading platen is controlled. The tensile strength is given by the equation 7·3.



(*a*)



(*b*)

Fig. 7·3

$$\sigma_t = \frac{2F}{\pi DL} \qquad \ldots(7{\cdot}3)$$

where σ_t = tensile stress at failure in kg/cm^2,

F = failure load in kg,

D = diameter of the specimen in cm,

L = length of the specimen in cm.

Failure takes place by splitting along a vertical diameter of the specimen.

7·5·2. Bending Test

With a diamond saw, a beam is cut out of the rock sample.

The specimen is supported at two ends as shown in Fig. 7·4. At the centre of the support a concentrated load is applied. The load is increased till failure takes place.

Tensile strength is obtained from equation 7·4.

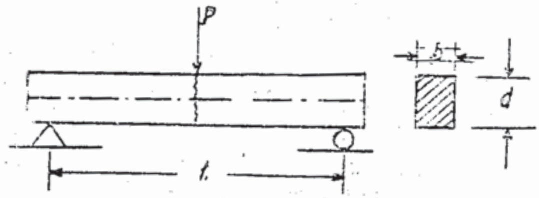


Fig. 7·4. Bending test.

$$\sigma_t = \frac{M}{I/n}$$

$$= \frac{P\,(l/2)}{\frac{1}{12}\,bd^3/n}$$

$$= \frac{6\,Pln}{bd^3} \quad \text{...(7·4)}$$

where

σ_t=Tensile strength in kg/cm²,
M=Bending moment,
P=Applied load in kg,
l=Length between supports,
b=Width of the specimen,
d=Depth (thickness) of the specimen,
n=Distance from the neutral axis to the farthermost fibre.

7·6. Flexural Strength Test

Test is also known as a modulus of rupture test or simple bending test. Object of this experiment is to ascertain the strength of rock in bending. Depending upon loading arrangement the test is designed as

(*a*) Three point load test,
(*b*) Four point load test.

The specimen is prepared in a rectangular prism form so that the cross-section of the specimen is rectangular. Thickness is generally more than the width. Length of the specimen is generally 10 times the thickness.

Three Point Load Test

In a three point loading method, the sample is supported at two ends as shown in Fig. 7·4. At the middle point of the span, a concentrated load is applied till failure takes place. The flexural strength is obtained by equation 7·5.

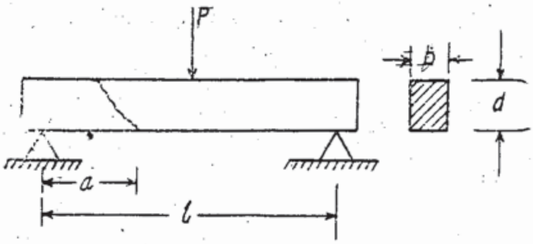


Fig. 7·5. Three point load test.

$$\sigma_f = \frac{3\,Pl}{2\,bd^3} \qquad \ldots(7{\cdot}5)$$

where P = Failure load in kg,
l = Length between supports,
b = Width of the specimen,
d = Thickness of the specimen.

If the specimen does not fail exactly under the load, then flexural strength is calculated by the formula

$$\sigma_f = \frac{3\,P.a}{bd^2} \qquad \ldots(7{\cdot}6)$$

where a = the average distance of the failure line from the nearest support as shown in Fig. 7·5.

Four Point Load Test

In the three point loading method, the failure occurs due to bending as well as shear. For determination of flexural strength the failure should be only by bending. For causing failure of the beam by bending only, the four point loading arrangement is shown in Fig. 7·6. When failure takes place within middle third of the span the flexural strength σ_f is given by equation 7·7.

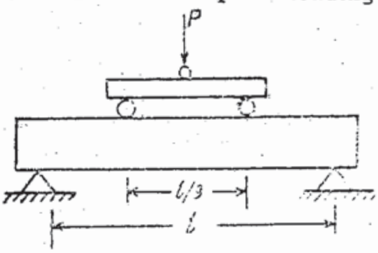


Fig. 7·6. Four point load test.

$$\sigma_f = \frac{Pl}{bd^2} \qquad \ldots(7{\cdot}7)$$

And if the failure is beyond the middle third it is obtained by equation 7·6.

7·7. Shear Strength Test

A knowledge of shear strength of a rock is necessary to deal with the situation where the rock mass is subjected to shear. A few problems dealing with shear stress are the stability of rock slopes, stability of a structure against sliding on its base etc.

In general there are two methods of test for evaluation of shear strength of rocks.

(*a*) Direct shear strength test.

(*b*) Indirect shear test which popularly is known as punch shear test.

Direct shear strength test further is done by two methods.

(*i*) Shear box test.

(*ii*) Shear test on rock cubes.

7·7·1. Shear Box Test

A rock specimen in the form of a rectangular prism is made of the standard size. Standard size means that the dimensions of the prism should be such that it can be put in the shear box. The box consists of two parts—an upper and a lower part. The lower part moves over rollers while through the upper part a fixed normal load is applied. Failure plane is defined in this test which is along meeting plane of the two halves. Hence while preparing a specimen, care should be taken that weakest plane of the rock mass specimen lies along the pre-determined plane of failure.

After putting the specimen in the shear box, a fixed normal load is applied on the sample through a loading pad A horizontal shear force is applied on the box. The horizontal force is increased till failure takes place along the shear plane as shown in Fig. 7·7.

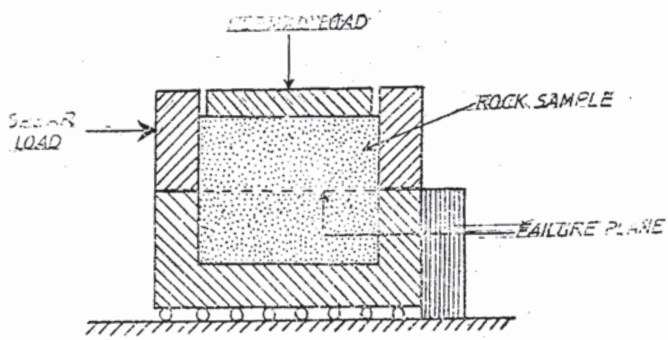


Fig. 7·7. Shear box test.

Normal stress $\sigma_n = \frac{P}{A}$

and shear stress $\tau = \frac{T}{A}$

where A = Cross-sectional area of the sample,

P = Normal load,

T = Horizontal or shear load.

If the normal stress σ_n on the sample is same as that expected on the rock mass in the field then shear stress τ at failure will be equivalent to shear strength of the rock mass.

If several tests are done at different normal loads then a graph is plotted between normal stress and the corresponding shear stress. The line joining the points is the coulomb failure line and its intercept with Y axis gives the cohesion value c and its inclination with horizontal *i.e.* X axis gives the angle of internal friction ϕ which are known as shear strength parameters. The relation is expressed in equation 7·8.

$$\tau = c + \sigma_n \tan \phi \quad ...(7·8)$$

If $c = 0$,

then $$\tau = s = \sigma_n \tan \phi \quad ...(7·9)$$

7·7·2. Direct Shear Test on Rock Cubes

The test is carried on a rock cube of a standard dimension. The length of the cube varies from 5 cm to 15 cm.

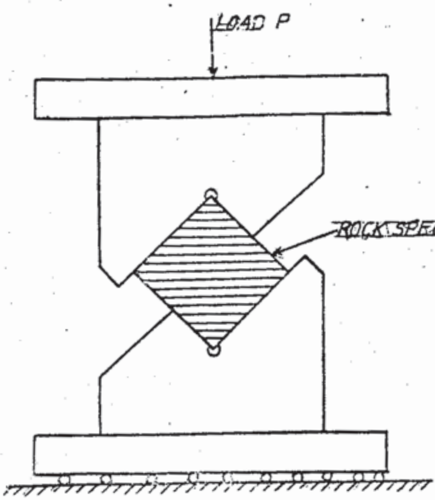


Fig. 7·8: Block shear test.

The cubical specimen is placed in the holder diagonally at an angle of 45° with the horizontal as shown in Fig. 7·8. The cube with the holder is placed between two loading platens of a compression machine The loading device transfers the vertical load to the rock specimen forcing the rock cube to shear along the predetermined place.

$$\text{The normal stress } \sigma_n = \frac{P \sin 45°}{A} \quad \ldots(7{\cdot}10)$$

$$\text{Shear stress } \quad \tau = \frac{P \cos 45°}{A} \quad \ldots(7{\cdot}11)$$

7·7·3. Punch Shear Test

In this test, the shear strength of a specimen is evaluated by punching shear. The sample is taken in the disc form of thickness t.

The test equipment consists of a piston shaped cylindrical jig having projected end. This cylindrical jig fits in a hollow cylindrical block. The disc shaped sample is placed at the bottom of the cylindrical block and the piston is put over the sample. Now whole arrangement is put between the platens of a loading machine and the load applied. The load P to punch the sample is noted.

The punching shear strength s is calculated from equation 7·12.

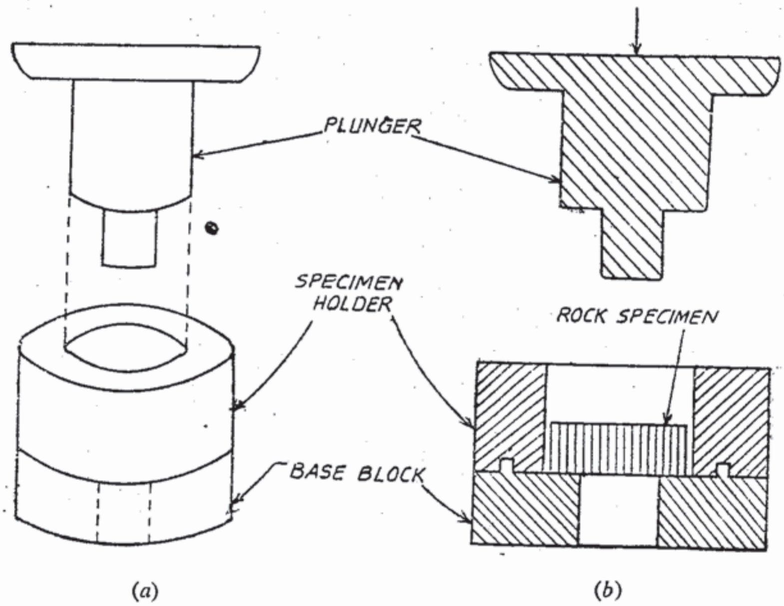


Fig. 7·9. Punch shear test.

$$s = \frac{P}{A} = \frac{P}{\pi d.t} \quad \ldots(7{\cdot}12)$$

where A = Circumferential shear area
t = Thickness of the disc
d = Diameter of the puncher.

Punching shear strength of dry rock specimen is approximately thrice the value of wet rock specimen. Although results vary widely from ± 28% to ± 37%, the test is done due to its simplicity.

7·8. Test for Elastic Constants

Different types of Elastic constants have been named earlier in this chapter. We know that within elastic range these canstants help in the determination of stresses within the body of the rock mass if corresponding strains are known.

When a tensile or compressive stress is applied to a specimen, within the range of elastic limit the ratio of stress to strain in the direction of applied stress is known as Young's modulus of elasticity or simply in short form "Modulus of elasticity". It is designated by letter "*E*" universally.

If a shear load is applied to a specimen, then the ratio of shear stress to the corresponding shear strain is known as the shear modulus or the modulus of rigidity and is commonly designated by the letter *G*.

When a specimen is loaded, axial strain takes place and this axial strain is accompanied by a lateral strain. If compressive load is applied to the specimen the specimen expands laterally and hence the lateral strain is positive whereas in case of a tensile load on the specimen, an extension of the specimen takes place and the specimen stretches due to which there is a shortening of its dimension in the lateral direction. Hence, the lateral strain in this case is negative. The ratio of lateral strain to axial strain is known as Poisson's Ratio and generally, it is denoted by symbol "ν".

In general, the elastic constants can be measured by two methods,

(*i*) Static methods, (*ii*) Dynamic methods.

7·8·1. Static Methods

The basic requirement for the evaluation of elastic constants is to stressing a test specimen and then to measure the corresponding strain. At the time of loading, care should be taken that the load being applied should not be so much, that specimen may cross the elastic limit. Hence, the elastic constants may be obtained by simple compression on test, Direct tension test, Brazilian test or Bending test.

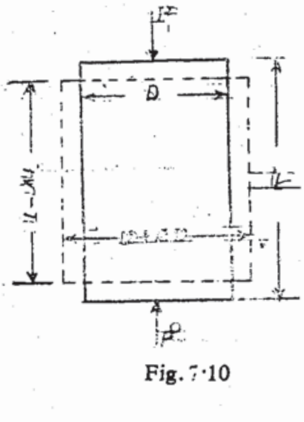


Fig. 7·10

If a prismatic specimen, of cross-sectional area *A* and length *l* is loaded with a compressive force *P* uniformly distributed over the area, then there shall be a contraction in its length parallel to the load and expansion at right angle to the load axis.

Axial strain $\epsilon = \frac{\Delta L}{L}$

and lateral strain $\epsilon' = \frac{\Delta D}{D}$.

where D is the diameter of the specimen.

Assuming the material to be linearly elastic, the modulus of elasticity in compression

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\epsilon}$$

$$= \frac{P/A}{\Delta L/L} = \frac{PL}{\Delta LA} \quad \ldots(7{\cdot}13)$$

and Poisson's ratio $\nu = \frac{\epsilon'}{\epsilon} = \frac{\Delta D/D}{\Delta L/L}$

$$= \frac{\Delta DL}{\Delta LD} \quad \ldots(7{\cdot}14)$$

Measurement of Elastic constants involves a measurement of stress and the corresponding strains. Stress can be known by the load applied and area of specimen on which the load is acting. Strain is known by knowing the original length of the specimen and the change in length due to the application of load. Change in length (*i.e.* deformation) measuring instruments are of three different types. Their grouping is done on the method of principle of their working. They are grouped as

(*a*) Mechanical strain gauges,

(*b*) Optical strain gauges,

(*c*) Electrical strain gauges.

The basic requirement of strain gauges are that they should be able to measure strain with an accuracy of 1μ. The gauge size should be small so that mounting on a small specimen may also be done easily. The response of the gauge should be linear to the deformation it measures. It should have remote reading facility. Electrical strain gauges satisfy all these requirements and hence, are mostly used for strain measurements.

7·8·1·1. Test Procedure

Value of the elastic constant is evaluated by an uniaxial compression, uniaxial tensile or flexural strength test. The choice depends on the type of loading expected on the rock mass in the actual field condition.

The specimen for the test is preferrably cylindrical and it is prepared in the same way as that for uniaxial compression test.

The specimen is kept between two platens of the loading machine. For measurement of a strain, the most suitable method is to affix electrical strain gauges on the specimen, but the easiest method is to put dial gauges one vertically and the other laterally to measure the vertical and the lateral deformations respectively. To account for unequal deformation more than one gauge may be put both vertically and laterally and average values are taken. The arrangement is shown in Fig. 7·11. If the loading arrangement is

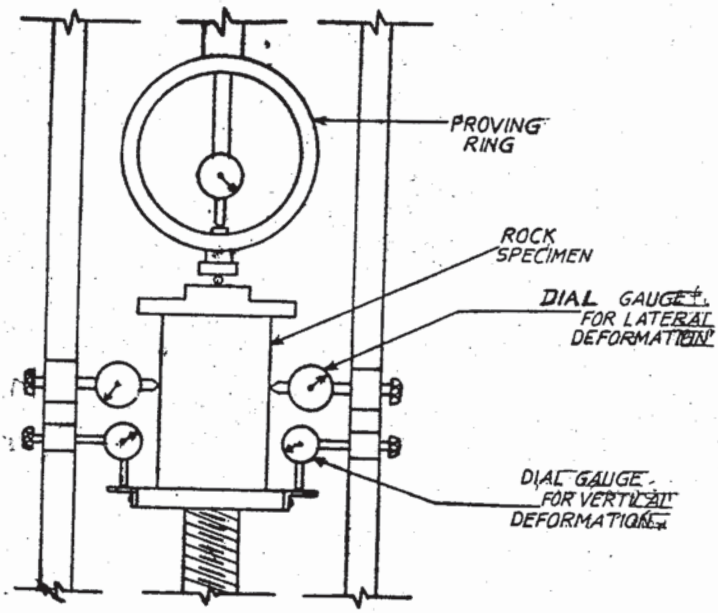


Fig. 7·11. Set up for measurement of vertical and lateral deformations.

stress controlled, then the rate of loading should be 0·5 to 1 MPa/sec ($\approx$5 to 10 kgf/sec). In strain controlled test the rate of loading should be less. During plastic deformation, the strain increases at even a constant load. Hence, a strain controlled loading is preferable.

After the test has been done, strains at different stages of loading are calculated and stress or strain curve is drawn. If dial gauges have been used for the measurement of deflection, calculation of strains are done with deflection readings. In case the electrical strain gauges have been used, the strains at different stages will be obtained directly.

The portion "l" of the total length of the specimen L, over which the change in length is measured, is known as base-length. When change in length is not measured along full length, or along width of the specimen but only over a limited portion, then only this limited length is taken as the base length in the calculation of strain.

The modulus of elasticity (E) is obtained by stress-strain graph in one of the following methods.

(*a*) The ratio of an ultimate compressive strength to the corresponding strain

i.e. $E = \frac{\sigma_{ult}}{\epsilon_l}$

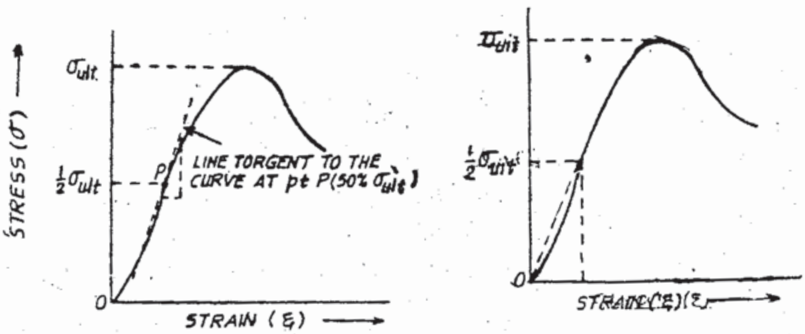


Fig. 7·12

(*b*) By noting down slope of the line which is tangent to the stress-strain curve (tangent modulus) at the 50% of the ultimate compressive strength.

(*c*) By secant modulus at 50% of ultimate compressive strength which means that the slope of the line which joins the point corresponding to 50% ultimate strength on the curve to the origin.

The modulus ratio is defined as the ratio of the tangent modulus at 50% ultimate strength to the ultimate strength. Description of a rock mass is also done on the basis of modulus ratio.

$$\text{Poisson's ratio} \quad \nu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{\epsilon_d}{\epsilon_l}$$

7·8·1·2. Young's Modulus by Brazilian Test

Modulus of elasticity and Poisson's ratio can be evaluated by Brazilian test also. At the centre of the sample, two electrical strain gauges—one in vertical (in line of loading) direction and the other in horizontal direction—are put respectively. Otherwise a 90° strain gauge rosette is put at the centre of the specimen. The load and corresponding vertical and horizontal strain readings are noted. E and ν can be obtained with following relations.

For a plane stress case

$$\nu = -\left(\frac{3\epsilon_H + \epsilon_V}{3\epsilon_V + \epsilon_H}\right) \qquad \text{...(7·15)}$$

$$E = -\frac{6P(1-\nu^2)}{\pi Dt(\epsilon_V + \epsilon_H)} \qquad \text{...(7·16)}$$

For a plane strain case,

$$\nu = -\left\{\frac{\epsilon_V + 3\epsilon_H}{2(\epsilon_V - \epsilon_H)}\right\} \qquad \text{...(7·17)}$$

$$E = -\left[\frac{6P(1-\nu)(1-2\nu)}{\pi Dt\{\nu\epsilon_H + (1-\nu)\epsilon_V\}}\right] \qquad \text{...(7·18)}$$

where P=Applied vertical load,
D=Diameter of the test specimen,
t=Thickness of the specimen,
ϵ_V=Strain at the centre along the vertical axis,
ϵ_H=Strain at the centre along horizontal axis.

For an accuracy within 5%, the gauge length should not be more than 0·07 D.

7·8·1·3. Young's Modulus by Bending Test

The value of Young's modulus can be known by bending test also. The loading may be three-point or four-point as shown in Fig. 7·5 and 7·6 respectively. As discussed earlier, four point loading is a case of pure bending and hence this method gives a more accurate result. Deflection at the centre of the beam is measured by a dial gauge.

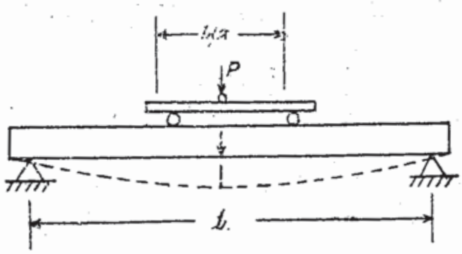


Fig. 7·13. Four point loading.

For a three point loading,

$$E = \frac{Pl^3}{4y.bd^3} \qquad \text{...(7·19)}$$

and for a four-point loading

$$E = \frac{3\,Pl^3}{14y\,bd^3} \qquad \ldots(7{\cdot}20)$$

where P=Vertical applied load,
b=Width of the specimen,
d=Thickness of the specimen,
l=Span of the beam (specimen)
and y=Deflection of the beam at the centre.

For the same rock, modulus of elasticity depends on the type of loading by which it has been determined. Hence, modulus of elasticity in compression, tension and bending tests differ. They have been related by equation 7·21 which was proposed by Adler (Adler 1970).

$$E_b = \frac{4E_cE_t}{(\sqrt{E_c}+\sqrt{E_t})^2} \qquad \ldots(7{\cdot}21)$$

where E_b, E_c and E_t are modulus of elasticity in bending, compression and tension respectively.

7·8·2. Dynamic Methods

Sometimes, the rock mass is subjected to a transient dynamic loading also. To know its behaviour or response for such a loading, it is necessary to know the elastic constants by dynamic methods. Then only its reaction to dynamic stresses can be ascertained.

The dynamic elastic constants are calculated by measurement of velocities of elastic waves. Details about different types of elastic waves have been discussed in the chapter dealing with dynamic properties of rocks. In a broad sense elastic constants of rock mass can be determined in field as well in laboratory also. Needless to say that field measurements are more realistic. However for a preliminary work or an approximate analysis, the laboratory methods can be used. The methods used to determine dynamic elastic constants in laboratories are

(*i*) The resonance method,

(*ii*) The ultrasonic pulse method.

7·8 2·1. Resonance Method

When a cylindrical or prismatic bar is given a longitudinal vibration, the length of the bar (specimen) contains an integral number n of half-wave lengths. If l is the length of the specimen and λ wave length of vibration, then

$$l = \frac{n\lambda}{2}$$

wave velocity
$$V = \lambda f = \frac{2lf}{n} \qquad \ldots(7{\cdot}22)$$

f is the resonant frequency of any mode of vibration.

The modulus of elasticity E is obtained by equation 7·23

$$E=\frac{1}{k}\left[\frac{2lf}{n}\right]\rho \qquad \ldots(7{\cdot}23)$$

where n=Number of modes of vibration

ρ=Density of specimen

k=Correction factor which depends on shape, size and Poisson's ratio of the specimen and wave length

A typical set up for the resonant method is shown in Fig. 7·14.

The sine wave oscillator and exciter are tuned to the resonant frequency of the rock. Mechanical vibrations are induced in the specimen and the specimen is caused to vibrate as a whole in one of its natural frequency modes. Mechanical vibrations of the specimen are converted to an electrical signal by the pick up and after amplification through the preamplifier fed to the oscilloscope. The output of the oscillator is directly fed to the oscilloscope also for a comparison of signal which passed through the specimen and after being received by pick up is fed to the oscilloscope. By observing amplitude and Lissajous figures on the oscilloscope the resonant frequency

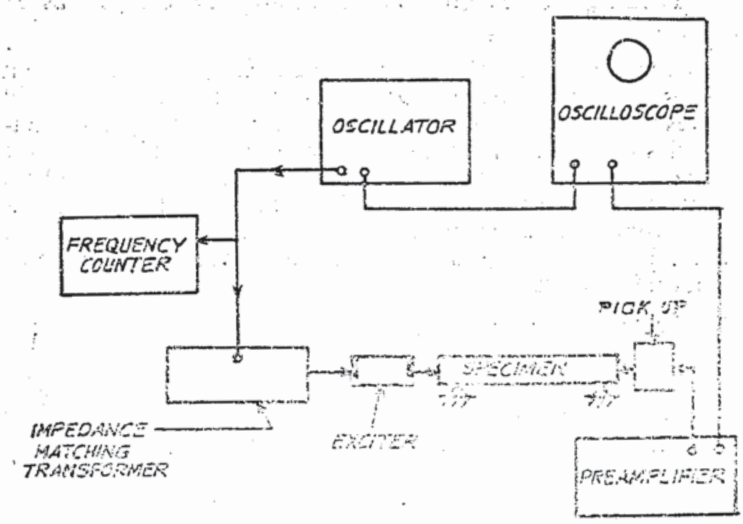


Fig. 7·14

can be identified. When resonance occurs, the resonance frequency can be read on a frequency counter. Knowing resonant frequency f, modulus of elasticity E can be calculated by equation 7·23.

7·8·2·2. Ultrasonic Pulse Method

In this method a mechanical impulse is given to the specimen which is in the form of cylindrical or prismatic bar. Time t required for the pulse to traverse through length l of the specimen is noted and velocity V of wave is calculated by the usual relation

$$V = l/t$$

The modulus of elasticity and Poisson's ratio are calculated by equation 7·24 and 7·25.

$$E = \frac{\rho V_s^2 (3V_l^2 - 4V_s^2)}{V_l^2 - V_s^2} \qquad \ldots(7{\cdot}24)$$

$$\nu = \frac{V_l^2 - 2V_s^2}{2 (V_l^2 - V_s^2)} \qquad \ldots(7{\cdot}25)$$

where ρ = Density of the rock mass

V_l = Longitudinal wave velocity

V_s = Shear wave velocity.

The set up for the measurement is shown in Fig. 7·15. A pulse generator which is also known as an exciter, generates a short duration electrical pulse and transmits it to the "driver" or "excitor" transducer. By the excitor transducer the electrical pulse is converted into a mechanical impulse which is transmitted to the specimen. The

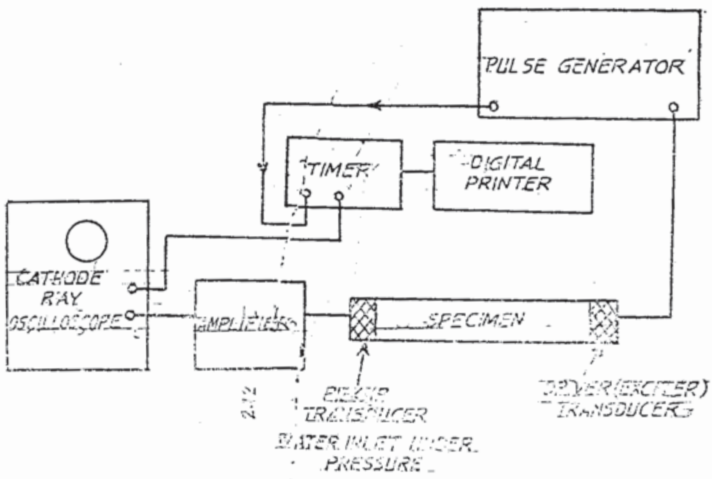


Fig. 7·15

pulse travels through the specimen and when it reaches at the other end, it is picked up by "pick up" transducer. The pick up transducer again changes the mechanical impulse to an electrical signals which after being amplified through the amplifier is fed to the cathode ray oscilloscope. The moment at which the wave reaches the oscilloscope. can be known by its screen. By an automatic arrangement in timer the time of starting of the pulse through the pulse generator and being received at the oscilloscope (after passing through the specimen) is noted down automatically and recorded by the digital printer. Thus knowing the length of the specimen and finding out time for the wave to travel through the specimen, velocity V of the wave is obtained by relation $V=l/t$. Then with help of equation 7·24 and 7·25 modulus of elasticity and Poisson's ratio is calculated.

Out of the two methods, the ultrasonic pulse method is better because it give a better result.

'In situ' method for determination of elastic constants will be discussed in the chapter "Dynamic Properties of Rocks".

8
Creep Behaviour and Rheological Models

8·1. Loading Diagrams

When a material is loaded, the stress-strain curve is linear initially and the path is reversible upto a particular limit, which is known as the elastic limit, beyond which if the material is loaded the curve is not reversible. For example, consider the two curves of Fig. 8·1.

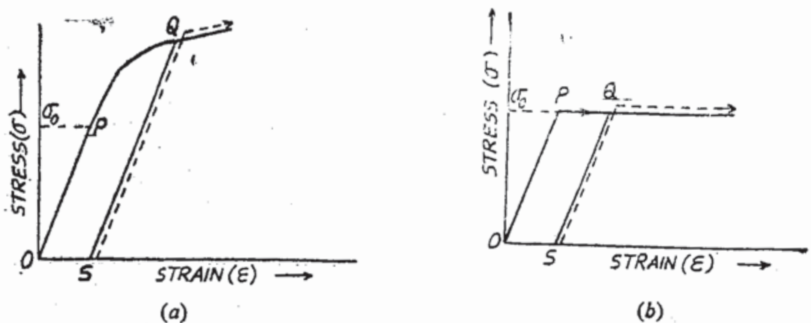


Fig. 8·1

Along the path OP, the behaviour is elastic *i.e.*, path is reversible. Once the point P is crossed *i.e.*, $\sigma > \sigma_0$ the path is no longer reversible. If the sample is unloaded, after reaching point Q, then the unloading path is given by QS which is parallel to OP. After unloading, the strain represented by OS is known as permanent strain and stress σ_0 is defined as the yield point. If the test sample is again loaded the path is reversible along SQ. But once limit Q is crossed again it becomes irreversible. Stress at Q represented by σ_Q is defined as current yield point. In figure (*a*) σ_Q is a function of permanent strain OS and illustrates work-hardening but in figure (*b*) the stress σ_Q is a constant and the material which shows such a property is known as perfectly plastic. On set of plasticity is indicated by the irreversibility of the path beyond the yield point P. Thus plasticity is said to occur if the stress crosses the yield point, beyond which the permanent strain appears.

Most of the rocks exhibits both instantaneous and delayed deformations when they are loaded, and therefore they are known as "viscoelastic".

8·2. Creep

Creep occurs due to plastic deformation. Hence, even at a constant load, there will be a deformation in the body with respect to

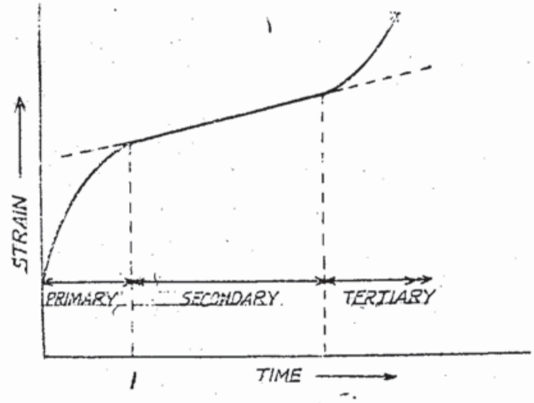


Fig. 8·2

time. And this phenomenon of increase in strain during the course of time under constant stress is defined as creep. Fig. 8·2 shows the result of a creep test in which stress increment is applied and held constant for record of increment in strain with time. The curve has got three parts. The first part is primary creep which just follows after application of the load increment and the strain decreases with time. In the secondary creep which is 2nd part of the curve, the strain ratio is constant and in the third part of the curve, which is known as tertiary creep, the strain rate increases with time until the specimen fails or ruptures.

8·3. Creep in Rocks

The creep phenomenon in rock is explained due to two reasons cracking and mass flow. In some types of rock (*e.g.* rock salt) it is due to cracking while in some (*e.g.* uncemented clay rock), it is due to a mass flow.

Because of the occurrence of dislocations, every crystal of the rock mass contains many microscopic regions of high internal stress. If a load is applied, these regions move easily which results in a measurable plastic deformation. Rise in tempesature of rock facilitates dislocation movements because of thermally activated vibrations of atomic and ionic particles about their equilibrium position. At a high temperature, these vibrations becomes so strong that a constant external load gives rise to a steady increasing deformation which is known as creep.

Some rocks such as gabbros, granites etc. show little creep when subjected to uniaxial compression. But in some rocks the creep strain exceeds the instantaneous elastic deformations. An extreme example of such a type is rock salt. In rock salt, creep *i.e.* deformation in rock with time at constant load has been observed to take place up to a considerable long time. Even at a lesser stress there is creep in rock salts. Hence, several research work has been reported for the study of creep behaviour in rock salts.

Some of the concepts derived from investigations regarding creep are summarised below :

1. The rate of creep for an opening decreases with age.
2. The rate of creep increases with the depth of the opening from the surface of the earth.
3. The plasticity of some of the rocks *e.g.*, salt rock, increases with increasing temperature, and, therefore, creep rate is greater at higher temperature.
4. Fine grained materials are more creep resistant than coarse-grained at a lower temperature, however it has been reported that elevation of temperature reverses the case.
5. Based on actual field data the creep rate in a mine has been expressed by

$$\frac{d\epsilon}{dt} = Ae^{-kt} \qquad ...(8{\cdot}1)$$

where $\frac{d\epsilon}{dt}$ = creep rate

A = constant

t = time.

Another equation based on triaxial extension creep test has been given by Stag.

$$\epsilon = K\sigma^n t^m \qquad ...(8{\cdot}2)$$

where ϵ = axial strain on a cylindrical test specimen

σ = stress difference in lb/in^2

t = time in hour

K, m and n = constants

For a particular test on salt rock the values of K, m and n have beed reported as $1{\cdot}87 \times 10^{-13}$, 0·36 and 2·98 respectively.

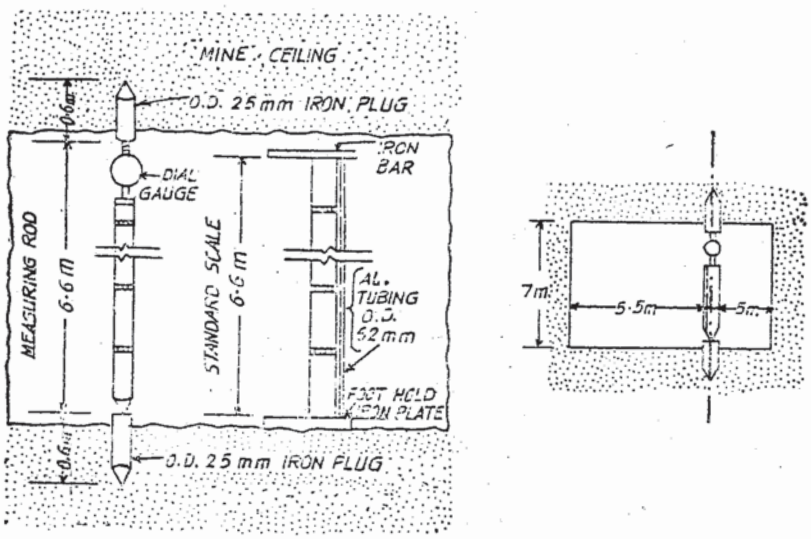
8·3·1. Measurement of Creep

The measurement of creep for a rock specimen can be done by subjecting a cylindrical specimen at a constant load and measuring the strain at different time intervals till the time strain curve becomes asymtotic. The load is increased in stages and for each stage of loading, strain is noted at different time intervals. The strain measurement equipment should be very accurate in order to measure very less values which occur during the creep process.

Measurement of creep has been done in field also. One of the typical measurement in salt rock has been reported by Reynolds and Gloyna (1961).

An opening of 10·5 m width and 7 m height as reported was made in the mine (opening made for mining purpose may also be used) and in this opening the creep measurement was done. For vertical creep measurement following procedure was made.

The measuring equipment consisted of a measuring rod with a dial gauge attached to one end and a standard-length reference rod 6·6 m long. Two pins were fixed at roof and floor of the opening respectively just vertically to each other. The distance between two pins was determined by comparing the dial gauge reading between the pins to the reading for the standard length rod. The set up s shown in Fig. 8·3. Observation of deflection in the dial gauge was made at different time intervals (of course, in years).



Mine floor Fig. 8·3 Mine opening

8·3·2. Estimation of Creep Deformation

If log of creep is plotted versus time on a semi-long plot the equation 8·1 will give a straight line plot. The constant A in the equation is the intersection of the line with y-axis and the rate constant k is the slope of the line divided by 0·434(1/2·232) to change the base of log. The resulting equation for the particular test which is plotted in Fig. 8·4 is

$$\text{Creep Rate} = \frac{d\epsilon}{dt}$$

$$= 0·03\, e^{-0·635\, t} \qquad \ldots(8·3)$$

An estimation of the total amount of deformation which will occur can be made by integrating eq. (8·3) between limits

$$t=0$$

and $$t=\infty,$$

Giving

$$\epsilon=\int_{t=0}^{t=\infty} d\epsilon,$$

$$=0.03\int_{t=0}^{t=\infty}(e^{-0.635\,t})\,dt$$

$$=4.7\%$$

This corresponds to an ultimate reduction of 33 cm in the heigh of an opening which is 7 m heigh.

The above procedure gives an idea of importance of creep study for rock masses. We have seen that creep is a response of strain to stress which is a time dependent movement of rock under sustained

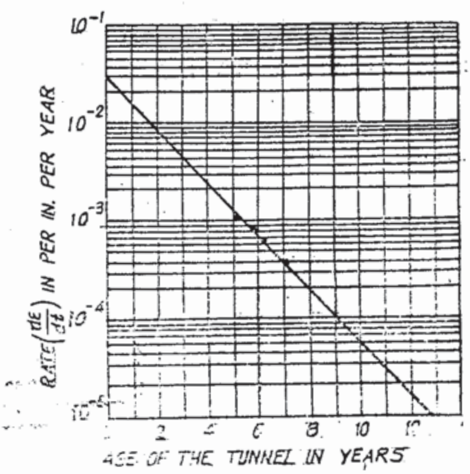


Fig. 8·4

load and this occurs due to a plastic displacement. Hence for problems relating to openings in rock mass at deeper strata, or at great depths, a knowledge of creep behaviour of rock mass is absolutely necessary. Because, at great depths such as mines, the rock mass is considered to be in plastic state due to huge overburden pressure and if an opening is made, creep will occur and its estimation is necessary for perfect design and performance.

It appears that no general equation exists which adequately defines completely the creep properties of all types of rock mass. This property varies from rock to rock and other factors also. Since the process of creep is non-linear because therock behaviour is changed by each new load increment and in order to calculate stresses and deformations in non-linear viscoelastic materials, the properties are determined and used as functions of stress. There is no ideally viscoelastic rock. However, theory of linear visco-elasticity are used incrementally to solve time dependent problems in a similar way as theory of linear elasticity is used to calculate stress-strain for time independent problems.

It is possible to fit creep curves empirically using exponential or power functions. If creep data are fitted in models comprising of springs and dashpots, the results of the solution can be used to a great extent.

8·4. Rheology and Rheological Models

Rheology is defined as the study of materials in a fluid state as a rate process. Hence rheological models are time-deformation-dependent stress models.

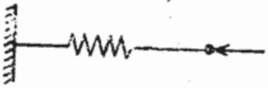
The simplest rheological model is the spring element or hookean model which obeys Hook's law. It is shown in Fig. 8·5 (*a*). If the extension of the spring under application of the applied stress is instantaneous, the relationship between stress and strain is described as follows,

$$\sigma = k\epsilon \qquad \ldots(8{\cdot}4)$$

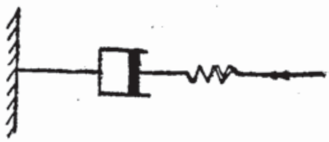
where σ = applied stress,

k = constant,

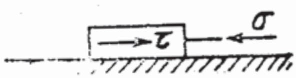
ϵ = strain.



(*a*) Hookean.



(*b*) Newtonian.



(*c*) Yield stress.

Fig. 8·5. Elementary rheological models.

If there is a dashpot analogous to the shock absorber in an automobile, it is known as Newtonian model as shown in Fig. 8·5 (*b*). For the stress-strain behaviour characterised by a constant rate of strain under the applied stress, the model is best represented in terms of a dashpot.

The strain is related to the stress through the following relation.

$$\sigma = n \frac{d\epsilon}{dt} \qquad \ldots(8{\cdot}5)$$

where η = viscosity in units of stress × time.

The condition of a minimum stress level causing strain or slip can be represented by a model analogous to the block sliding down a plane. Such a model is known as yield stress model as shown in Fig. 8·5 (*c*).

$\sigma < \sigma_0$ for $\epsilon = 0$

$\sigma > \sigma_0$ for finite movement of strain ϵ

σ_0 = frictional resistance

σ = constant maximum value

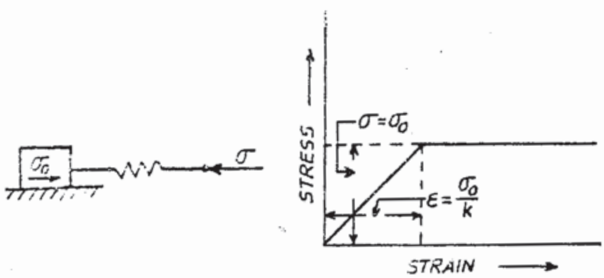
The primary problem in the use of the elementary Hooken and Newtonian model is the choice of the coefficients. The spring constant in the Hookean model describes the elastic nature of the spring. The choice of spiring constant becomes critical, since this should not only describe the elastic deformation at loading but should also represent a linear stress-strain rebound.

It is to be noted that behaviour of rock mass can not be described by any of the models. Since there are no ideally linear viscoelastic rock, their behaviour can be described by composite models, which are combinations of the fundamental models as decribed above. The degree of accuracy between predictions based upon mathematical formulation and observed behaviour depends on the accuracy with which a composite model is selected.

8·5. Different Rheological Models

8·5·1. The St. Venant Model

St. Venant Model is obtained by combining the Hooken model



(*a*) Model Fig. 8·6 (*b*) Stress∼Strain curve with Yield Stress model in series. The method of loading is such that spring can carry only a compression stress. σ can be increased linearly with corresponding strains until $\sigma=\sigma_0$, when the yield point is reached. Fig. 8·6 shows the model.

8·5·2. The Kelvin Model

A parallel coupling of a Hooken and a Newtonian models makes the Kelvin model as shown in Fig. 8·7. This model is also known as the Voigt model.

For the parallel coupling

$$\epsilon_{Hookean}=\epsilon_{Newtonian}$$

and

$$\sigma=\sigma_{Hookean}+\sigma_{Newtonian}$$

Hence, the mathematical formulation for stress would be

$$\sigma=k\epsilon+\eta\ \frac{d\varepsilon}{dt} \quad \text{...(8·6)}$$

or

$$\frac{\sigma}{\eta}=\frac{k}{\eta}\ \epsilon+\frac{d\epsilon}{dt} \quad \text{...(8·7)}$$

The general form of solution of the differential equation

$$N=Mx+\frac{dx}{dy} \text{ is given by}$$

$$x=e^{-\int Mdy}\left(\int Ne^{+\int Mdy}+C\right)$$

Hence, solution of the equation (8·7) is given by

$$\epsilon=e^{-\int \frac{k}{\eta}dt}\left[\int \frac{\sigma}{\eta}\ e^{\int \frac{k}{\eta}dt}+C\right] \quad \text{...(8·8)}$$

$$\epsilon=0$$

when

$$t=0 \text{ for } \sigma$$

$$\sigma=\text{constant}$$

$$=\sigma_c$$

$\therefore$

$$0=\frac{\sigma_c}{k}+C$$

or

$$C=-\frac{\sigma_c}{k}$$

Hence the eq. (8·8) can be written in general from as

$$\varepsilon=e^{\frac{-kt}{n}}\left[\frac{\sigma}{k}\ e^{\frac{-kt}{\eta}}-\frac{\sigma}{k}\right]$$

and for

$$\sigma=\sigma_c$$

$$\epsilon=\frac{\sigma_c}{k}\left(1-e^{\frac{-kt}{\eta}}\right) \qquad ...(8·9)$$

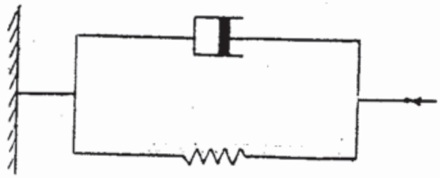
The strain-time curve for equation 8·9 is shown in Fig. 8·7. At time t_1, if σ_c is released then

$$\epsilon=\epsilon_1$$

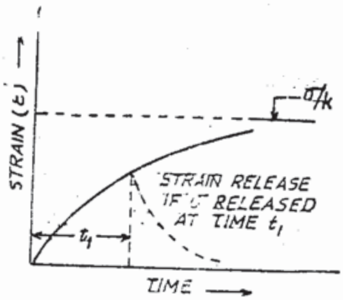
and the equation for the relaxation curve is

$$\epsilon=\epsilon_1\, e^{\frac{-k(t-t_1)}{\eta}} \qquad ...(8·10)$$

A suddenly applied constant shear stress causes shear strain at an exponentially decreasing rate approaching zero as time increases.



(a) Model.



(b) Time-strain relationship.

Fig. 8·7

8·5·3. The Maxwell Model

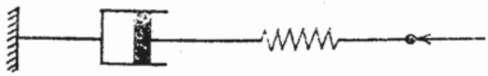
The Maxwell model consists of a Hookean and a Newtonian model in series as shown in Fig. 8·8. Since σ is common to both the elements,

$$\epsilon_{Total}=\epsilon_{Hookean}+\epsilon_{Newtonian}$$

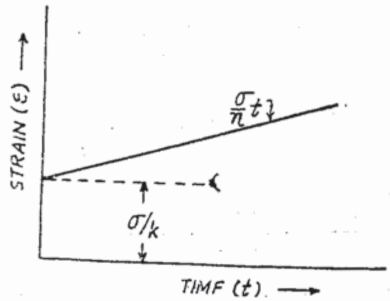
Putting values of strain

$$\epsilon=\frac{\sigma}{k}+\frac{\sigma t}{\eta} \qquad ...(8·11)$$

The equation represents an instantaneous deformation due to an applied load followed by a time dependent deformation given by the dashpot. The strain-time diagram is shown in Fig. 8·8 (*b*).



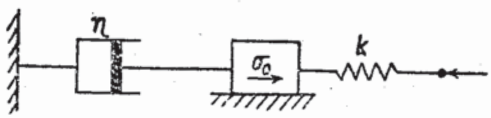
(*a*) Model.



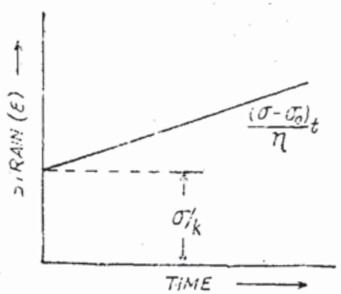
(*b*) Time~strain curve.
Fig. 8·8

8·5·4. The Bingham Model

If it is required to impose a restriction on the strain beyond the yield point stress, then a Newtonian model can be put in series with St. Venant model. The resulting model is Bingham model as shown in Fig. 8·9.



(*a*) Model.



(*b*) Time~strain curve.
Fig. 8·9

As long as $\sigma < \sigma_0$, $\epsilon = \frac{\sigma}{k}$. Once the yield stress is reached $\sigma > \sigma_0$ and the friction block will move. But the dashpot will start influencing the deformation. Ths strain now can be given by

$$\epsilon_{Total} = \epsilon_{Hookean} + \epsilon_{Newtonian}$$

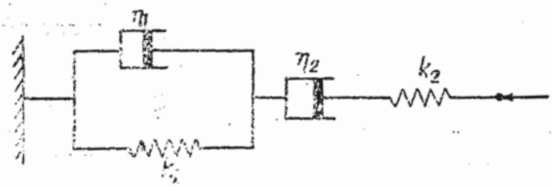
or

$$\epsilon_T = \epsilon_{Spring} + \epsilon_{Dashpot}$$

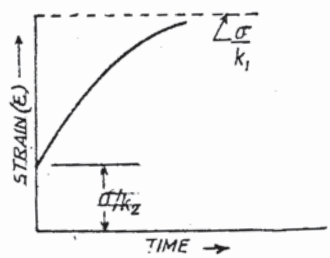
$$\therefore, \quad \epsilon = \frac{\sigma}{k} + \frac{(\sigma - \sigma_0)t}{\eta} \qquad \ldots(8{\cdot}12)$$

8·5·5. The Burger Model

The model is shown in Fig. 8·10. Since it combines the Kelvin and Maxwell models in series, the solution of this rheological model is the sum of the solutions of the two models.



(*a*) Model.



(*b*) Time~strain curve.

Fig. 8·10

$$\epsilon = \frac{\sigma}{k_2} + \frac{\sigma \cdot t}{\eta_2} + \frac{\sigma}{k_1}\left[1 - e^{\frac{k_1 \cdot t}{\eta_1}}\right] \ldots(8{\cdot}13)$$

In view of the nature of the general creep curve shown in Fig. 8·2, the Burger Model is the simplest model that can be used to evaluate the strain up to the starting of tertiary creep. Other types of models can also be tried but the Burger model describes most of the practical cases in rock flow problems.