Chapter 4

Measurement and introduction to Pythagoras' theorem

What you will learn

Australian curriculum

MEASUREMENT AND GEOMETRY

Using units of measurement

Choose appropriate units of measurement for area and volume and convert from one unit to another (ACMMG200)

Find perimeters and areas of parallelograms, rhombuses and kites (ACMMG196)

Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area (ACMMG197)

Develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume (ACMMG198)

Solve problems involving duration, including using 12- and 24-hour time within a single time zone (ACMMG199)

NUMBER AND ALGEBRA

Real numbers

Investigate the concept of irrational numbers, including π (ACMNA186)

Online resources

- Chapter pre-test
- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to HOTmaths Australian Curriculum courses

The wheels are turning

Civilisations in ancient and modern times have used measurement to better understand the world in which they live and work. The circle, for example, in the form of a wheel helped civilisations gain mobility, and modern society to develop machines. For thousands of years mathematicians have studied the properties of the circle including such measurements as its circumference and area.

The ancient civilisations knew of the existence of a special number (which we know as pi) that links a circle's radius with its circumference and area. It was the key to understanding the precise geometry of a circle, but they could only guess its value. We now know that pi is a special number that has an infinite number of decimal places with no repeated pattern. From a measurement perspective, pi is the distance a wheel with diameter 1 unit will travel in one full turn.

4A Length and perimeter CONSOLIDATING

Interactive
Widgets
HOTsheets
Walkthroughs

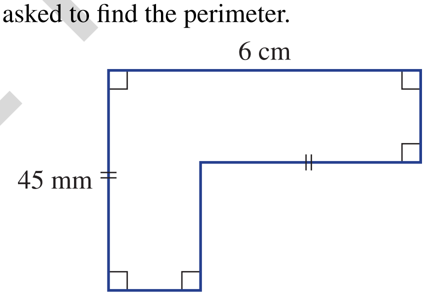
For thousands of years, civilisations have found ways to measure length. The Egyptians, for example, used the cubit (length of an arm from the elbow to the tip of the middle finger), the Romans used the pace (5 feet) and the English developed their imperial system using inches, feet, yards and miles. The modern-day system used in Australia (and most other countries) is the metric system, which was developed in France in the 1790s and is based on the unit called the metre. We use units of length to describe the distance between two points, or the distance around the outside of a shape, called the perimeter.

The Romans would have measured the perimeter of the Colosseum in paces.

Let's start: Provide the perimeter

In this diagram some of the lengths are given. Three students were asked to find the perimeter.

- Will says that you cannot work out some lengths and so the perimeter cannot be found.
- Sally says that there is enough information and the answer is $9 + 12 = 21$ cm.
- Greta says that there is enough information but the answer is $90 + 12 = 102$ cm.

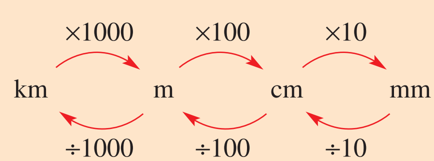


Who is correct?

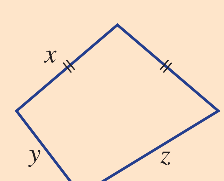
Discuss how each person arrived at their answer.

Key ideas

- The common metric units of length include the **kilometre** (km), **metre** (m), **centimetre** (cm) and **millimetre** (mm)



- **Perimeter** is the distance around a closed shape.
  - All units must be of the same type when calculating the perimeter.
  - Sides with the same type of markings (dashes) are of equal length.



$P = 2x + y + z$

Example 1 Converting length measurements

Convert these lengths to the units shown in the brackets.

a 5.2 cm (mm) **b** 85 000 cm (km)

SOLUTION

a $5.2\text{ cm} = 5.2 \times 10$
$= 52\text{ mm}$

EXPLANATION

1 cm = 10 mm so multiply by 10.

$\text{cm} \xrightarrow{\times 10} \text{mm}$

SOLUTION

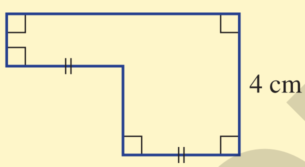
b $85\,000\text{ cm} = 85\,000 \div 100 \div 1000$
$= 0.85\text{ km}$

EXPLANATION

1 m = 100 cm and 1 km = 1000 m so divide by 100 and 1000.

Example 2 Finding perimeters

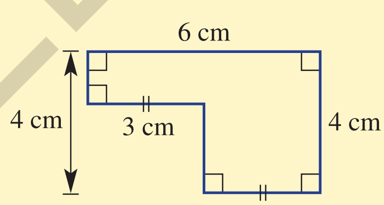
Find the perimeter of this shape.



SOLUTION

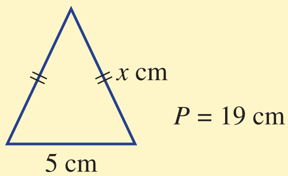
$P = 2 \times (3 + 3) + 2 \times 4$
$= 12 + 8$
$= 20\text{ cm}$

EXPLANATION



Example 3 Finding an unknown length

Find the unknown value x in this triangle if the perimeter is 19 cm.



SOLUTION

$2x + 5 = 19$

$2x = 14$

$x = 7$

EXPLANATION

$2x + 5$ makes up the perimeter.

$2x$ is the difference between 19 and 5.

If $2x = 14$ then $x = 7$ since $2 \times 7 = 14$.

Exercise 4A

1–4	4	—

UNDERSTANDING

1 Write the missing number in these sentences.

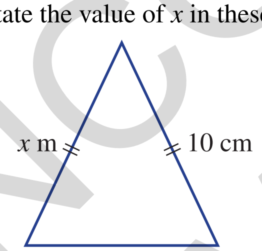
a There are ___ mm in 1 cm.
b There are ___ cm in 1 m.
c There are ___ m in 1 km.
d There are ___ cm in 1 km.
e There are ___mm in 1 m.
f There are ___ mm in 1 km.

2 Evaluate the following.

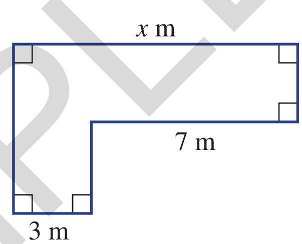
a 10×100
b 100×1000
c $10 \times 100 \times 1000$

3 State the value of x in these diagrams.

a



b



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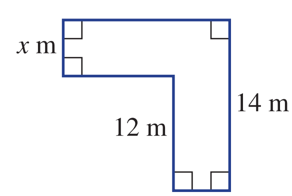


4 Find the perimeter of these quadrilaterals.

a Square with side length 3 m.
b Rectangle with side lengths 4 cm and 7 cm.
c Rhombus with side length 2.5 m.
d Parallelogram with side lengths 10 km and 12 km.
e Kite with side lengths 0.4 cm and 0.3 cm.
f Trapezium with side lengths 1.5 m, 1.1 m, 0.4 m and 0.6 m.

5–7(½) 5–7(½) 5–7(½)

FLUENCY

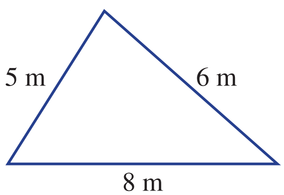
Example 1

5 Convert these measurements to the units shown in the brackets.

a 3 cm (mm) **b** 6.1 m (cm) **c** 8.93 km (m) **d** 3 m (mm)
e 0.0021 km (m) **f** 320 mm (cm) **g** 9620 m (km) **h** 38 000 cm (km)
i 0.0043 m (mm) **j** 0.0204 km (cm) **k** 23098 mm (m) **l** 342 000 cm (km)
m 194 300 mm (m) **n** 10 000 mm (km) **o** 0.02403 m (mm) **p** 994 000 mm (km)

Example 2

6 Find the perimeter of these shapes.

a


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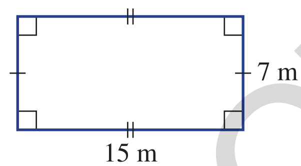

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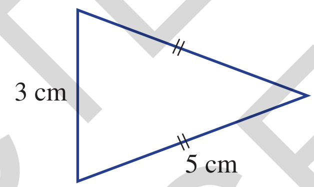
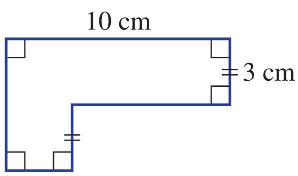


d


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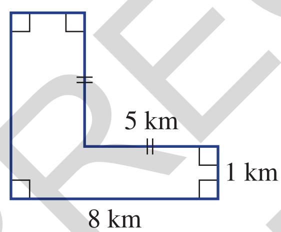
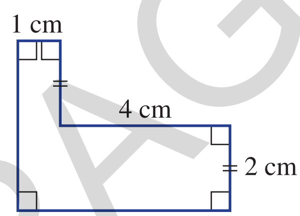


f


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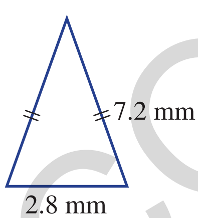

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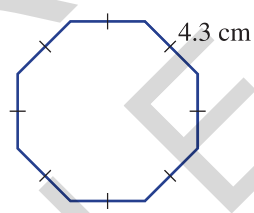

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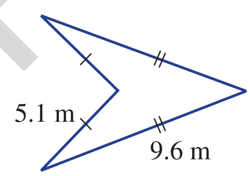
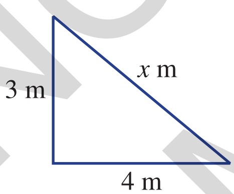
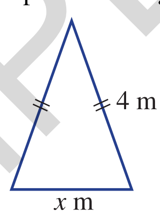


Example 3

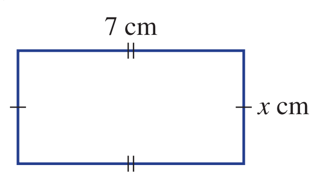
7 Find the unknown value x in these shapes with the given perimeter (P).

a


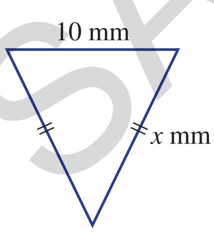
$P = 12$ m

b


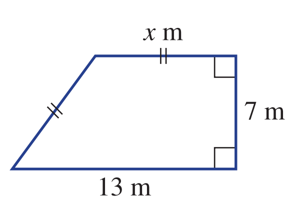
$P = 10$ m

c


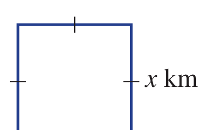
$P = 22$ cm

d


$P = 46$ mm

e


$P = 39$ m

f


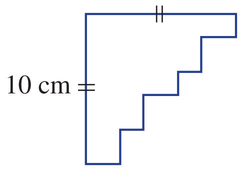
$P = 26$ km

4A

8, 9	8–11	9–12

PROBLEM-SOLVING

8 Find the perimeter of these shapes. Give your answers in cm and assume that angles that look right-angled are 90°.

a


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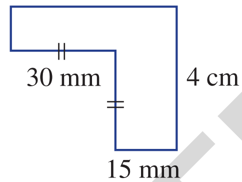
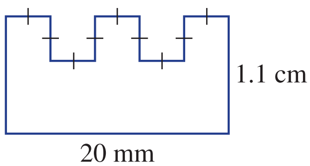


c


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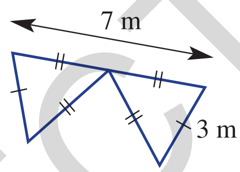

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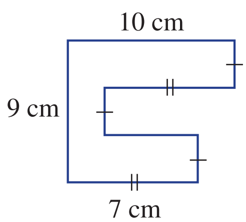
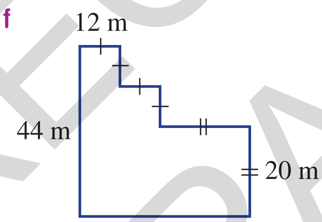
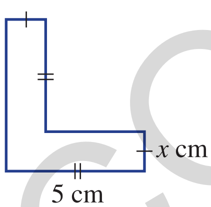
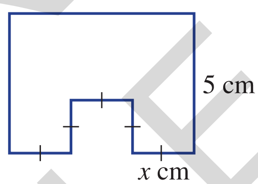


f


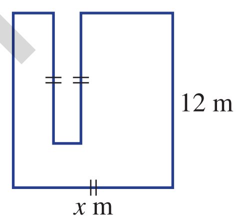
9 Find the unknown value x in these diagrams. All angles are 90°.

a


$P = 24$ cm

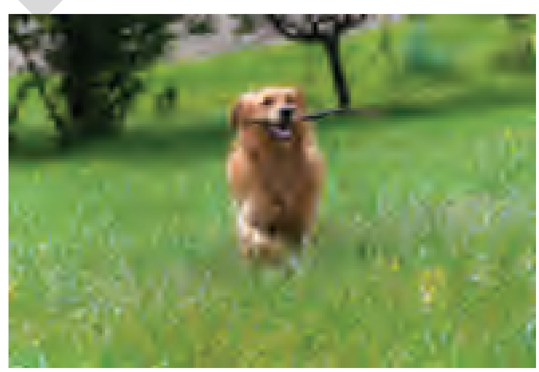
b


$P = 34$ cm

c


$P = 60$ m

10 Jennifer needs to fence her country house block to keep her dog in. The block is a rectangle with length 50 m and width 42 m. Fencing costs $13 per metre. What will be the total cost of fencing?

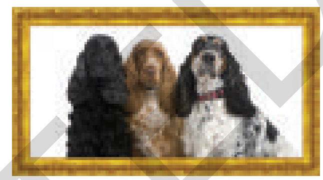


4A

PROBLEM-SOLVING

11 Gillian can jog 100 metres in 24 seconds. How long will it take her to jog 2 km? Give your answer in minutes.

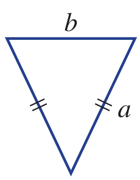
12 A rectangular picture of length 65 cm and width 35 cm is surrounded by a frame of width 5 cm. What is the perimeter of the framed picture?



REASONING

13	13	13, 14

13 Write down rules using the given letters for the perimeter of these shapes, e.g. $P = a + 2b$. Assume that angles that look right-angled are 90°.

a


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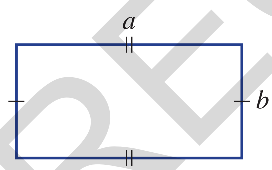

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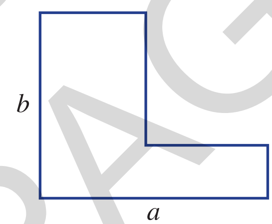

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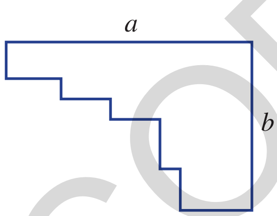

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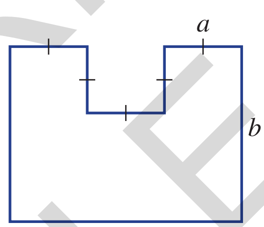

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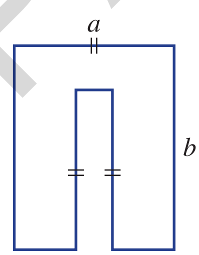

14 Write a rule for x in terms of its perimeter P, e.g. $x = P - 10$.

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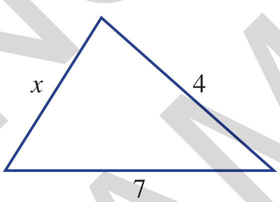

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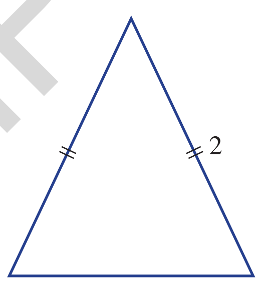

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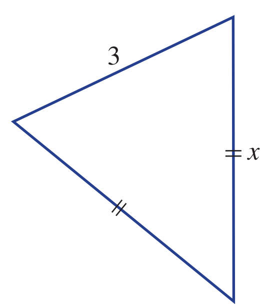

d


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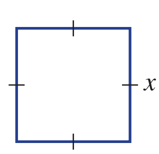

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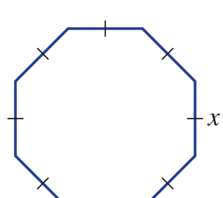


4A

Disappearing squares

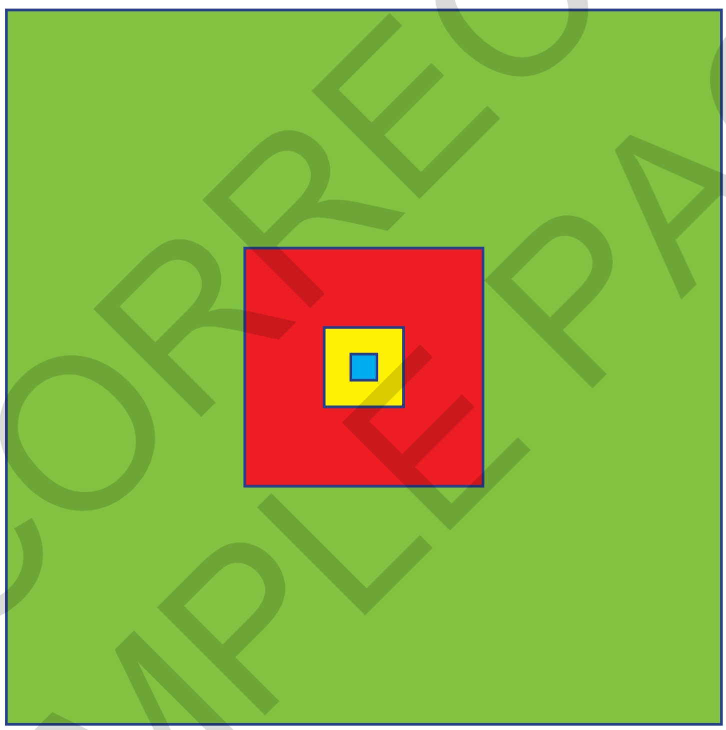
—	—	15

ENRICHMENT

15 A square is drawn with a particular side length. A second square is drawn inside the square so that its side length is one-third that of the original square. Then a third square is drawn, with side length of one-third that of the second square and so on.

a What is the minimum number of squares that would need to be drawn in this pattern (including the starting square), if the innermost square has a perimeter of less than 1 hundredth the perimeter of the outermost square?

b Imagine now if the situation is reversed and each square's perimeter is 3 times larger than the next smallest square. What is the minimum number of squares that would be drawn in total if the perimeter of the outermost square is to be at least 1000 times the perimeter of the innermost square?



4B Circumference of a circle

Interactive

Widgets

HOTsheets

Walkthroughs

Since the ancient times, people have known about a special number that links a circle's diameter to its circumference. We know this number as pi (π). Pi is a mathematical constant that appears in formulas relating to circles, but it is also important in many other areas of mathematics. The actual value of pi has been studied and approximated by ancient and more modern civilisations over thousands of years. The Egyptians knew pi was slightly more than 3 and approximated it to be $\frac{256}{81} \approx 3.16$. The Babylonians used $\frac{25}{8} = 3.125$ and the ancient Indians used $\frac{339}{108} \approx 3.139$.

It is believed that Archimedes of Syracus (287–212 BCE) was the first person to use a mathematical technique to evaluate pi. He was able to prove that pi was greater than $\frac{223}{71}$ and less than $\frac{22}{7}$. In 480 AD, the Chinese mathematician Zu Chongzhi showed that pi was close to $\frac{355}{113} \approx 3.1415929$, which is accurate to seven decimal places.

Before the use of calculators, the fraction $\frac{22}{7}$ was commonly used as a good and simple approximation to pi. Interestingly, mathematicians have been able to prove that pi is an irrational number, which means that there is no fraction that can be found that is exactly equal to pi. If the exact value of pi was written down as a decimal, the decimal places would continue forever with no repeated pattern.

Archimedes of Syracus (287–212 BCE)

Let's start: Discovering pi

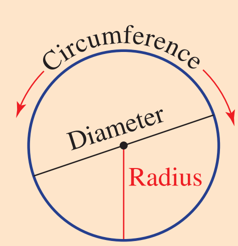
Here are the diameters and circumferences for three circles correct to two decimal places. Use a calculator to work out the value of Circumference ÷ Diameter and put your results in the third column. Add your own circle measurements by measuring the diameter and circumference of circular objects such as a can.

Diameter d (mm)	Circumference C (mm)	$C \div d$
4.46	14.01	
11.88	37.32	
40.99	128.76	
Add your own	Add your own	

- What do you notice about the numbers $C \div d$ in the third column?
- Why might the numbers in the third column vary slightly from one set of measurements to another?
- What rule can you write down which links C with d?

Key ideas

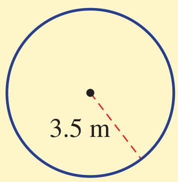
- Features of a circle
  - **Diameter** (d) is the distance across the centre of a circle.
  - **Radius** (r) is the distance from the centre to the circle. Note $d = 2r$.
- **Circumference** (C) is the distance around a circle.
  - $C = 2\pi r$ or $C = \pi d$
- **Pi** (π) ≈ 3.14159 (correct to five decimal places)
  - Common approximations include 3.14 and $\frac{22}{7}$.
  - A more precise estimate for pi can be found on most calculators or on the internet.



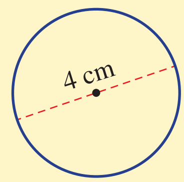
Example 4 Finding the circumference with a calculator

Find the circumference of these circles correct to two decimal places. Use a calculator for the value of pi.

a



b

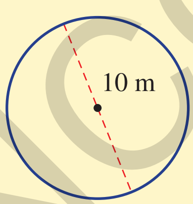


SOLUTION	EXPLANATION
a $C = 2\pi r$	
$= 2 \times \pi \times 3.5$	Since r is given, you can use $C = 2\pi r$.
$= 7\pi$	Alternatively use $C = \pi d$ with $d = 7$.
$= 21.99$ m (to 2 d.p.)	Round off as instructed.
b $C = \pi d$	
$= \pi \times 4$	Substitute into the rule $C = \pi d$ or use
$= 4\pi$	$C = 2\pi r$ with $r = 2$.
$= 12.57$ cm (to 2 d.p.)	Round off as instructed.

Example 5 Finding circumference without a calculator

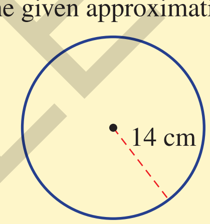
Calculate the circumference of these circles using the given approximation of π.

a



$\pi = 3.14$

b



$\pi = \frac{22}{7}$

SOLUTION	EXPLANATION
a $C = \pi d$	Use $\pi = 3.14$ and multiply mentally. Move the decimal point one place to the right.
$= 3.14 \times 10$	
$= 31.4$ m	Alternatively use $C = 2\pi r$ with $r = 5$.
b $C = 2\pi r$	Use $\pi = \frac{22}{7}$ and cancel the 14 with the 7
$= 2 \times \frac{22}{7} \times 14$	before calculating the final answer.
$= 88$ cm	$2 \times \frac{22}{7} \times 14 = 2 \times 22 \times 2$

Exercise 4B

1–4	4	—

UNDERSTANDING

1 Evaluate the following using a calculator and round to two decimal places.

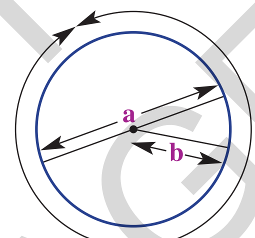
a $\pi \times 5$ **b** $\pi \times 13$ **c** $2 \times \pi \times 3$ **d** $2 \times \pi \times 37$

2 Write down the value of π correct to:

a one decimal place **b** two decimal places

c three decimal places

3 Name the features of the circle as shown.



4 A circle has circumference (C) 81.7 m and diameter (d) 26.0 m correct to one decimal place. Calculate $C \div d$. What do you notice?

5(½), 6, 7	5(½), 6, 7	5(½), 6, 7

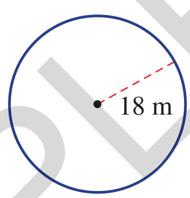
FLUENCY

Example 4

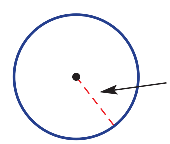
5 Find the circumference of these circles correct to two decimal places. Use a calculator for the value of pi.

a

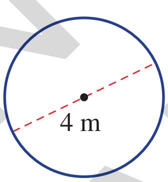
b



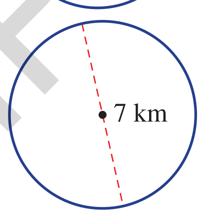
c



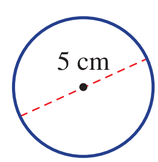
d



e



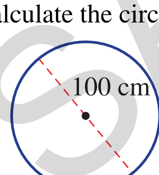
f



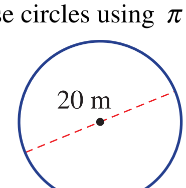
Example 5a

6 Calculate the circumference of these circles using $\pi = 3.14$.

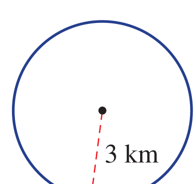
a



b



c

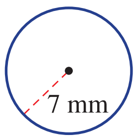


4B

FLUENCY

Example 5b

7 Calculate the circumference of these circles using $\pi = \frac{22}{7}$.

a


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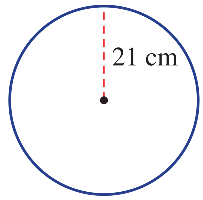
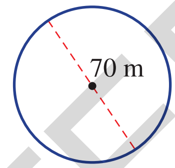


c


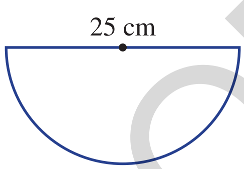
PROBLEM-SOLVING

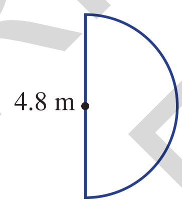
8–10 9–12 11–13

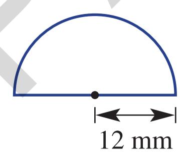
8 A water tank has a diameter of 3.5 m. Find its circumference correct to one decimal place.

9 An athlete trains on a circular track of radius 40 m and jogs 10 laps each day, 5 days a week. How far does he jog each week? Round the answer to the nearest whole number of metres.

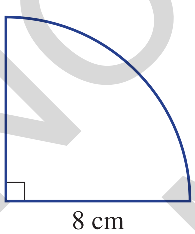
10 These shapes are semicircles. Find the perimeter of these shapes including the straight edge and round the answer to two decimal places.

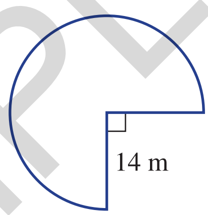
a


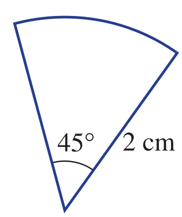
b


c


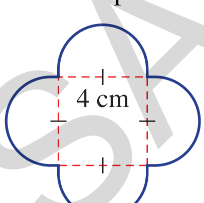
11 Calculate the perimeter of these diagrams correct to two decimal places.

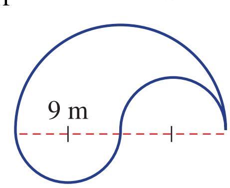
a


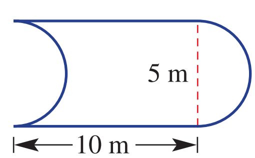
b


c


12 Calculate the perimeter of these shapes correct to two decimal places.

a


b


c


4B

PROBLEM-SOLVING

13 Here are some student's approximate circle measurements. Which students are likely to have incorrect measurements?

	r	C
Mick	4 cm	25.1 cm
Svenya	3.5 m	44 m
Andre	1.1 m	13.8 m

REASONING 14 | 14, 15 | 15–17

14 Explain why the rule $C = 2\pi r$ is equivalent to (i.e. the same as) $C = \pi d$.

15 It is more precise in mathematics to give 'exact' values for circle calculations in terms of π, e.g. $C = 2 \times \pi \times 3$ gives $C = 6\pi$. This gives the final exact answer and is not written as a rounded decimal. Find the exact answers for Question 5 in terms of π.

16 Find the exact answers for Question 12 above in terms of π.

17 We know that $C = 2\pi r$ or $C = \pi d$.

a Rearrange these rules to write a rule for:
  i r in terms of C
  ii d in terms of C

b Use the rules you found in part a to find the following correct to two decimal places.
  i The radius of a circle with circumference 14 m
  ii The diameter of a circle with circumference 20 cm

Memorising pi

ENRICHMENT — | — | 18

18 The box shows π correct to 100 decimal places. The Guinness World record for the most number of digits of π recited from memory is held by Lu Chao, a Chinese student. He recited 67 890 digits non-stop over a 24-hour period.

3.1415926535 8979323846 2643383279 5028841971 6939937510
5820974944 5923078164 0628620899 8628034825 3421170679

Challenge your friends to see who can remember the most number of digits in the decimal representation of π.

Number of digits memorised	Report
10+	A good show
20+	Great effort
35+	Superb
50+	Amazing memory
100 000	World record

4C Area

Interactive
Widgets
HOTsheets
Walkthroughs

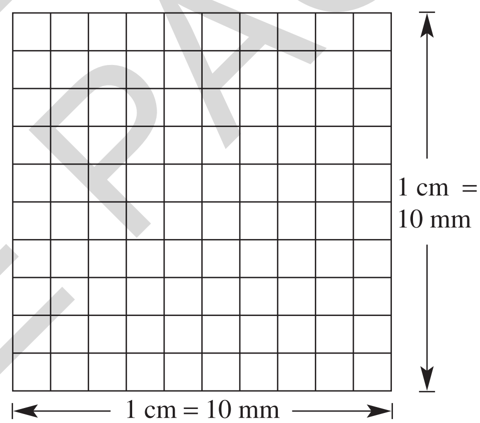
Area is a measure of surface and is often referred to as the amount of space contained inside a two-dimensional space. Area is measured in square units and the common metric units are square millimetres (mm^2), square centimetres (cm^2), square metres (m^2), square kilometres (km^2) and hectares (ha). The hectare is often used to describe area of land, since the square kilometre for such areas is considered to be too large a unit and the square metre too small. A school football oval might be about 1 hectare for example and a small forest might be about 100 hectares.

Large wheat farms in Australia can range from 1000 to 15 000 hectares.

Let's start: Squares of squares

Consider this enlarged drawing of one square centimetre divided into square millimetres.

- How many square millimetres are there on one edge of the square centimetre?
- How many square millimetres are there in total in 1 square centimetre?
- What would you do to convert between mm^2 and cm^2 or cm^2 and mm^2 and why?
- Can you describe how you could calculate the number of square centimetres in one square metre and how many square metres in one square kilometre? What diagrams would you use to explain your answer?



Key ideas

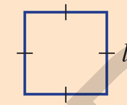
- The common metric units for area include:
  - square millimetres (mm^2)
  - square centimetres (cm^2)
  - square metres (m^2)
  - square kilometres (km^2)
  - hectares (ha)

$$\text{ha} \underset{\div 10\,000}{\overset{\times 10\,000}{\rightleftharpoons}} m^2$$

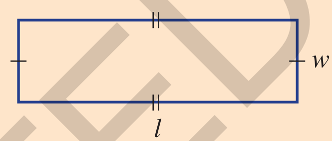
$$km^2 \underset{\div 1000^2 = 1\,000\,000}{\overset{\times 1000^2 = 1\,000\,000}{\rightleftharpoons}} m^2 \underset{\div 100^2 = 10\,000}{\overset{\times 100^2 = 10\,000}{\rightleftharpoons}} cm^2 \underset{\div 10^2 = 100}{\overset{\times 10^2 = 100}{\rightleftharpoons}} mm^2$$

Key ideas

- Area of squares, rectangles and triangles
  - Square $A = l \times l = l^2$

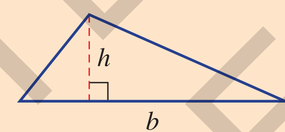


  - Rectangle $A = l \times w = lw$

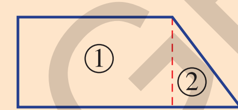


  - Triangle $A = \frac{1}{2} \times b \times h = \frac{1}{2}bh$

    The dashed line which gives the height is **perpendicular** (at right angles) to the base.



- Areas of **composite shapes** can be found by adding or subtracting the area of more basic shapes.



Example 6 Converting units of area

Convert these area measurements to the units shown in the brackets.

a 0.248 m^2 (cm^2)

b 3100 mm^2 (cm^2)

SOLUTION

EXPLANATION

a $0.248\ m^2 = 0.248 \times 10\,000$

$= 2480\ cm^2$

$1\ m^2 = 100^2\ cm^2$

$= 10\,000\ cm^2$

Multiply since you are changing to a smaller unit.

$$m^2 \xrightarrow{\times 100^2} cm^2$$

b $3100\ mm^2 = 3100 \div 100$

$= 31\ cm^2$

$1\ cm^2 = 10^2\ mm^2$

$= 100\ mm^2$

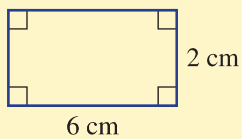
Divide since you are changing to a larger unit.

$$cm^2 \xleftarrow{\div 10^2} mm^2$$

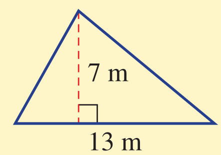
Example 7 Finding areas of rectangles and triangles

Find the area of these shapes.

a



b



SOLUTION

a $A = lw$

$= 6 \times 2$

$= 12\text{ cm}^2$

b $A = \frac{1}{2}bh$

$= \frac{1}{2} \times 13 \times 7$

$= 45.5\text{ m}^2$

EXPLANATION

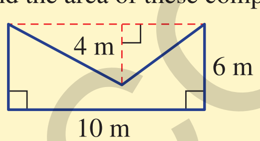
Write the formula for the area of a rectangle and substitute $l = 6$ and $w = 2$.

Remember that the height is measured using a line that is perpendicular to the base.

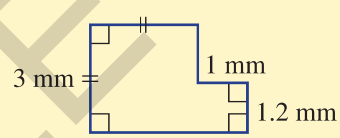
Example 8 Finding areas of composite shapes

Find the area of these composite shapes using addition or subtraction.

a



b



SOLUTION

a $A = lw - \frac{1}{2}bh$

$= 10 \times 6 - \frac{1}{2} \times 10 \times 4$

$= 60 - 20$

$= 40\text{ m}^2$

b $A = l^2 + lw$

$= 3^2 + 1.2 \times 1$

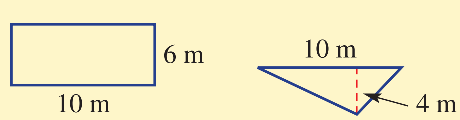
$= 9 + 1.2$

$= 10.2\text{ mm}^2$

EXPLANATION

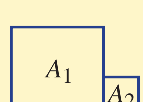
The calculation is done by subtracting the area of a triangle from the area of a rectangle.

Rectangle – triangle



The calculation is done by adding the area of a rectangle to the area of a square.

Area $= A_1 + A_2$



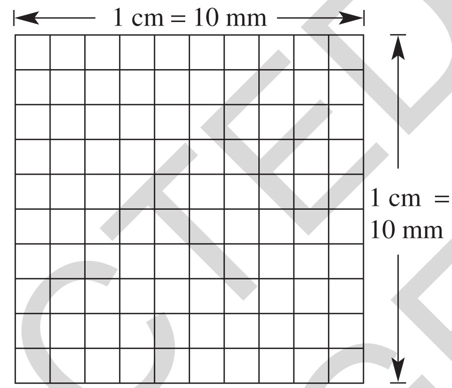
Exercise 4C

1–3	3	—

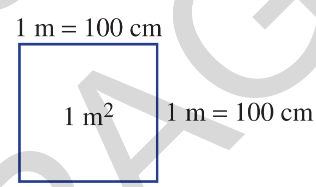
UNDERSTANDING

1 By considering the given diagrams answer the questions.

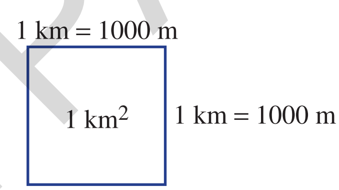
a **i** How many mm^2 in 1 cm^2?
ii How many mm^2 in 4 cm^2?
iii How many cm^2 in 300 mm^2?



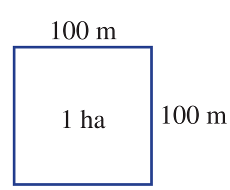
b **i** How many cm^2 in 1 m^2?
ii How many cm^2 in 7 m^2?
iii How many m^2 in 40 000 cm^2?



c **i** How many m^2 in 1 km^2?
ii How many m^2 in 5 km^2?
iii How many km^2 in 2 500 000 m^2?

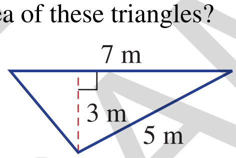


d **i** How many m^2 in 1 ha?
ii How many m^2 in 3 ha?
iii How many ha in 75 000 m^2?

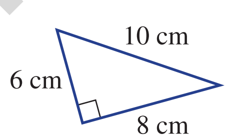


2 Which length measurements would be used for the *base* and the *height* (in that order) to find the area of these triangles?

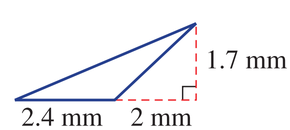
a



b



c



3 One hectare is how many square metres?

4C

4–6(½) | 4–6(½) | 4–6(½)

FLUENCY

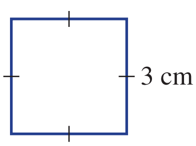
Example 6

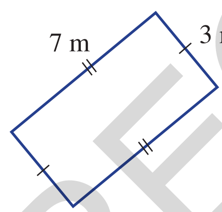
4 Convert these area measurements to the units shown in the brackets.

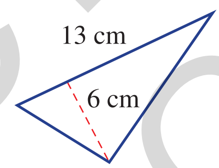
- a $2\ cm^2$ (mm^2)
- b $7\ m^2$ (cm^2)
- c $0.5\ km^2$ (m^2)
- d 3 ha (m^2)
- e $0.34\ cm^2$ (mm^2)
- f $700\ cm^2$ (m^2)
- g $3090\ mm^2$ (cm^2)
- h $0.004\ km^2$ (m^2)
- i $2000\ cm^2$ (m^2)
- j $450\,000\ m^2$ (km^2)
- k $4000\ m^2$ (ha)
- l $3210\ mm^2$ (cm^2)
- m $320\,000\ m^2$ (ha)
- n $0.0051\ m^2$ (cm^2)
- o $0.043\ cm^2$ (mm^2)
- p $4802\ cm^2$ (m^2)
- q $19\,040\ m^2$ (ha)
- r $2933\ m^2$ (ha)
- s 0.0049 ha (m^2)
- t 0.77 ha (m^2)
- u 2.4 ha (m^2)

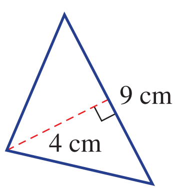
Example 7

5 Find the areas of these squares, rectangles and triangles.

a


b


c


d


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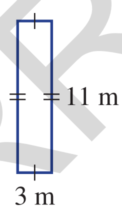

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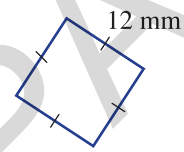

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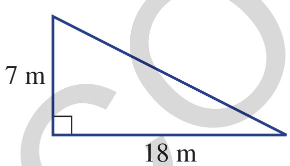

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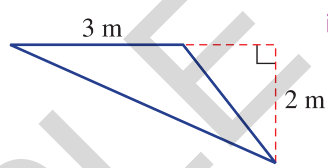

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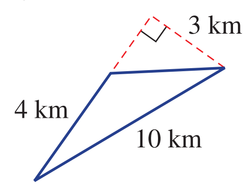
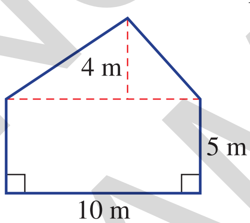


Example 8

6 Find the area of these composite shapes by using addition or subtraction.

a


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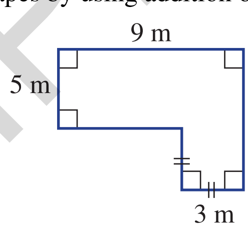

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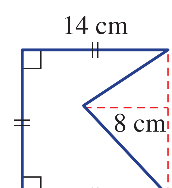

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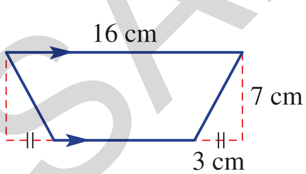

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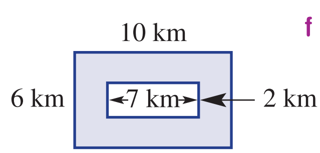
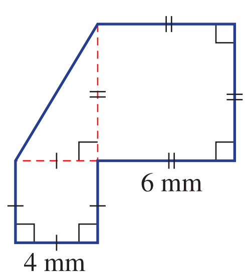


(Find the shaded area)

f


7, 8	7–10(½)	7–10(½), 11, 12

PROBLEM-SOLVING

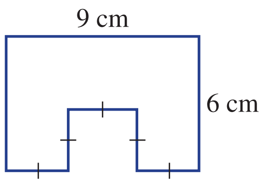
7 Use your knowledge of area units to convert these measurements to the units shown in the brackets.

a 0.2 m^2 (mm^2) **b** 0.000 043 km^2 (cm^2) **c** 374 000 cm^2 (km^2)

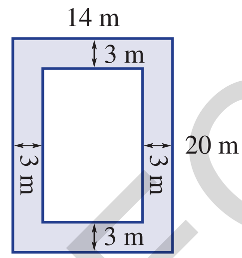
d 10 920 mm^2 (m^2) **e** 0.000 000 2 ha (cm^2) **f** 1 000 000 000 mm^2 (ha)

8 Find the area of these composite shapes. You may need to determine some side lengths first. Assume that angles that look right-angled are 90°.

a

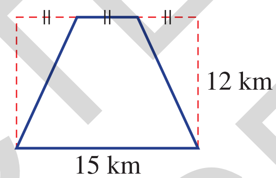


b



(Find the shaded area)

c



9 Find the side length of a square if its area is:

a 36 m^2 **b** 2.25 cm^2

10 **a** Find the area of a square if its perimeter is 20 m.

b Find the area of a square if its perimeter is 18 cm.

c Find the perimeter of a square if its area is 49 cm^2.

d Find the perimeter of a square if its area is 169 m^2.

11 A triangle has area 20 cm^2 and base 4 cm. Find its height.

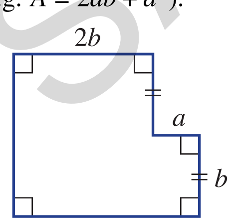
12 Paint costs $12 per litre and can only be purchased in a full number of litres. One litre of paint covers an area of 10 m^2. A rectangular wall is 6.5 m long and 3 m high and needs two coats of paint. What will be the cost of paint for the wall?

13	13, 14	13–15

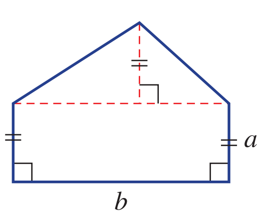
REASONING

13 Write down expressions for the area of these shapes in simplest form using the letters a and b (e.g. $A = 2ab + a^2$).

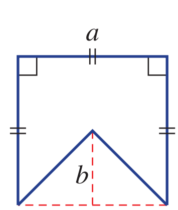
a



b



c



4C

REASONING

14 Using only whole numbers for length and width, answer the following questions.

a How many distinct (different) rectangles have an area of 24 square units?

b How many distinct squares have an area of 16 square units?

15 Write down rules for:

a the width of a rectangle (w) with area A and length l

b the side length of a square (l) with area A

c the height of a triangle (h) with area A and base b

The acre

—	—	16

ENRICHMENT

16 Two of the more important imperial units of length and area that are still used today are the mile and the acre. Many of our country and city roads, farms and house blocks were divided up using these units.

Here are some conversions

1 square mile = 640 acres

1 mile ≈ 1.609344 km

1 hectare = 10 000 m^2

a Use the given conversions to find:

i the number of square kilometres in 1 square mile (round to two decimal places)

ii the number of square metres in 1 square mile (round to the nearest whole number)

iii the number of hectares in 1 square mile (round to the nearest whole number)

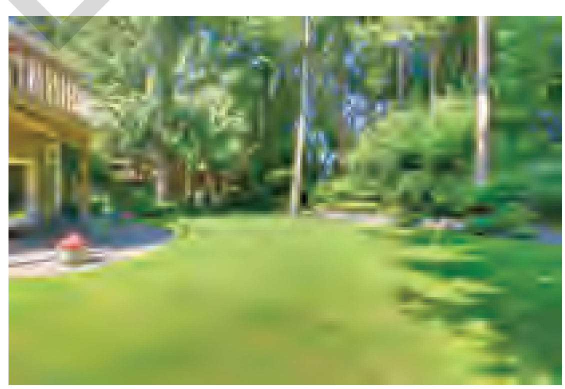
iv the number of square metres in 1 acre (round to the nearest whole number)

v the number of hectares in 1 acre (round to one decimal place)

vi the number of acres in 1 hectare (round to one decimal place)

b A dairy farmer has 200 acres of land. How many hectares is this? (Round your answer to the nearest whole number.)

c A house block is 2500 m^2. What fraction of an acre is this? (Give your answer as a percentage rounded to the nearest whole number.)



4D Area of special quadrilaterals

Interactive
Widgets
HOTsheets
Walkthroughs

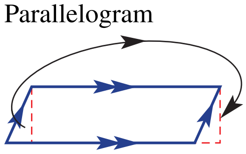
The formulas for the area of a rectangle and a triangle can be used to develop the area of other special quadrilaterals. These quadrilaterals include the parallelogram, the rhombus, the kite and the trapezium. Knowing the formulas for the area of these shapes can save a lot of time dividing shapes into rectangles and triangles.

The area of each quadrilateral needs to be calculated to work out how many pavers are needed.

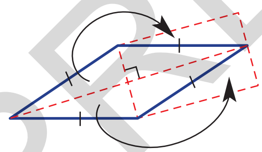
Let's start: Developing formulas

These diagrams contain clues as to how you might find the area of the shape using only what you know about rectangles and triangles. Can you explain what each diagram is trying to tell you?

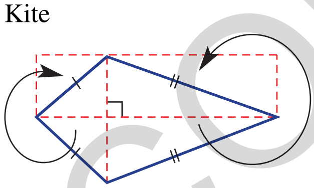
- Parallelogram



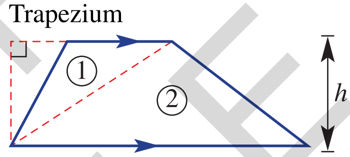
- Rhombus



- Kite

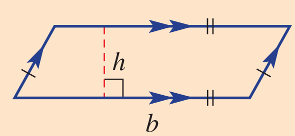


- Trapezium

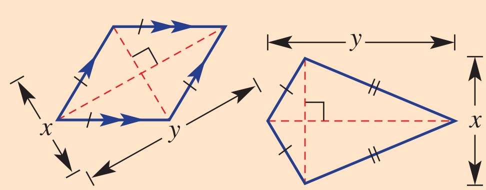


Key ideas

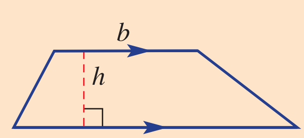
- Area of a **parallelogram**
  Area = base × perpendicular height
  or $A = bh$



- Area of a **rhombus** or **kite**
  Area $= \frac{1}{2} \times$ diagonal $x \times$ diagonal y
  or $A = \frac{1}{2}xy$



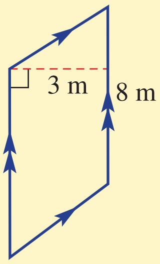
- Area of a **trapezium**
  Area $= \frac{1}{2} \times$ sum of parallel sides × perpendicular height
  or $A = \frac{1}{2}(a + b)h$



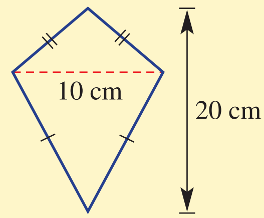
Example 9 Finding areas of special quadrilaterals

Find the area of these shapes.

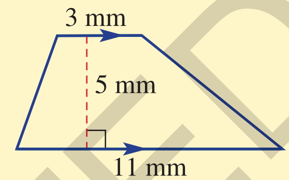
a



b



c



SOLUTION

EXPLANATION

a $A = bh$

$= 8 \times 3$

$= 24\ \text{m}^2$

The height is measured at right angles to the base.

b $A = \frac{1}{2}xy$

$= \frac{1}{2} \times 10 \times 20$

$= 100\ \text{cm}^2$

Use the formula $A = \frac{1}{2}xy$ since both diagonals are given. This formula can also be used for a rhombus.

c $A = \frac{1}{2}(a+b)h$

$= \frac{1}{2} \times (11+3) \times 5$

$= \frac{1}{2} \times 14 \times 5$

$= 35\ \text{mm}^2$

The two parallel sides are 11 mm and 3 mm in length. The perpendicular height is 5 mm.

Exercise 4D

UNDERSTANDING		
1, 2	2	—

1 Find the value of A using these formulas and given values.

a $A = bh\ (b = 2, h = 3)$

b $A = \frac{1}{2}xy\ (x = 5, y = 12)$

c $A = \frac{1}{2}(a+b)h\ (a = 2, b = 7, h = 3)$

d $A = \frac{1}{2}(a+b)h\ (a = 7, b = 4, h = 6)$

4D

UNDERSTANDING

2 Complete these sentences.

a A perpendicular angle is ________ degrees.

b In a parallelogram, you find the area using a base and the ____________.

c The two diagonals in a kite or a rhombus are ________________.

d To find the area of a trapezium you multiply $\frac{1}{2}$ by the sum of the two ____________ sides and then by the ______________ height.

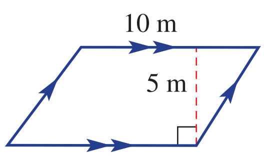
e The two special quadrilaterals that have the same area formula using diagonal lengths x and y are the _____________ and the _____________.

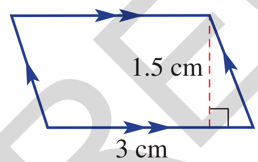
3(½), 4 | 3(½), 4 | 3(½), 4

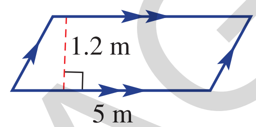
FLUENCY

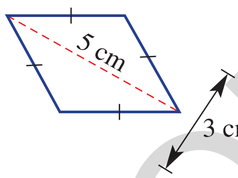
Example 9

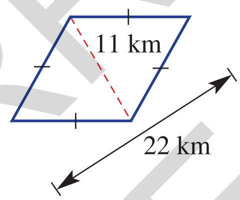
3 Find the area of these special quadrilaterals. First state the name of the shape.

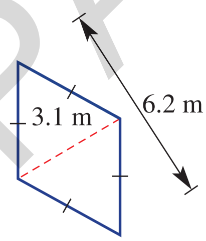
a 

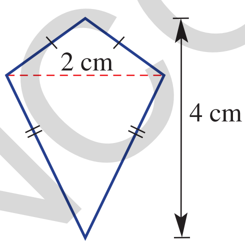
b 

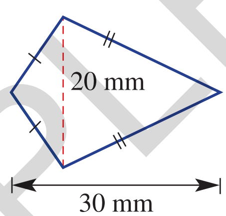
c 

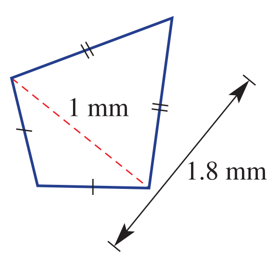
d 

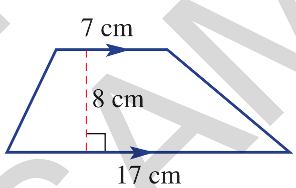
e 

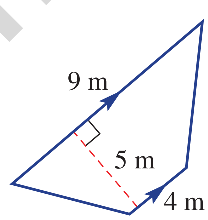
f 

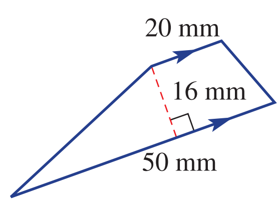
g 

h 

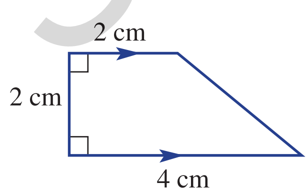
i 

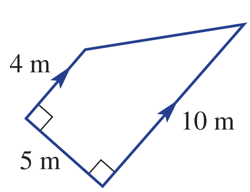
j 

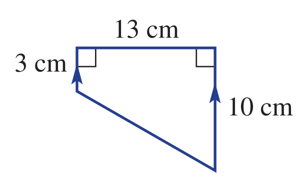
k 

l 

4 These trapeziums have one side at right angles to the two parallel sides. Find the area of each.

a 

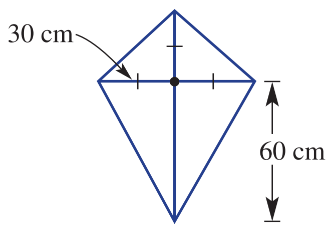
b 

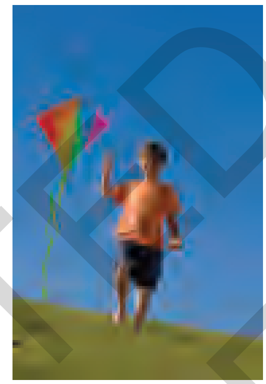
c 

4D

PROBLEM-SOLVING 5, 6 | 5–7 | 6–8

5 A flying kite is made from four centre rods all connected near the middle of the kite as shown. What area of plastic, in square metres, is needed to cover the kite?





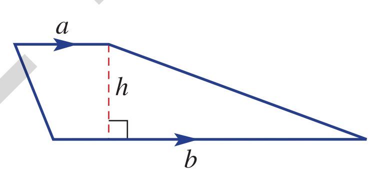
6 A parallelogram has an area of 26 m^2 and its base length is 13 m. What is its perpendicular height?

7 A landscape gardener charges \$20 per square metre of lawn. A lawn area is in the shape of a rhombus and its diagonals are 8 m and 14.5 m. What would be the cost of laying this lawn?

8 The parallel sides of a trapezium are 2 cm apart and one of the sides is 3 times the length of the other. If the area of the trapezium is 12 cm^2, what are the lengths of the parallel sides?

REASONING 9 | 9, 10 | 10, 11

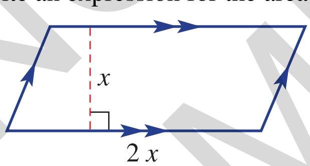
9 Consider this shape.



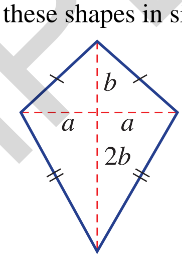
a What type of shape is it?
b Find its area if $a = 5$, $b = 8$, and $h = 3$.
All measurements are in cm.

10 Write an expression for the area of these shapes in simplest form (e.g. $A = 2a + 3ab$).

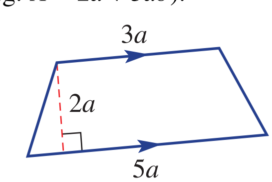
a



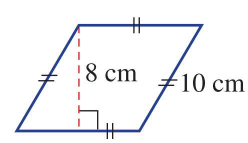
b



c



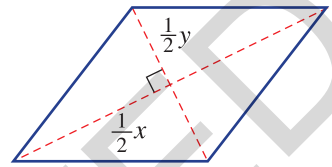
11 Would you use the formula $A = \frac{1}{2}xy$ to find the area of this rhombus? Explain.



Proof	—	—	12, 13

12 Complete these proofs to give the formula for the area of a rhombus and a trapezium.

a Rhombus



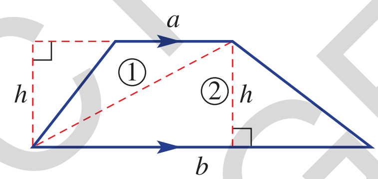
$A = 4$ triangle areas

$= 4 \times \frac{1}{2} \times \text{base} \times \text{height}$

$= 4 \times \frac{1}{2} \times \_\_\_\_\_ \times \_\_\_\_\_$

$= \_\_\_\_\_$

b Trapezium 1



$A =$ Area (triangle 1) + Area (triangle 2)

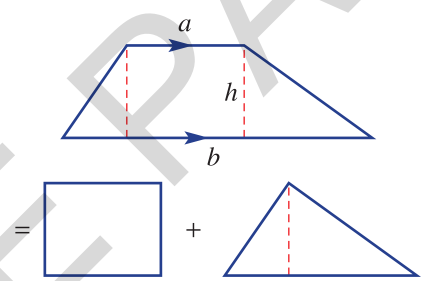
$= \frac{1}{2} \times \text{base}_1 \times \text{height}_1 + \frac{1}{2} \times \text{base}_2 \times \text{height}_2$

$= \frac{1}{2} \times \_\_\_\_\_ \times \_\_\_\_\_ + \frac{1}{2} \times \_\_\_\_\_ \times \_\_\_\_\_$

$= \_\_\_\_\_ + \_\_\_\_\_$

$= \_\_\_\_\_$

c Trapezium 2



$A =$ Area (rectangle) + Area (triangle)

$= \text{length} \times \text{width} + \frac{1}{2} \times \text{base} \times \text{height}$

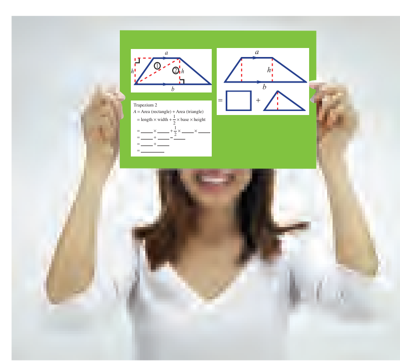
$= \_\_\_\_\_ \times \_\_\_\_\_ + \frac{1}{2} \times \_\_\_\_\_ \times \_\_\_\_\_$

$= \_\_\_\_\_ + \_\_\_\_\_ - \_\_\_\_\_$

$= \_\_\_\_\_ + \_\_\_\_\_$

$= \_\_\_\_\_$

13 Design an A4 poster for one of the proofs in Question **12** to be displayed in your class.

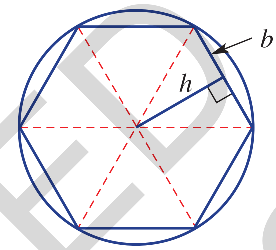


4E Area of a circle

We know that the link between the perimeter of a circle and its radius has challenged civilisations for thousands of years. Similarly people have studied the link between a circle's radius and its area.

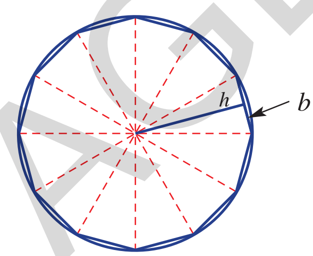
Archimedes (287–212 BCE) attempted to calculate the exact area of a circle using a particular technique involving limits. If a circle is approximated by a regular hexagon, then the approximate area would be the sum of the areas of 6 triangles with base b and height h.

So $A \approx 6 \times \frac{1}{2}bh$



Hexagon ($n = 6$)
$A = 6 \times \frac{1}{2}bh$

If the number of sides (n) on the polygon increases, the approximation would improve. If n approaches infinity, the error in estimating the area of the circle would diminish to zero.



Dodecagon ($n = 12$)
$A = 12 \times \frac{1}{2}bh$

Proof

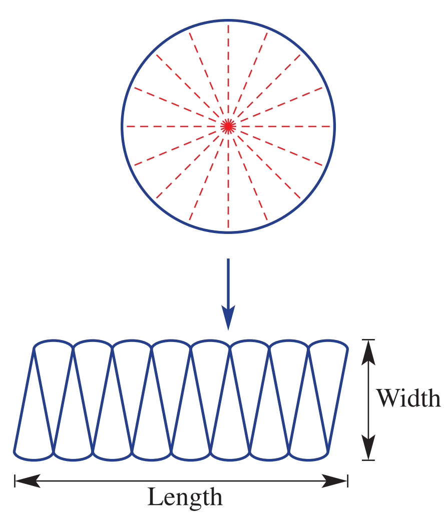
$$\begin{aligned} A &= n \times \frac{1}{2}bh \\ &= \frac{1}{2} \times nb \times h \\ &= \frac{1}{2} \times 2\pi r \times r \\ &= \pi r^2 \end{aligned}$$

(As n approaches ∞, nb limits to $2\pi r$ as nb is the perimeter of the polygon, and h limits to r.)

Let's start: Area as a rectangle

Imagine a circle cut into small sectors and arranged as shown.

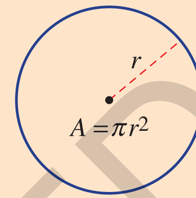
Now try to imagine how the arrangement on the right would change if the number of sector divisions was not 16 (as shown) but a much higher number.



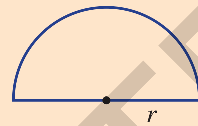
- What would the shape on the right look like if the number of sector divisions was a very high number? What would the length and width relate to in the original circle?
- Try to complete this proof.
  A = length × width
  $= \frac{1}{2} \times$ _____ $\times r$
  = __________

Key ideas

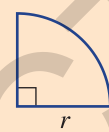
- The ratio of the area of a circle to the square of its radius is equal to π.
  $\frac{A}{r^2} = \pi$ so $A = \pi r^2$



- A half circle is called a **semicircle**.
  $A = \frac{1}{2}\pi r^2$



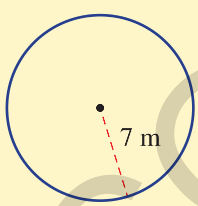
- A quarter circle is called a **quadrant**.
  $A = \frac{1}{4}\pi r^2$



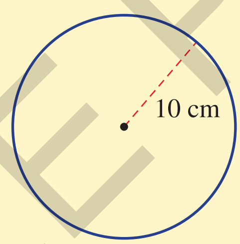
Example 10 Finding circle areas without technology

Find the area of these circles using the given approximate value of π.

a $\pi = \frac{22}{7}$



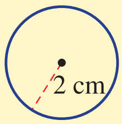
b $\pi = 3.14$



SOLUTION	EXPLANATION
a $A = \pi r^2$	Always write the rule.
$= \frac{22}{7} \times 7^2$	Use $\pi = \frac{22}{7}$ and $r = 7$.
$= 154\ \text{m}^2$	$\frac{22}{7} \times 7 \times 7 = 22 \times 7$
b $A = \pi r^2$	Use $\pi = 3.14$ and substitute $r = 10$.
$= 3.14 \times 10^2$	3.14×10^2 is the same as 3.14×100
$= 314\ \text{cm}^2$	

Example 11 Finding circle areas using a calculator

Use a calculator to find the area of this circle correct to two decimal places.



SOLUTION

$A = \pi r^2$

$= \pi \times 2^2$

$= 12.57$ cm^2 (to 2 d.p.)

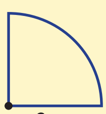
EXPLANATION

Use the π button on the calculator and enter $\pi \times 2^2$ or $\pi \times 4$.

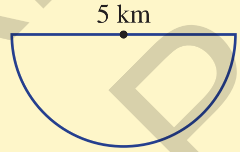
Example 12 Finding areas of semicircles and quadrants

Find the area of this quadrant and semicircle correct to two decimal places.

a



b



SOLUTION

a $A = \frac{1}{4} \times \pi r^2$

$= \frac{1}{4} \times \pi \times 3^2$

$= 7.07$ m^2 (to 2 d.p.)

b $r = \frac{5}{2} = 2.5$

$A = \frac{1}{2} \times \pi r^2$

$= \frac{1}{2} \times \pi \times 2.5^2$

$= 9.82$ km^2 (to 2 d.p.)

EXPLANATION

a The area of a quadrant is $\frac{1}{4}$ the area of a circle with the same radius.

b The radius is half the diameter.

The area of a semicircle is $\frac{1}{2}$ the area of a circle with the same radius.

Exercise 4E

1–3	3	—

UNDERSTANDING

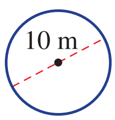
1 Evaluate without the use of a calculator.

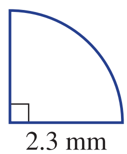
a 3.14×10 b 3.14×4 c $\frac{22}{7} \times 7$ d $\frac{22}{7} \times 7^2$

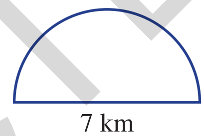
2 Use a calculator to evaluate these to two decimal places.

a $\pi \times 5^2$ b $\pi \times 13^2$ c $\pi \times 3.1^2$ d $\pi \times 9.8^2$

3 What radius length (r) would be used to help find the area of these shapes?

a


b


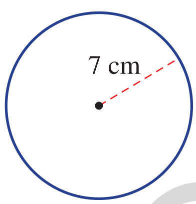
c


4–6(½)	4–6(½)	4–6(½)

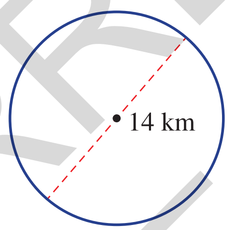
FLUENCY

Example 10

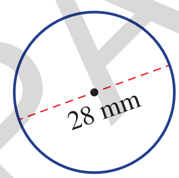
4 Find the area of these circles, using the given approximate value of π.

a


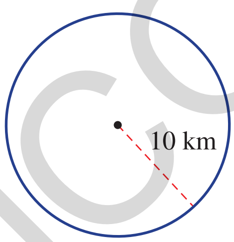
$\pi = \frac{22}{7}$

b


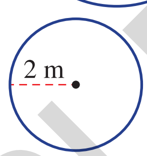
$\pi = \frac{22}{7}$

c


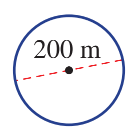
$\pi = \frac{22}{7}$

d


$\pi = 3.14$

e


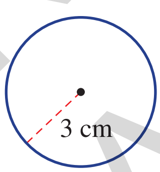
$\pi = 3.14$

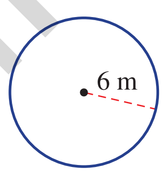
f


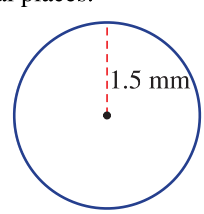
$\pi = 3.14$

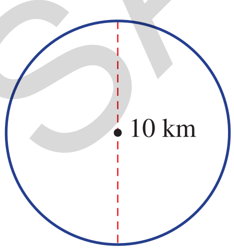
Example 11

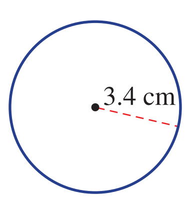
5 Use a calculator to find the area of these circles correct to two decimal places.

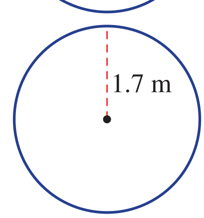
a


b


c


d


e


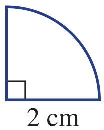
f


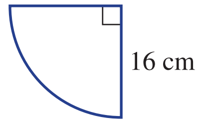
4E

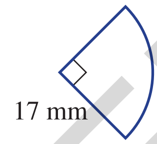
FLUENCY

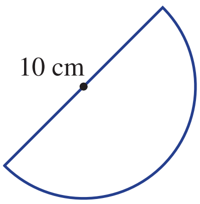
Example 12

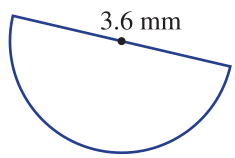
6 Find the area of these quadrants and semicircles correct to two decimal places.

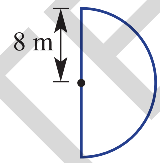
a


b


c


d


e


f


7, 8	9–11	10–12

PROBLEM-SOLVING

7 A pizza tray has a diameter of 30 cm. Calculate its area to the nearest whole number of cm^2.

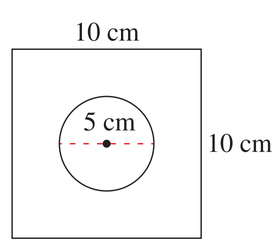
8 A tree trunk is cut to reveal a circular cross-section of radius 60 cm. Is the area of the cross-section more than 1 m^2 and, if so, by how much? Round your answer to the nearest whole number of cm^2.

9 A circular oil slick has a diameter of 1 km. The newspaper reported an area of more than 1 km^2. Is the newspaper correct?

10 Two circular plates have radii 12 cm and 13 cm. Find the difference in their area correct to two decimal places.

11 Which has the largest area, a circle of radius 5 m, a semicircle of radius 7 m or a quadrant of radius 9 m?

12 A square of side length 10 cm has a hole in the middle. The diameter of the hole is 5 cm. What is the area remaining? Round the answer to the nearest whole number.



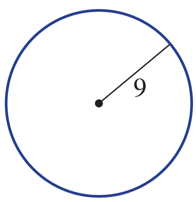
13	13, 14	14, 15

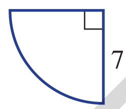
REASONING

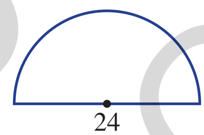
13 A circle has radius 2 cm.

a Find the area of the circle using $\pi = 3.14$.
b Find the area if the radius is doubled to 4 cm.
c What is the effect on the area if the radius is doubled?
d What is the effect on the area if the radius is tripled?
e What is the effect on the area if the radius is quadrupled?
f What is the effect on the area if the radius is multiplied by n?

14 The area of a circle with radius 2 could be written exactly as $A = \pi \times 2^2 = 4\pi$. Write the exact area of these shapes.

a


b


c


15 We know that the diameter d of a circle is twice the radius r, i.e. $d = 2r$ or $r = \frac{1}{2}d$.

a Substitute $r = \frac{1}{2}d$ into the rule $A = \pi r^2$ to find a rule for the area of a circle in terms of d.
b Use your rule from part **a** to check that the area of a circle with diameter 10 m is 25π m^2.

Reverse problems

—	—	16

ENRICHMENT

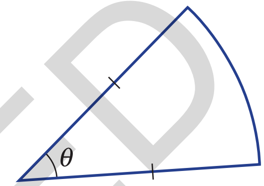
16 Reverse the rule $A = \pi r^2$ to find the radius in these problems.

a If $A = 10$, use your calculator to show that $r \approx 1.78$.
b Find the radius of circles with these areas. Round the answer to two decimal places.
  i 17 m^2 **ii** 4.5 km^2 **iii** 320 mm^2
c Can you write a rule for r in terms of A? Check that it works for the circles defined in part **b**.

4F Sectors and composite shapes EXTENDING

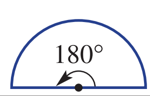
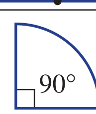
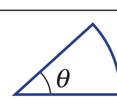
Interactive
Widgets
HOTsheets
Walkthroughs

A slice of pizza or a portion of a round cake cut from the centre forms a shape called a sector. The area cleaned by a windscreen wiper could also be thought of as a difference of two sectors with the same angle but different radii. Clearly the area of a sector depends on its radius, but it also depends on the angle between the two straight edges.



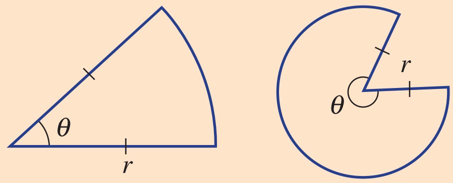
Let's start: The sector area formula

Complete this table to develop the rule for finding the area of a sector.

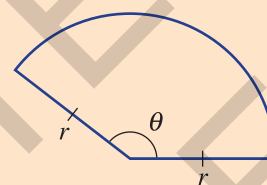
Angle	Fraction of area	Area rule	Diagram
180°	$\frac{180}{360} = \frac{1}{2}$	$A = \frac{1}{2} \times \pi r^2$	 180°
90°	$\frac{90}{360} = $ ____	$A = $ ____ $\times \pi r^2$	 90°
45°			
30°			
θ		$A = $ ____ $\times \pi r^2$	 θ

Key ideas

- A **sector** is formed by dividing a circle with two radii.



- A sector's area is determined by calculating a fraction of the area of a circle with the same radius.
  - Fraction is $\frac{\theta}{360}$
  - Sector area $= \frac{\theta}{360} \times \pi r^2$



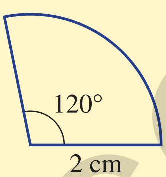
- The area of a **composite shape** can be found by adding or subtracting the areas of more basic shapes.

$A = lw + \frac{1}{2}\pi r^2$

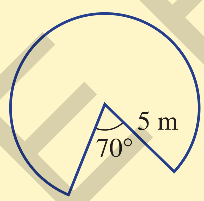
Example 13 Finding areas of sectors

Find the area of these sectors correct to two decimal places.

a



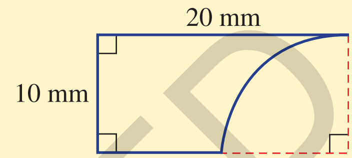
b



SOLUTION	EXPLANATION
a $A = \frac{\theta}{360} \times \pi r^2$	First write the rule for the area of a sector.
$= \frac{120}{360} \times \pi \times 2^2$	Substitute $\theta = 120$ and $r = 2$. Note that $\frac{120}{360}$ simplifies to $\frac{1}{3}$.
$= \frac{1}{3} \times \pi \times 4$	
$= 4.19\text{ cm}^2$ (to 2 d.p.)	
b $\theta = 360 - 70 = 290$	First calculate the angle inside the sector and remember that a revolution is 360°. Then substitute $\theta = 290$ and $r = 5$.
$A = \frac{\theta}{360} \times \pi r^2$	
$= \frac{290}{360} \times \pi \times 5^2$	
$= 63.27\text{ m}^2$ (to 2 d.p.)	

Example 14 Finding areas of composite shapes

Find the area of this composite shape correct to the nearest whole number of mm^2.



SOLUTION	EXPLANATION
a $A = lw - \frac{1}{4}\pi r^2$ $= 20 \times 10 - \frac{1}{4} \times \pi \times 10^2$ $= 200 - 25\pi$ $= 121$ mm^2 (to nearest whole number)	The area can be found by subtracting the area of a quadrant from the area of a rectangle.

Exercise 4F

UNDERSTANDING	1–3	3	—

1 Simplify these fractions.

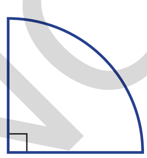
a $\frac{180}{360}$ **b** $\frac{90}{360}$ **c** $\frac{60}{360}$ **d** $\frac{45}{360}$

2 Evaluate the following using a calculator. Give your answer correct to two decimal places.

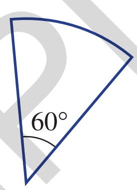
a $\frac{80}{360} \times \pi \times 2^2$ **b** $\frac{20}{360} \times \pi \times 7^2$ **c** $\frac{210}{360} \times \pi \times 2.3^2$

3 What fraction of a circle in simplest form is shown by these sectors?

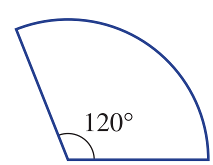
a



b



c

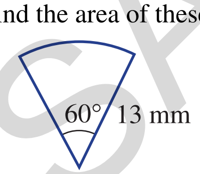


FLUENCY	4(½), 5, 6(½)	4(½), 5, 6(½)	4(½), 5, 6(½)

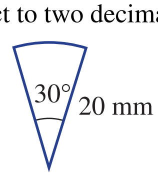
Example 13a

4 Find the area of these sectors correct to two decimal places.

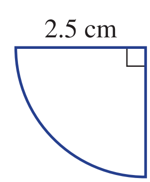
a



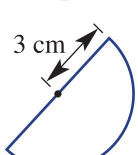
b



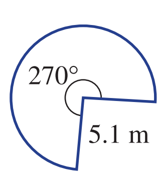
c



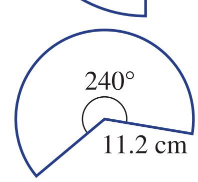
d



e

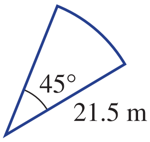


f

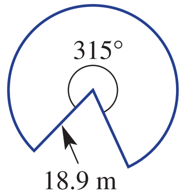


FLUENCY

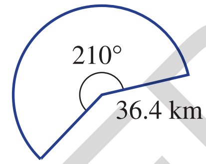
g



h



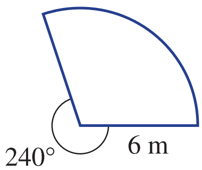
i



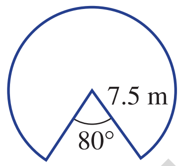
Example 13b

5 Find the area of these sectors correct to two decimal places.

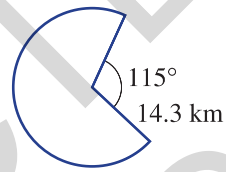
a



b



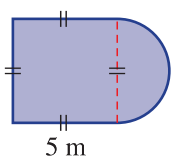
c



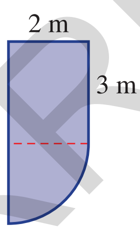
Example 14

6 Find the areas of these composite shapes using addition or subtraction. Round the answer to two decimal places.

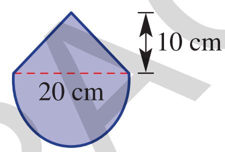
a



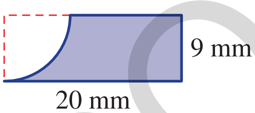
b



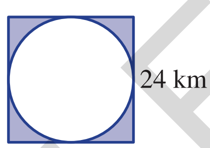
c



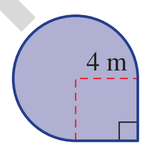
d



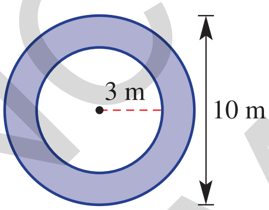
e



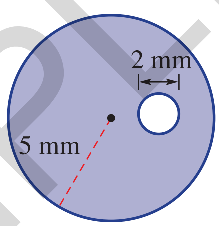
f



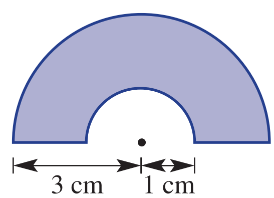
g



h



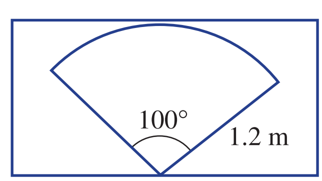
i



7–9 8–10 8–10

PROBLEM-SOLVING

7 A simple bus wiper blade wipes an area over 100° as shown. Find the area wiped by the blade correct to two decimal places.

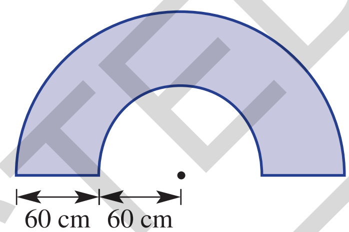


4F

PROBLEM-SOLVING

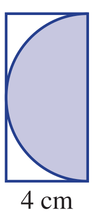
8 At Buy-by-the-sector Pizza they offer a sector of a 15 cm radius pizza with an angle of 45° or a sector of a 13 cm radius pizza with an angle of 60°. Which piece gives the bigger area and by how much? Round the answer to two decimal places.

9 An archway is made up of an inside and outside semicircle as shown. Find the area of the arch correct to the nearest whole cm^2.

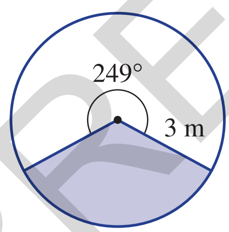


10 What percentage of the total area is occupied by the shaded region in these diagrams? Round the answer to one decimal place.

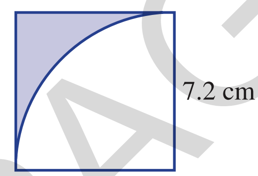
a



b



c

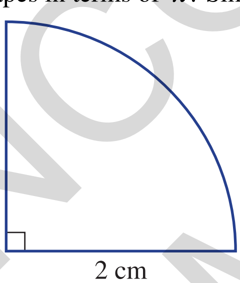


11(½)	11(½)	11(½), 12

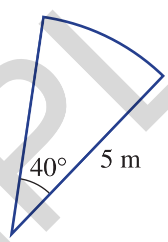
REASONING

11 An exact area measure in terms of π might look like $\pi \times 2^2 = 4\pi$. Find the exact area of these shapes in terms of π. Simplify your answer.

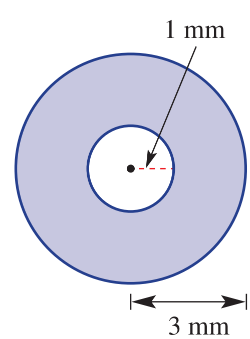
a



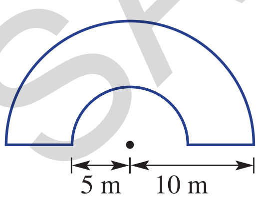
b



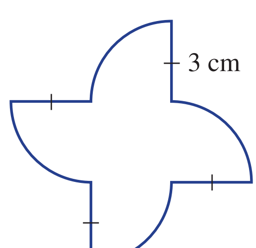
c Find the shaded area



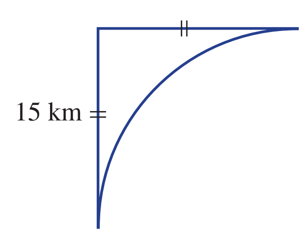
d



e



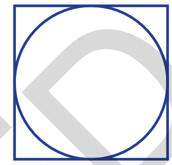
f



4F

REASONING

12 Consider the percentage of the area occupied by a circle inside a square and touching all sides as shown.



a If the radius of the circle is 4 cm, find the percentage of area occupied by the circle. Round the answer to one decimal place.

b Repeat part **a** for a radius of 10 cm. What do you notice?

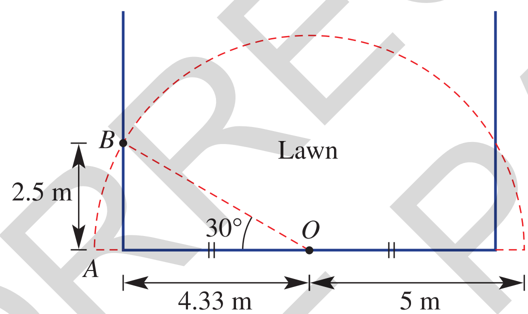
c Can you prove that the percentage area is always the same for any radius r? Hint: Find the percentage area using the pronumeral r for the radius.

Sprinkler waste

—	—	13

ENRICHMENT

13 A rectangular lawn area has a 180° sprinkler positioned in the middle of one side as shown.



a Find the area of the sector OAB correct to two decimal places.

b Find the area watered by the sprinkler outside the lawn area correct to two decimal places.

c Find the percentage of water wasted, giving the answer correct to one decimal place.

4G Surface area of a prism EXTENDING

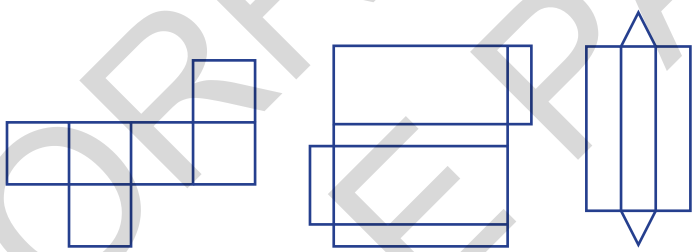
Interactive
Widgets
HOTsheets
Walkthroughs

Many problems in three dimensions can be solved by looking at the problem or parts of the problem in two dimensions. Finding the surface area of a solid is a good example of this, as each face can usually be redrawn in two-dimensional space. The approximate surface area of the walls of an unpainted house, for example, could be calculated by looking at each wall separately and adding to get a total surface area.

Let's start: Possible prisms

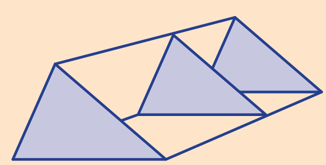
Here are three nets that fold to form three different prisms.

- Can you draw and name the prisms?
- Try drawing other nets of these prisms that are a different shape to the nets given here.

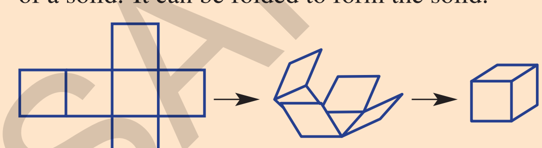


Key ideas

- A **prism** is a polyhedron with a constant (uniform) **cross-section**.
  - The cross-section is parallel to the two identical (congruent) ends.
  - The other sides are parallelograms (or rectangles for right prisms).



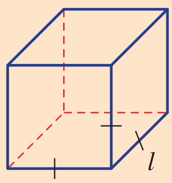
- A **net** is a two-dimensional representation of all the surfaces of a solid. It can be folded to form the solid.



Key ideas

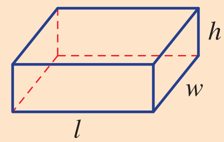
- The **total surface area** (TSA) of a prism is the sum of the areas of all its faces.

Cube



$TSA = 6l^2$

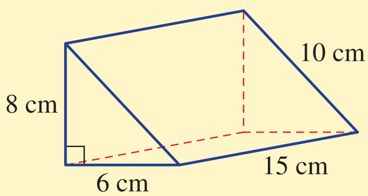
Rectangular prism



$TSA = 2lw + 2lh + 2wh$

Example 15 Finding surface areas

Find the surface area of this prism.



SOLUTION

Area of 2 triangular ends

$A = 2 \times \frac{1}{2} \times bh$

$= 2 \times \frac{1}{2} \times 6 \times 8$

$= 48 \text{ cm}^2$

Area of 3 rectangles

$A = (6 \times 15) + (8 \times 15) + (10 \times 15)$

$= 360 \text{ cm}^2$

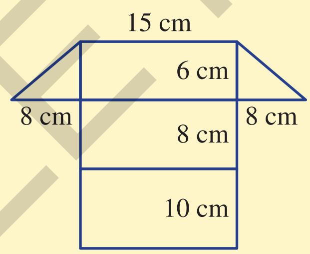
Surface area

$TSA = 48 + 360$

$= 408 \text{ cm}^2$

EXPLANATION

One possible net is



Work out the area of each shape or group of shapes and find the sum of their areas to obtain the total surface area.

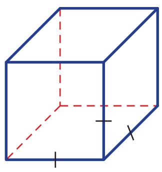
The surface area of this chocolate can be estimated by a similar process.

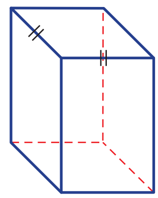
Exercise 4G

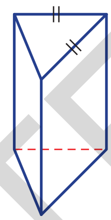
1–3	3	—

UNDERSTANDING

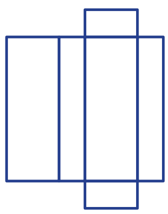
1 How many faces are there on these prisms? Also name the types of shapes that make the different faces.

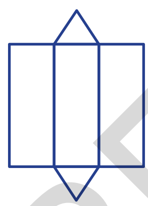
a 

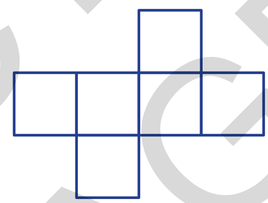
b 

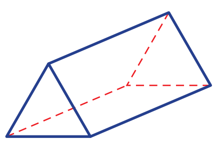
c 

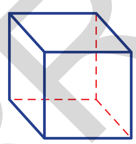
2 Match the net to its solid.

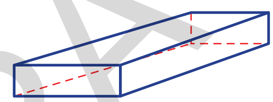
a 

b 

c 

A 

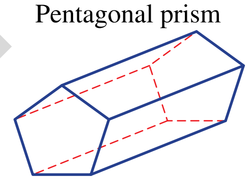
B 

C 

3 How many rectangular faces are on these solids?

a Triangular prism
b Rectangular prism
c Hexagonal prism
d Pentagonal prism

Pentagonal prism

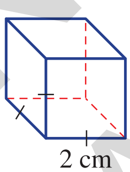


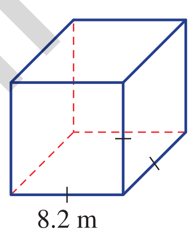
4(½)	4(½), 5	4(½), 5

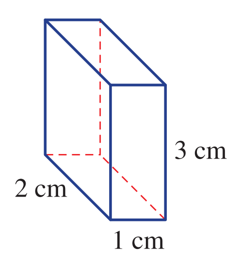
FLUENCY

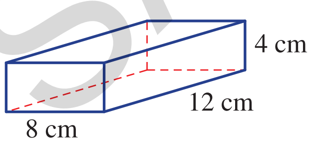
Example 15

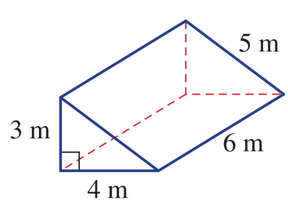
4 Find the surface area of these right prisms.

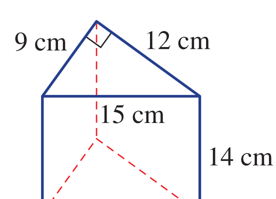
a 

b 

c 

d 

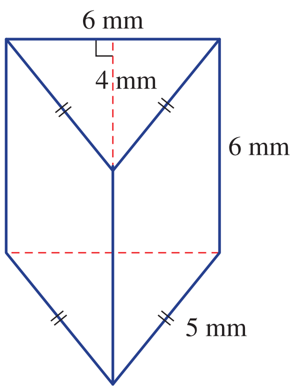
e 

f 

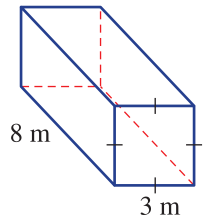
4G

FLUENCY

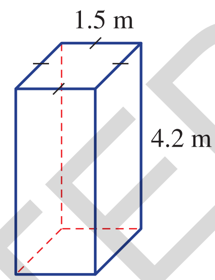
g



h



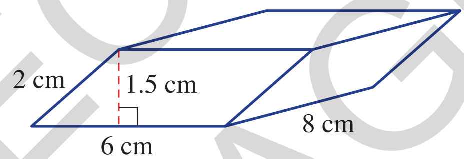
i



5 This prism has two end faces that are parallelograms.

a Use $A = bh$ to find the combined area of the two ends.

b Find the surface area of the prism.



6, 7 | 6–8 | 7–9

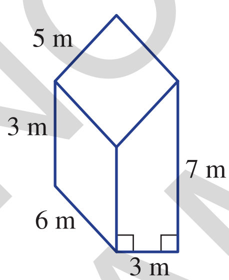
PROBLEM-SOLVING

6 An open box (with no lid) is in the shape of a cube and is painted on the outside including the base. What surface area is painted if the side length of the box is 20 cm?

7 A book 20 cm long, 15 cm wide and 3 cm thick is covered in plastic. What area of plastic is needed for to cover 1000 books? Convert your answer to m^2.

8 Find the surface area of these solids.

a



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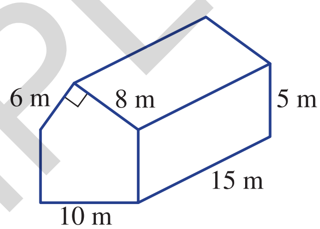
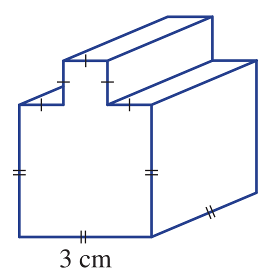
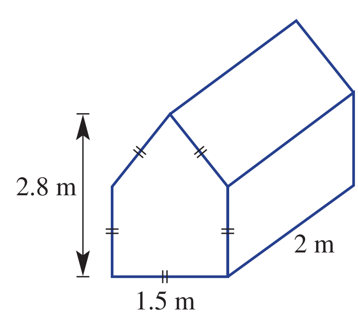


c



9 The floor, sides and roof of this tent are made from canvas at a cost of \$5 per square metre. The tent's dimensions are shown in the diagram. What is the cost of the canvas for the tent?

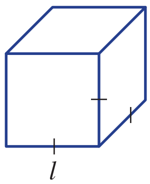


4G

REASONING	10	10, 11	10, 11

10 Write down the rule for the surface area for these right prisms in simplest form.

a



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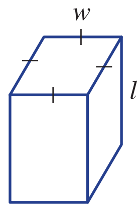
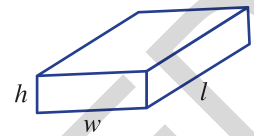


c



11 A cube of side length 1 cm has a surface area of 6 cm^2.

a What is the effect on the surface area of the cube if:

i its side length is doubled?

ii its side length is tripled?

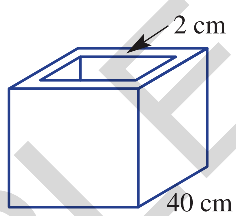
iii its side length is quadrupled?

b Do you notice a pattern from your answers to part **a**? What effect would multiplying the side length by a factor of n have on the surface area?

ENRICHMENT	—	—	12

The thick wooden box

12 An open box (with no lid) in the shape of a cube is made of wood that is 2 cm thick. Its outside side length is 40 cm.



a Find its total surface area both inside and out.

b If the box was made with wood that is 1 cm thick, what would be the increase in surface area?

4H Volume and capacity

Interactive

Widgets

HOTsheets

Walkthroughs

Volume is a measure of the space occupied by a three-dimensional object. It is measured in cubic units. Common metric units for volume given in abbreviated form include mm^3, cm^3, m^3 and km^3. We also use mL, L, kL and ML to describe volumes of fluids or gas. The volume of space occupied by a room in a house for example might be calculated in cubic metres (m^3) or the capacity of fuel tanker might be measured in litres (L) or kilolitres (kL).


A fuel tanker can carry about 32 000 litres.

Let's start: Packing a shipping container

There are 250 crates of apples to be shipped from Australia to Japan. Each crate is 1 m long, 1 m wide and 1 m high. The shipping container used to hold the crates is 12 m long, 4 m wide and 5 m high.

The fruit picker says that the 250 crates will 'fit in, no problems'. The forklift driver says that the 250 crates will 'just squeeze in'. The truck driver says that 'you will need more than one shipping container'.

- Explain how the crates might be packed into the container. How many will fit into one end?
- Who (the fruit picker, forklift driver or truck driver) is the most accurate? Explain your choice.
- What size shipping container and what dimensions would be required to take all 250 crates with no space left over? Is this possible or practical?

Key ideas

- **Volume** is measured in cubic units. Common metric units are:
  - cubic millimetres (mm^3)
  - cubic centimetres (cm^3)
  - cubic metres (m^3)
  - cubic kilometres (km^3)

$$\begin{tikzcd} km^3 \arrow[r, bend left, "\times 1000^3"] & m^3 \arrow[l, bend left, "\div 1000^3"] \arrow[r, bend left, "\times 100^3"] & cm^3 \arrow[l, bend left, "\div 100^3"] \arrow[r, bend left, "\times 10^3"] & mm^3 \arrow[l, bend left, "\div 10^3"] \end{tikzcd}$$

- **Capacity** is the volume of fluid or gas that a container can hold.

  Common metric units are:
  - millilitre (mL)
  - litre (L)
  - kilolitre (kL)
  - megalitre (ML)

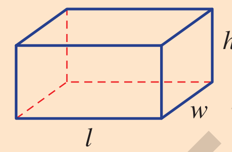
$$\begin{tikzcd} ML \arrow[r, bend left, "\times 1000"] & kL \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & L \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & mL \arrow[l, bend left, "\div 1000"] \end{tikzcd}$$

- Some common conversions are:
  - $1\ mL = 1\ cm^3$
  - $1\ L = 1000\ mL$
  - $1\ kL = 1000\ L = 1\ m^3$

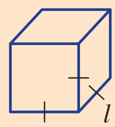
Key ideas

- Volume of a rectangular prism
  - Volume = length × width × height

    $V = lwh$

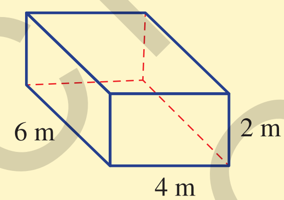


- Volume of a cube
  - $V = l^3$



Example 16 Finding the volume of a rectangular prism

Find the volume of this rectangular prism.



SOLUTION

$V = lwh$

$= 6 \times 4 \times 2$

$= 48\text{ m}^3$

EXPLANATION

First write the rule and then substitute for the length, width and height. Any order will do since $6 \times 4 \times 2 = 4 \times 6 \times 2 = 2 \times 4 \times 6$ etc.

Example 17 Finding capacity

Find the capacity, in litres, for a container that is a rectangular prism 20 cm long, 10 cm wide and 15 cm high.

SOLUTION

$V = lwh$

$= 20 \times 10 \times 15$

$= 3000\text{ cm}^3$

$= 3000 \div 1000$

$= 3\text{ L}$

EXPLANATION

First calculate the volume of the container in cm^3.

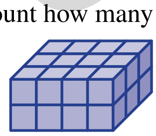
Then convert to litres using $1\text{ L} = 1000\text{ cm}^3$.

Exercise 4H

1–3	3	—

UNDERSTANDING

1 Count how many cubic units are shown in these cube stacks.

a


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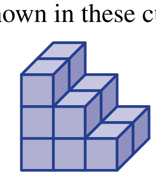

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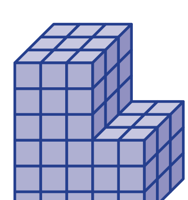


4H

UNDERSTANDING

2 Find the missing number.

a $3 \times 4 \times 8 =$ _____
b _____ $\times 5 \times 20 = 600$
c $8 \times$ _____ $\times 12 = 192$
d $20 \times 2 \times$ _____ $= 200$

3 Write the missing number in the following unit conversions.

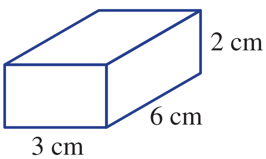
a 1 L = _____ mL
b _____ kL = 1000 L
c 1000 kL = _____ ML
d 1 mL = _____ cm^3
e 1000 cm^3 = _____ L
f 1 m^3 = _____ L

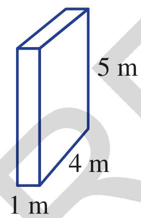
4–6(½)	4–6(½)	4–6(½)

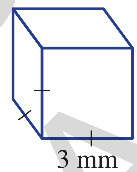
FLUENCY

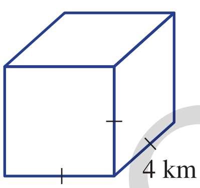
Example 16

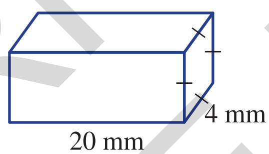
4 Find the volume of these rectangular prisms.

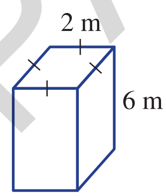
a 

b 

c 

d 

e 

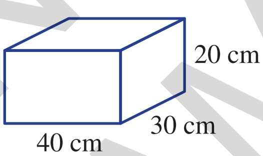
f 

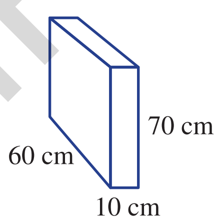
5 Convert the measurements to the units shown in the brackets.

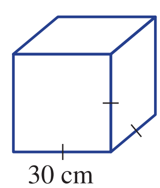
a 2 L (mL)
b 5 kL (L)
c 0.5 ML (kL)
d 3000 mL (L)
e 4 mL (cm^3)
f 50 cm^3 (mL)
g 2500 cm^3 (L)
h 5.1 L (cm^3)

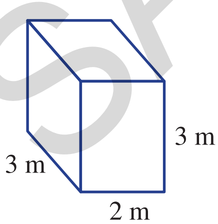
Example 17

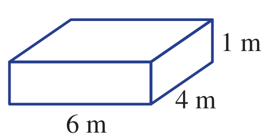
6 Find the capacity of these containers, converting your answer to litres.

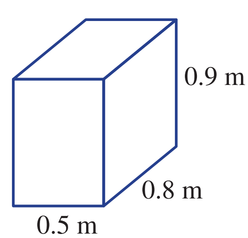
a 

b 

c 

d 

e 

f 

4H

7–9	8–10	9–11

PROBLEM-SOLVING

7 A oil tanker has a capacity of 60 000 m^3.

a What is the ship's capacity in:

i litres?

ii kilolitres?

iii megalitres?

b If the tanker leaks oil at a rate of 300 000 litres per day, how long will it take for all the oil to leak out? Assume the ship started with full capacity.

8 Water is being poured into a fish tank at a rate of 2 L every 10 seconds. The tank is 1.2 m long by 1 m wide by 80 cm high. How long will it take to fill the tank? Give the answer in minutes.

9 A city skyscraper is a rectangular prism 50 m long, 40 m wide and 250 m high.

a What is the total volume in m^3?

b What is the total volume in ML?

10 If 1 kg is the mass of 1 L of water, what is the mass of water in a full container that is a cube with side length 2 m?

11 Using whole numbers only, give all the possible dimensions of rectangular prisms with the following volume. Assume the units are all the same.

a 12 cubic units b 30 cubic units c 47 cubic units

12	12, 13	13, 14

REASONING

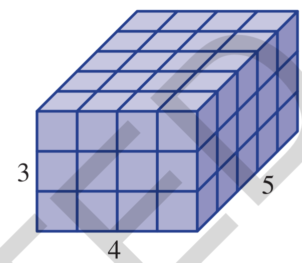
12 Explain why a rectangular prism of volume 46 cm^3 cannot have all its side lengths (length, width and height) as whole numbers greater than 1. Assume all lengths are in centimetres.

13 How many cubic containers, with side lengths that are a whole number of centimetres, have a capacity of less than 1 litre?

4H

REASONING

14 Consider this rectangular prism.



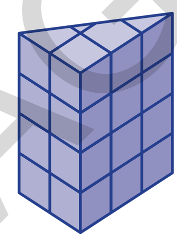
a How many cubes are in the base layer?
b What is the area of the base?
c What do you notice about the two answers from above? How can this be explained?
d If A represents the area of the base, explain why the rule $V = Ah$ can be used to find the volume of a rectangular prism.
e Could any side of a rectangular prism be considered to be the base when using the rule $V = Ah$? Explain.

ENRICHMENT

Halving rectangular prisms — — 15

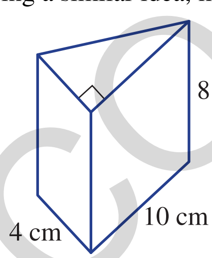
15 This question looks at using half of a rectangular prism to find the volume of a triangular prism.

a Consider this triangular prism.



i Explain why this solid could be thought of as half a rectangular prism.
ii Find its volume.

b a Using a similar idea, find the volume of these prisms.

i


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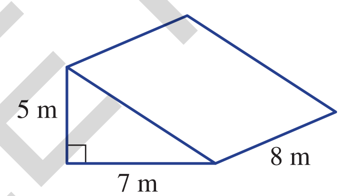
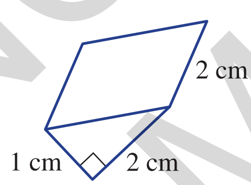


iii


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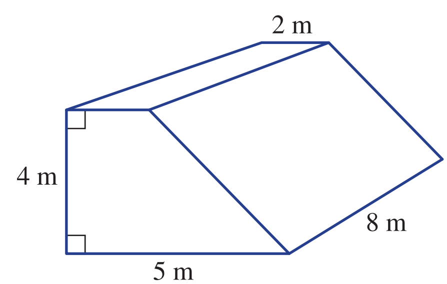

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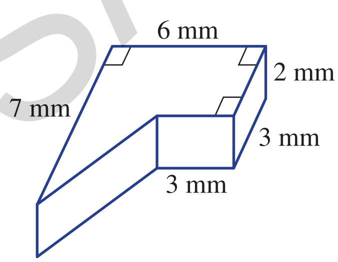

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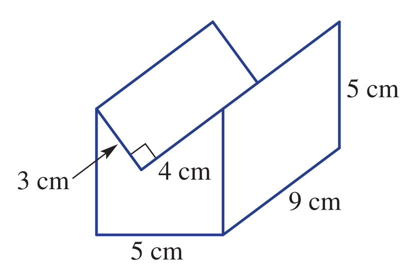
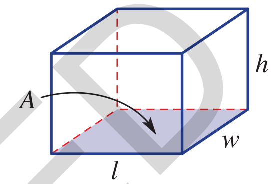


4I Volume of prisms and cylinders

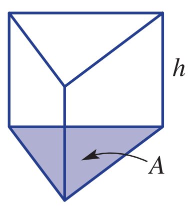
Interactive
Widgets
HOTsheets
Walkthroughs

We know that for a rectangular prism its volume V is given by the rule $V = lwh$. Length × width (lw) gives the number of cubes on the base, but it also tells us the area of the base A. So $V = lwh$ could also be written as $V = Ah$.

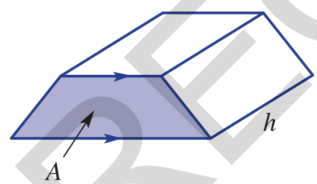


The rule $V = Ah$ can also be applied to prisms that have different shapes as their bases. One condition, however, is that the area of the base must represent the area of the cross-section of the solid. The height h is measured perpendicular to the cross-section. Note that a cylinder is *not* a prism as it does not have sides that are parallelograms; however, it can be treated like a prism when finding its volume because it has a constant cross-section, a circle.

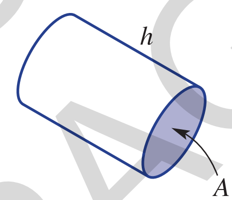
Here are some examples of two prisms and a cylinder with A and h marked.



Cross-section is a triangle



Cross-section is a trapezium



Cross-section is a circle

Let's start: Drawing prisms

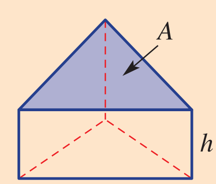
Try to draw prisms (or cylinders) that have the following shapes as their cross-sections.

- Circle
- Trapezium
- Parallelogram
- Triangle
- Pentagon

The cross-section of a prism should be the same size and shape along the entire length of the prism. Check this property on your drawings.

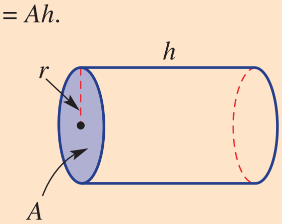
Key ideas

- A **prism** is a polyhedron with a constant (uniform) cross-section.
  - The sides joining the two congruent ends are parallelograms.
  - A right prism has rectangular sides joining the congruent ends.



- Volume of a prism = Area of cross-section × perpendicular height or $V = Ah$.
- Volume of a **cylinder** $= Ah = \pi r^2 \times h = \pi r^2 h$

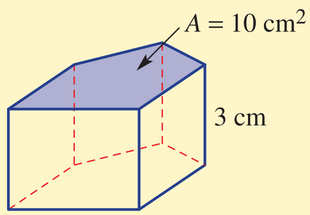
  So $V = \pi r^2 h$



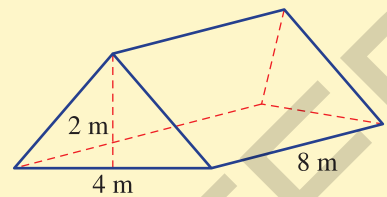
Example 18 Finding the volumes of prisms

Find the volumes of these prisms.

a



b



SOLUTION

a $V = Ah$
$= 10 \times 3$
$= 30$ cm^3

b $V = Ah$
$= \left(\frac{1}{2} \times 4 \times 2\right) \times 8$
$= 32$ m^3

EXPLANATION

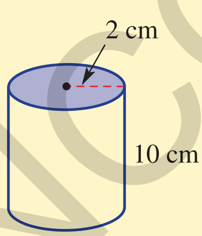
a Write the rule and substitute the given values of A and h, where A is the area of the cross-section.

b The cross-section is a triangle, so use $A = \frac{1}{2}bh$ with base 4 m and height 2 m.

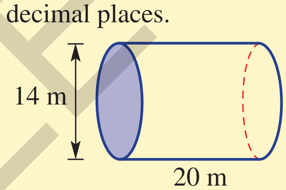
Example 19 Finding the volume of a cylinder

Find the volumes of these cylinders, rounding to two decimal places.

a



b



SOLUTION

a $V = \pi r^2 h$
$= \pi \times 2^2 \times 10$
$= 125.66$ cm^3 (to 2 d.p.)

b $V = \pi r^2 h$
$= \pi \times 7^2 \times 20$
$= 3078.76$ m^3 (to 2 d.p.)

EXPLANATION

a Write the rule and then substitute the given values for π, r and h.
Round as required.

b The diameter is 14 m so the radius is 7 m.
Round as required.

Exercise 4I

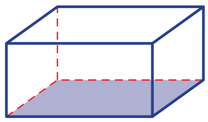
1–3	2	—

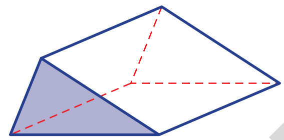
UNDERSTANDING

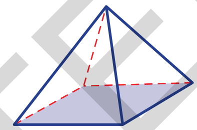
1 For these solids:

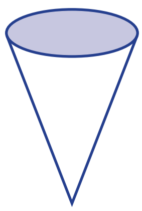
i state whether or not it looks like a prism

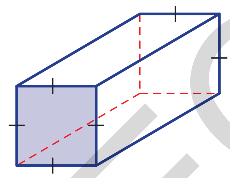
ii if it is a prism, state the shape of its cross-section.

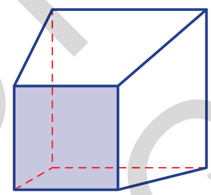
a 

b 

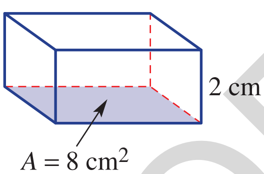
c 

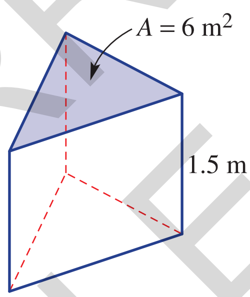
d 

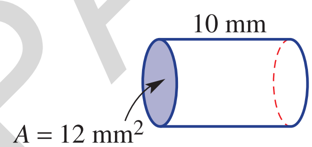
e 

f 

2 For these prisms and cylinder, state the value of A and the value of h that could be used in the rule $V = Ah$ to find the volume of the solid.

a 

b 

c 

3 Evaluate the following.

a $2 \times 5 \times 4$

b $\frac{1}{2} \times 3 \times 4 \times 6$

c $\frac{1}{2}(2 + 3) \times 4$

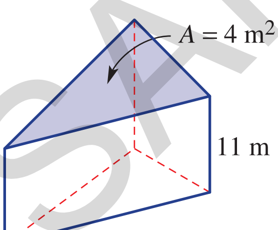
d $3.14 \times 10^2 \times 20$

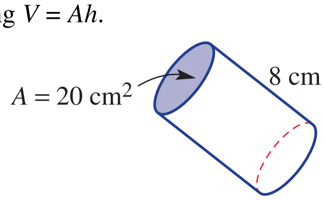
4–6(½)	4–6(½)	4–6(½)

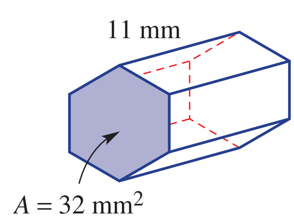
FLUENCY

Example 18a

4 Find the volume of these solids using $V = Ah$.

a 

b 

c 

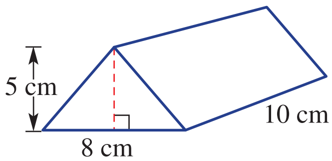
4I

FLUENCY

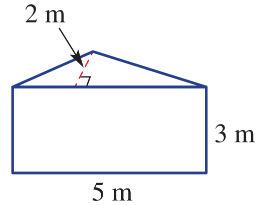
Example 18b

5 Find the volume of these prisms.

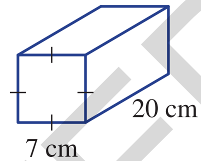
a



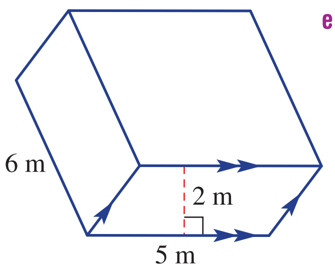
b



c



d



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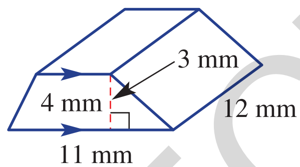

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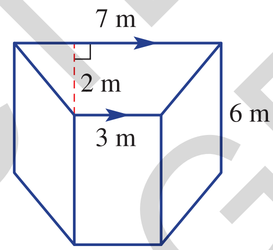
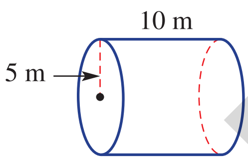


Example 19

6 Find the volume of these cylinders. Round the answer to two decimal places.

a



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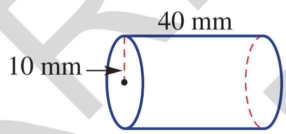

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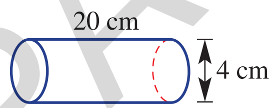
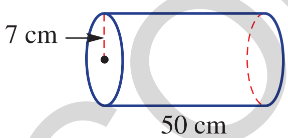


d



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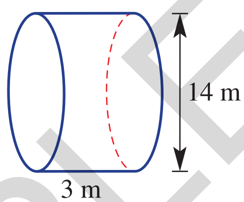

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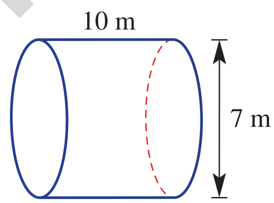
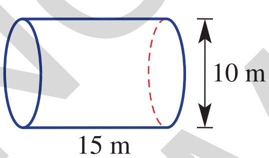
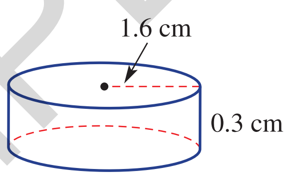


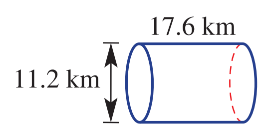
g



h



i



7, 8	8–10	9–11

PROBLEM-SOLVING

7 A cylindrical tank has a diameter of 3 m and height 2 m.

a Find its volume in m^3 correct to three decimal places.

b What is the capacity of the tank in litres?

8 Jack looks at buying either a rectangular water tank with dimensions 3 m by 1 m by 2 m or a cylindrical tank with radius 1 m and height 2 m.

a Which tank has the greater volume?

b What is the difference in the volume correct to the nearest litre?

4I

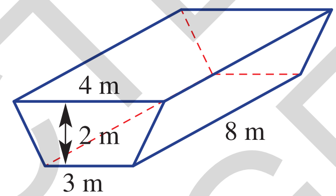
PROBLEM-SOLVING

9 Susan pours water from a full 4 L container into a number of water bottles for a camp hike. Each water bottle is a cylinder with radius 4 cm and height 20 cm. How many bottles can be filled completely?

10 There are 80 liquorice cubes stacked in a cylindrical glass jar. The liquorice cubes have a side length of 2 cm and the glass jar has a radius of 5 cm and a height of 12 cm. How much air space remains in the jar of liquorice cubes? Give the answer correct to two decimal places.

11 A swimming pool is a prism with a cross-section that is a trapezium as shown. The pool is being filled at a rate of 1000 litres per hour.

a Find the capacity of the pool in litres.
b How long will it take to fill the pool?



12	12, 13	13, 14

REASONING

12 Using exact values (e.g. 10π cm^3) calculate the volume of cylinders with these dimensions.

a Radius 2 m and height 5 m
b Radius 10 cm and height 3 cm
c Diameter 8 mm and height 9 mm
d Diameter 7 m and height 20 m

13 A cylinder has a volume of 100 cm^3. Give three different combinations of radius and height measurements that give this volume. Give these lengths correct to two decimal places.

14 A cube has side length x metres and a cylinder has a radius also of x metres and height h. What is the rule linking x and h if the cube and the cylinder have the same volume?

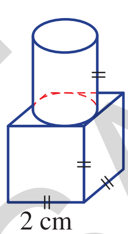
Complex composites

—	—	15

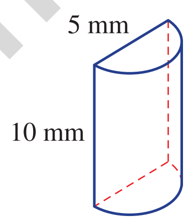
ENRICHMENT

15 Use your knowledge of volumes of prisms and cylinders to find the volume of these composite solids. Round the answer to two decimal places where necessary.

a



b



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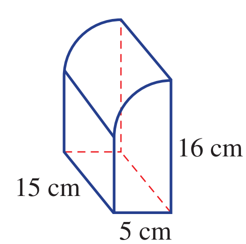

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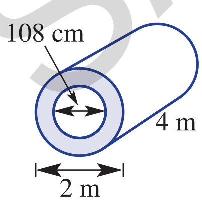

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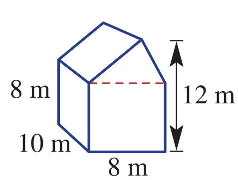

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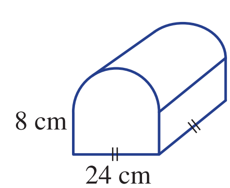


Progress quiz

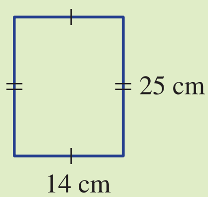
4A **1** Convert:

a 6.4 m into mm **b** 180 cm into m

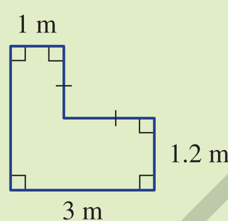
c 97000 cm into km **d** $2\frac{1}{2}$ m into cm

4A **2** Find the perimeter of these shapes.

a



b



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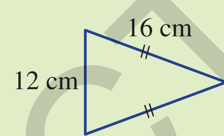
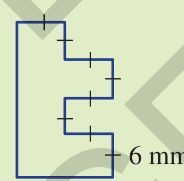
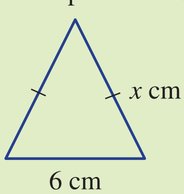


d

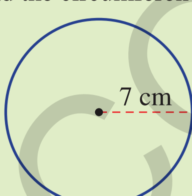


4A **3** If the perimeter of a triangle is 24 cm, find the value of x.

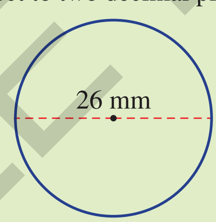


4B/E **4** Find the circumference and area of these circles, correct to two decimal places.

a



b



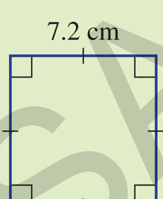
4C **5** Convert these area measurements to the units shown in brackets.

a 4.7 m^2 (cm^2) **b** 4100 mm^2 (cm^2)

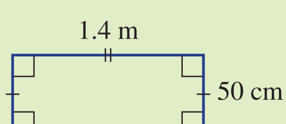
c 5000 m^2 (ha) **d** 0.008 km^2 (m^2)

4C/D **6** Find the area of these shapes.

a



b



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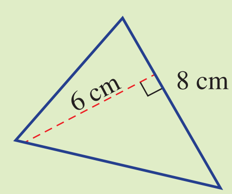

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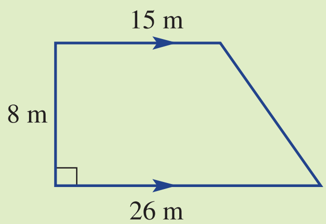

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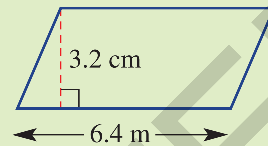

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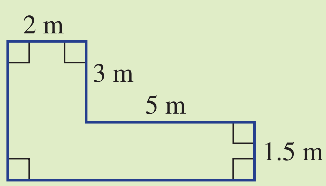
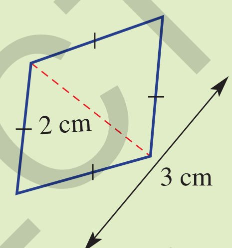
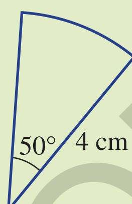


g


4F

7 Find the area and perimeter of these sectors. Round to two decimal places.

Ext

a


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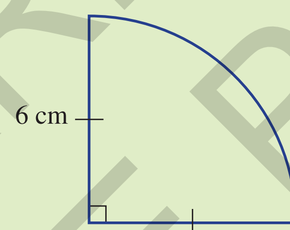

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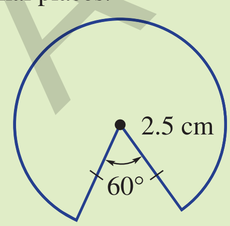
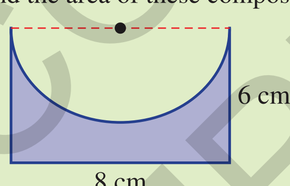


4F

8 Find the area of these composite shapes. Correct to two decimal places.

Ext

a


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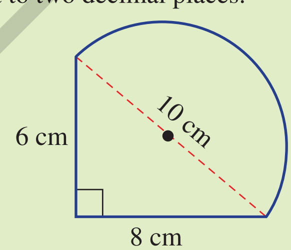
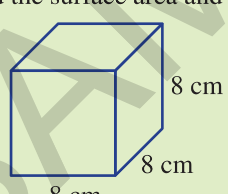


4G/H/I

9 Find the surface area and volume of these prisms.

Ext

a


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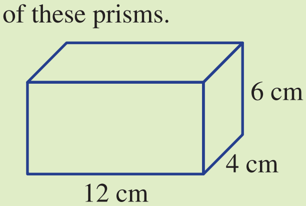

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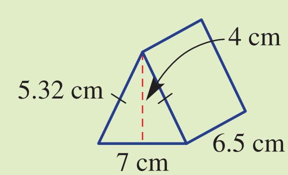
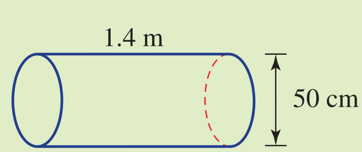


4I

10 For this cylinder, find:

a the volume in m^3 correct to four decimal places

b the capacity in litres to one decimal place.



4J Time

Interactive

Widgets

HOTsheets

Walkthroughs

Time in minutes and seconds is based on the number 60. Other units of time, including the day and year, are defined by the rate at which the Earth spins on its axis and the time that the Earth takes to orbit the Sun.


The Earth takes 1 year to orbit the Sun.

The origin of the units seconds and minutes dates back to the ancient Babylonians, who used a base 60 number system. The 24-hour day dates back to the ancient Egyptians, who described the day as 12 hours of day and 12 hours of night. Today, we use a.m. (*ante meridiem*, which is Latin for 'before noon') and p.m. (*post meridiem*, which is Latin for 'after noon') to represent the hours before and after noon (midday). During the rule of Julius Caesar, the ancient Romans introduced the solar calendar, which recognised that the Earth takes about $365\frac{1}{4}$ days to orbit the Sun. This gave rise to the leap year, which includes one extra day (in February) every 4 years.

Let's start: Knowledge of time

Do you know the answers to these questions about time and the calendar?

- When is the next leap year?
- Why do we have a leap year?
- Which months have 31 days?
- Why are there different times in different countries or parts of a country?
- What do BCE (or BC) and CE (or AD) mean on time scales?

Key ideas

- The standard unit of time is the **second** (s).
- Units of time include:
  - 1 **minute** (min) = 60 seconds (s)
  - 1 **hour** (h) = 60 minutes (min)
  - 1 **day** = 24 hours (h)
  - 1 **week** = 7 days
  - 1 **year** = 12 months

$$\begin{tikzcd} \text{day} \arrow[r, bend left, "\times 24"] & \text{hour} \arrow[l, bend left, "\div 24"] \arrow[r, bend left, "\times 60"] & \text{minute} \arrow[l, bend left, "\div 60"] \arrow[r, bend left, "\times 60"] & \text{second} \arrow[l, bend left, "\div 60"] \end{tikzcd}$$

- Units of time smaller than a second.
  - millisecond = 0.001 second (1000 milliseconds = 1 second)
  - microsecond = 0.000 001 second (1 000 000 microseconds = 1 second)
  - nanosecond = 0.000 000 001 second (1 000 000 000 nanoseconds = 1 second)
- a.m. or p.m. is used to describe the 12 hours before and after noon (midday).
- **24-hour time** shows the number of hours and minutes after midnight.
  - 0330 is 3:30 a.m.
  - 1530 is 3:30 p.m.

Key ideas

- The Earth is divided into 24 time zones (one for each hour).
  - Twenty-four 15° lines of longitude divide the Earth into its time zones. Time zones also depend on a country's borders and its proximity to other countries. (See map on pages 231–232 for details.)
  - Time is based on the time in a place called Greenwich, United Kingdom, and this is called Coordinated Universal Time (UTC) or Greenwich Mean Time (GMT).
  - Places east of Greenwich are ahead in time.
  - Places west of Greenwich are behind in time.
- Australia has three time zones:
  - Eastern Standard Time (EST), which is UTC plus 10 hours.
  - Central Standard Time (CST), which is UTC plus 9.5 hours.
  - Western Standard Time (WST), which is UTC plus 8 hours.

Example 20 Converting units of time

Convert these times to the units shown in brackets.

a 3 days (minutes) **b** 30 months (years)

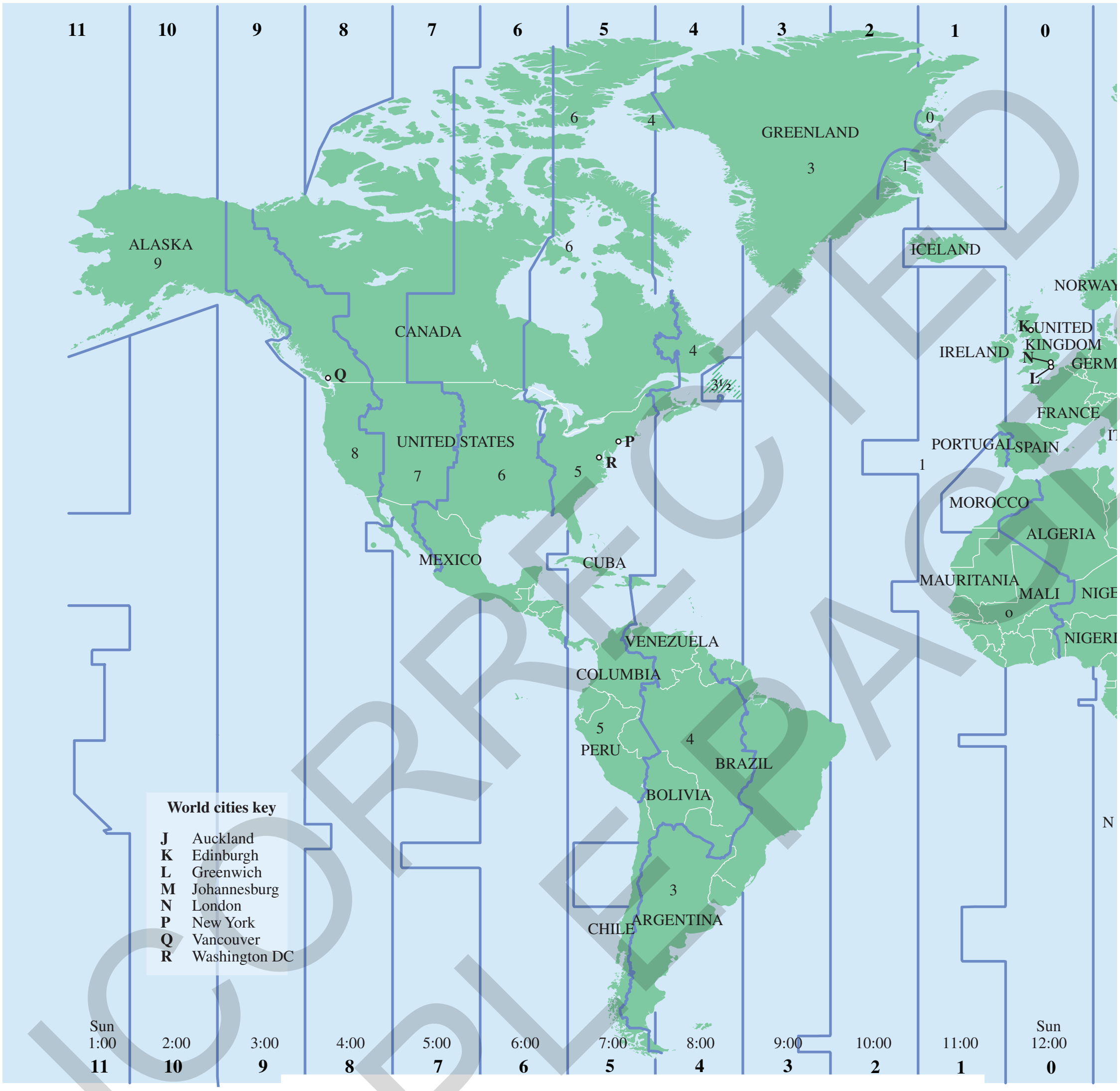
SOLUTION	EXPLANATION
a $3\text{ days} = 3 \times 24\text{ h}$ $= 3 \times 24 \times 60\text{ min}$ $= 4320\text{ min}$	1 day = 24 hours 1 hour = 60 minutes
b $30\text{ months} = 30 \div 12\text{ years}$ $= 2\frac{1}{2}\text{ years}$	There are 12 months in 1 year.

Example 21 Using 24-hour time

Write these times using the system given in brackets.

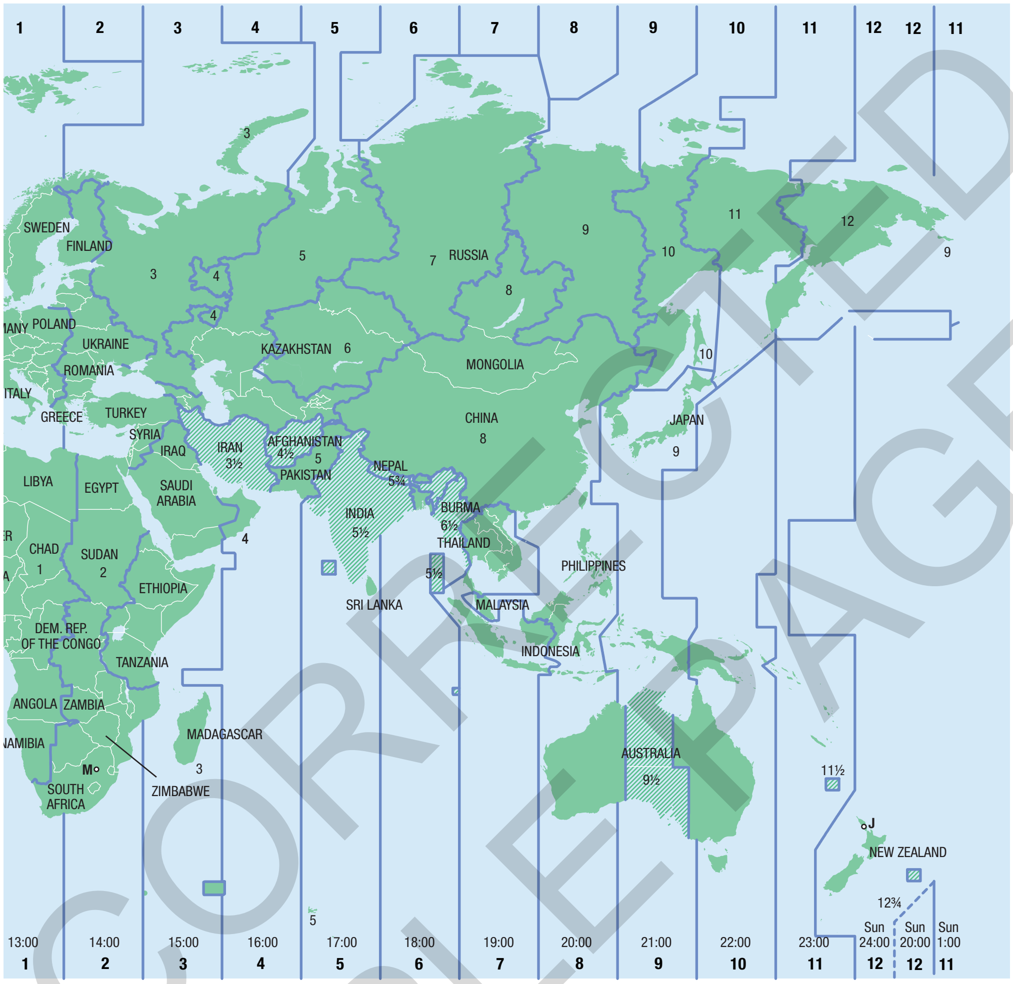
a 4:30 p.m. (24-hour time) **b** 1945 (a.m./p.m.)

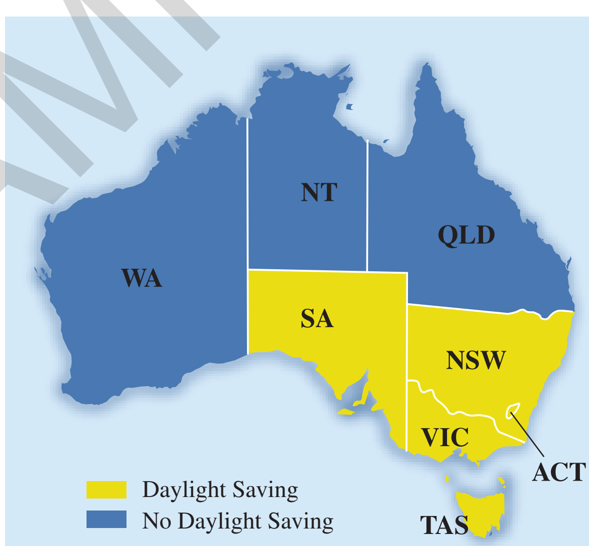
SOLUTION	EXPLANATION
a 4:30 p.m. = 1200 + 0430 = 1630 hours	Since the time is p.m., add 12 hours to 0430 hours.
b 1945 hours = 7:45 p.m.	Since the time is after 1200 hours, subtract 12 hours.



Australian cities key

A Adelaide
B Alice Springs
C Brisbane
D Cairns
E Canberra, ACT
F Darwin
G Hobart
H Melbourne
I Perth


1 2 3 4 5 6 7 8 9 10 11 12 12 11
SWEDEN
FINLAND
POLAND
UKRAINE
ROMANIA
ITALY
GREECE
TURKEY
SYRIA
IRAQ
IRAN
3½
AFGHANISTAN
4½
PAKISTAN
NEPAL
5¾
INDIA
5½
SRI LANKA
KAZAKHSTAN
RUSSIA
MONGOLIA
CHINA
JAPAN
BURMA
6½
THAILAND
PHILIPPINES
MALAYSIA
INDONESIA
AUSTRALIA
9½
11½
NEW ZEALAND
12¾
LIBYA
EGYPT
SAUDI ARABIA
CHAD
SUDAN
ETHIOPIA
DEM. REP. OF THE CONGO
TANZANIA
ANGOLA
ZAMBIA
NAMIBIA
MADAGASCAR
ZIMBABWE
SOUTH AFRICA
13:00 14:00 15:00 16:00 17:00 18:00 19:00 20:00 21:00 22:00 23:00 Sun 24:00 Sun 20:00 Sun 1:00
1 2 3 4 5 6 7 8 9 10 11 12 12 11


WA
NT
QLD
SA
NSW
VIC
ACT
TAS
Daylight Saving
No Daylight Saving

Example 22 Using time zones

Coordinated Universal Time (UTC) and is based on the time in Greenwich, United Kingdom. Use the world time zone map (on pages 231–232) to answer the following.

a When it is 2 p.m. UTC, find the time in these places.

- **i** France
- **ii** China
- **iii** Queensland
- **iv** Alaska

b When it is 9:35 a.m. in New South Wales, Australia, find the time in these places.

- **i** Alice Springs
- **ii** Perth
- **iii** London
- **iv** central Greenland

SOLUTION	EXPLANATION
a **i** 2 p.m. + 1 hour = 3 p.m.	Use the time zone map to see that France is to the east of Greenwich and is in a zone that is 1 hour ahead.
ii 2 p.m. + 8 hours = 10 p.m.	From the time zone map, China is 8 hours ahead of Greenwich.
iii 2 p.m. + 10 hours = 12 a.m.	Queensland uses Eastern Standard Time, which is 10 hours ahead of Greenwich.
iv 2 p.m. – 9 hours = 5 a.m.	Alaska is to the west of Greenwich, in a time zone that is 9 hours behind.
b **i** 9:35 a.m. $-\frac{1}{2}$ hour = 9:05 a.m.	Alice Springs uses Central Standard Time, which is $\frac{1}{2}$ hour behind Eastern Standard Time.
ii 9:35 a.m. – 2 hours = 7:35 a.m.	Perth uses Western Standard Time, which is 2 hours behind Eastern Standard Time.
iii 9:35 a.m. – 10 hours = 11:35 p.m. (the day before)	UTC (time in Greenwich, United Kingdom) is 10 hours behind EST.
iv 9:35 a.m. – 13 hours = 8:35 p.m. (the day before)	Central Greenland is 3 hours behind UTC in Greenwich, so is 13 hours behind EST.

Exercise 4J

1–3	3	—

UNDERSTANDING

1 From options **A** to **F**, match up the time units with the most appropriate description.

- **a** single heartbeat
- **b** 40 hours of work
- **c** duration of a university lecture
- **d** bank term deposit
- **e** 200 m run
- **f** flight from Australia to the UK

- **A** 1 hour
- **B** 1 minute
- **C** 1 day
- **D** 1 week
- **E** 1 year
- **F** 1 second

2 Find the number of:

- **a** seconds in 2 minutes
- **b** minutes in 180 seconds
- **c** hours in 120 minutes
- **d** minutes in 4 hours
- **e** hours in 3 days
- **f** days in 48 hours
- **g** weeks in 35 days
- **h** days in 40 weeks

3 What is the time difference between these times?

- **a** 12 p.m. and 6:30 p.m.
- **b** 12 a.m. and 10:45 a.m.
- **c** 12 a.m. and 4:20 p.m.
- **d** 11 a.m. and 3:30 p.m.

4–10(½)	4–11(½)	4–11(½)

FLUENCY

Example 20

4 Convert these times to the units shown in brackets.

- **a** 3 h (min)
- **b** 10.5 min (s)
- **c** 240 s (min)
- **d** 90 min (h)
- **e** 6 days (h)
- **f** 72 h (days)
- **g** 1 week (h)
- **h** 1 day (min)
- **i** 14 400 s (h)
- **j** 20 160 min (weeks)
- **k** 2 weeks (min)
- **l** 24 h (s)
- **m** 5000 milliseconds (s)
- **n** 2 500 000 microseconds (s)
- **o** 7 000 000 000 nanoseconds (s)
- **p** 0.4 s (milliseconds)
- **q** 0.000 002 7 s (microseconds)
- **r** 0.000 000 003 s (nanoseconds)

5 Write the time for these descriptions.

- **a** 4 hours after 2:30 p.m.
- **b** 10 hours before 7 p.m.
- **c** $3\frac{1}{2}$ hours before 10 p.m.
- **d** $7\frac{1}{2}$ hours after 9 a.m.
- **e** $6\frac{1}{4}$ hours after 11:15 a.m.
- **f** $1\frac{3}{4}$ hours before 1:25 p.m.

Example 21

6 Write these times using the system shown in brackets.

- **a** 1:30 p.m. (24-hour)
- **b** 8:15 p.m. (24-hour)
- **c** 10:23 a.m. (24-hour)
- **d** 11:59 p.m. (24-hour)
- **e** 0630 hours (a.m./p.m.)
- **f** 1300 hours (a.m./p.m.)
- **g** 1429 hours (a.m./p.m.)
- **h** 1938 hours (a.m./p.m.)
- **i** 2351 hours (a.m./p.m.)

4J

FLUENCY

7 Round these times to the nearest hour.
a 1:32 p.m. b 5:28 a.m. c 1219 hours d 1749 hours

8 What is the time difference between these time periods?
a 10:30 a.m. and 1.20 p.m. b 9:10 a.m. and 3:30 p.m. c 2:37 p.m. and 5:21 p.m.
d 10:42 p.m. and 7:32 a.m. e 1451 and 2310 hours f 1940 and 0629 hours

Example 22a

9 Use the time zone map on pages 231–232 to find the time in the following places, when it is 10 a.m. UTC.
a Spain b Turkey c Tasmania d Darwin
e Argentina f Peru g Alaska h Portugal

Example 22b

10 Use the time zone map on pages 231–232 to find the time in these places, when it is 3:30 p.m. in Victoria.
a United Kingdom b Libya c Sweden
d Perth e Japan f central Greenland
g Alice Springs h New Zealand

11 What is the time difference between these pairs of places?
a United Kingdom and Kazakhstan b South Australia and New Zealand
c Queensland and Egypt d Peru and Angola (in Africa)
e Mexico and Germany

12–14 14–17 17–21

PROBLEM-SOLVING

12 A scientist argues that dinosaurs died out 52 million years ago, whereas another says they died out 108 million years ago. What is the difference in their time estimates?

13 Three essays are marked by a teacher. The first takes 4 minutes and 32 seconds to mark, the second takes 7 minutes and 19 seconds, and the third takes 5 minutes and 37 seconds. What is the total time taken to complete marking the essays?

4J

PROBLEM-SOLVING

14 Adrian arrives at school at 8:09 a.m. and leaves at 3:37 p.m. How many hours and minutes is Adrian at school?

15 On a flight to Europe, Janelle spends 8 hours and 36 minutes on a flight from Melbourne to Kuala Lumpur, Malaysia, 2 hours and 20 minutes at the airport at Kuala Lumpur, and then 12 hours and 19 minutes on a flight to Geneva, Switzerland. What is Janelle's total travel time?

16 A phone plan charges 11 cents per 30 seconds. The 11 cents are added to the bill at the beginning of every 30-second block of time.

a What is the cost of a 70-second call?

b What is the cost of a call that lasts 6 minutes and 20 seconds?

17 A doctor earns \$180 000 working 40 weeks per year, 5 days per week, 10 hours per day. What does the doctor earn in each of these time periods?

a per day
b per hour
c per minute
d per second (in cents)

18 A 2 hour football match starts at 2:30 p.m. Eastern Standard Time (EST) in Newcastle, NSW. What time will it be in United Kingdom when the match finishes?

19 If the date is 29 March and it is 3 p.m. in Perth, what is the time and date in these places?

a Italy
b Alaska
c Chile

20 Monty departs on a 20 hour flight from Brisbane to London, United Kingdom, at 5 p.m. on 20 April. Give the time and date of his arrival in London.

21 Elsa departs on an 11 hour flight from Johannesburg, South Africa, to Perth at 6:30 a.m. on 25 October. Give the time and date of her arrival in Perth.

	22	22–23	23–25

REASONING

22 When there are 365 days in a year, how many weeks are there in a year? Round your answer to two decimal places.

23 a To convert from hours to seconds, what single number do you multiply by?
b To convert from days to minutes, what single number do you multiply by?
c To convert from seconds to hours, what single number do you divide by?
d To convert from minutes to days, what single number do you divide by?

24 Assuming there are 365 days in a year and my birthday falls on a Wednesday this year, on what day will my birthday fall in 2 years' time?

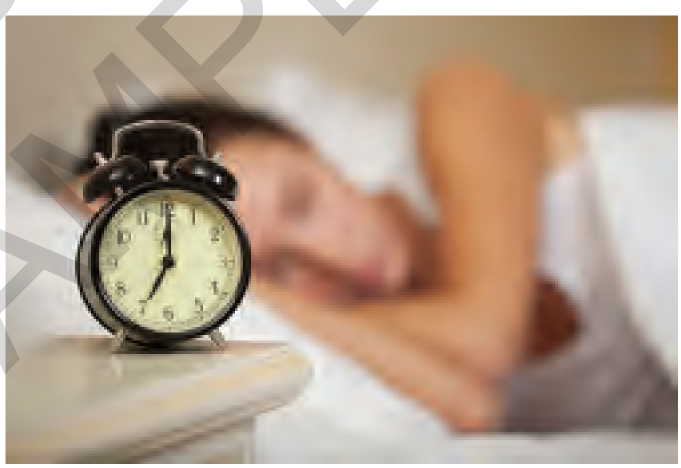
25 a Explain why you gain time when you travel from Australia to Europe.
b Explain why you lose time when you travel from Germany to Australia.
c Explain what happens to the date when you fly from Australia to Canada across the International Date Line.

Daylight saving

	—	—	26

ENRICHMENT

26 Use the internet to investigate how daylight saving affects the time in some places. Write a brief report discussing the following points.
a i Name the States in Australia that use daylight saving.
ii Name five other countries that use daylight saving.
b Describe how daylight saving works, why it is used and what changes have to be made to our clocks.
c Describe how daylight saving in Australia affects the time difference between time zones. Use New South Wales and Greece as an example.



4K Introduction to Pythagoras' theorem EXTENDING

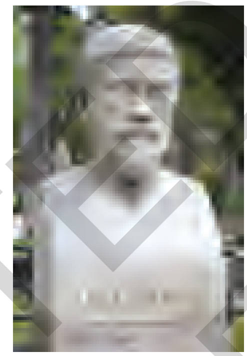
Interactive

Widgets

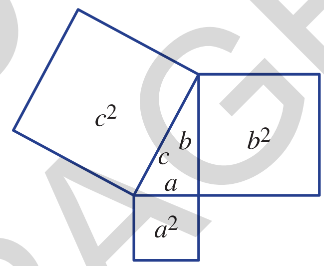
HOTsheets

Walkthroughs

Pythagoras was a philosopher in ancient Greece who lived in the 6th century BCE. He studied astronomy, mathematics, music and religion, but is most well known for the famous Pythagoras' theorem. Pythagoras was known to provide a proof for the theorem that bears his name, and methods to find Pythagorean triples, which are sets of three whole numbers that make up the sides of right-angled triangles.



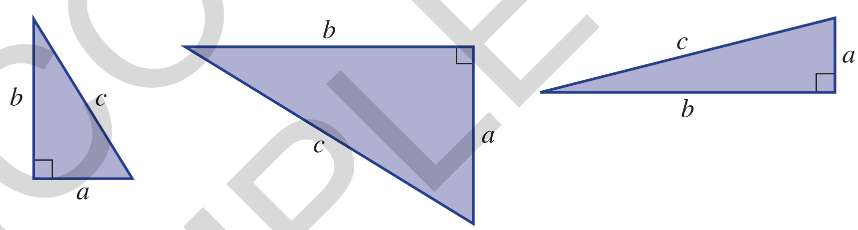
The ancient Babylonians, 1000 years before Pythagoras' time, and the Egyptians also knew that there was a relationship between the sides of a right-angled triangle. Pythagoras, however, was able to clearly explain and prove the theorem using mathematical symbols. The ancient theorem is still one of the most commonly used theorems today.



Pythagoras' theorem states that the square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides. An illustration of the theorem includes squares drawn on the sides of the right-angled triangle. The area of the larger square (c^2) is equal to the sum of the two smaller squares ($a^2 + b^2$).

Let's start: Discovering Pythagoras' theorem

Use a ruler to measure the sides of these right-angled triangles to the nearest mm. Then complete the table.

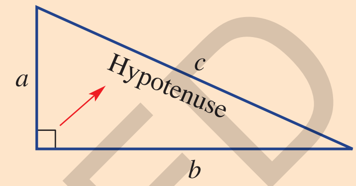


a	b	c	a^2	b^2	c^2

- Can you see any relationship between the numbers in the columns for a^2 and b^2 and the number in the column for c^2?
- Can you write down this relationship as an equation?
- Explain how you might use this relationship to calculate the value of c if it was unknown.
- Research how you can cut the two smaller squares (with areas a^2 and b^2) to fit the pieces into the larger square (with area c^2).

Key ideas

- The **hypotenuse**
  - It is the longest side of a right-angled triangle.
  - It is opposite the right angle.
- **Pythagoras' theorem**
  - The square of the length of the hypotenuse is the sum of the squares of the lengths of the other two shorter sides.
  - $a^2 + b^2 = c^2$ or $c^2 = a^2 + b^2$
- A **Pythagorean triple** (or triad) is a set of three numbers which satisfy Pythagoras' theorem.



Example 23 Checking Pythagorean triples

Decide if the following are Pythagorean triples.

a 6, 8, 10 **b** 4, 5, 9

SOLUTION

a $a^2 + b^2 = 6^2 + 8^2$
$= 36 + 64$
$= 100\ (= 10^2)$
$\therefore$ 6, 8, 10 is a Pythagorean triple.

b $a^2 + b^2 = 4^2 + 5^2$
$= 16 + 25$
$= 41\ (\neq 9^2)$
$\therefore$ 4, 5, 9 is not a Pythagorean triple.

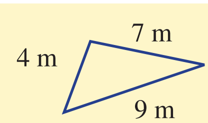
EXPLANATION

a Let $a = 6, b = 8$ and $c = 10$ and check that $a^2 + b^2 = c^2$

b $a^2 + b^2 = 41$ and
$9^2 = 81$ so
$a^2 + b^2 \neq c^2$

Example 24 Deciding if a triangle has a right angle

Decide if this triangle has a right angle.



SOLUTION

$a^2 + b^2 = 4^2 + 7^2$
$= 16 + 49$
$= 65\ (\neq 9^2 = 81)$

EXPLANATION

Check to see if $a^2 + b^2 = c^2$. In this case $a^2 + b^2 = 65$ and $c^2 = 81$ so the triangle is not right angled.

Exercise 4K

1–4	4	—

UNDERSTANDING

1 Calculate these squares and sums of squares.

a 3^2　b 5^2　c 12^2　d 1.5^2

e $2^2 + 4^2$　f $3^2 + 7^2$　g $6^2 + 11^2$　h $12^2 + 15^2$

2 Decide if these equations are true or false.

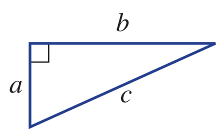
a $2^2 + 3^2 = 4^2$　b $6^2 + 8^2 = 10^2$　c $7^2 + 24^2 = 25^2$

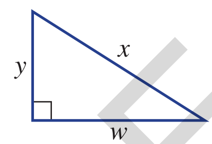
d $5^2 - 3^2 = 4^2$　e $6^2 - 3^2 = 2^2$　f $10^2 - 5^2 = 5^2$

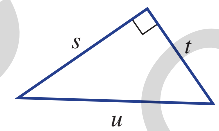
3 Write the missing words in this sentence.

The __________ is the longest side of a right-angled __________.

4 Which letter represents the length of the hypotenuse in these triangles?

a


b


c


5(½), 6, 7(½)	5(½), 6, 7(½)	5(½), 6, 7(½)

FLUENCY

Example 23

5 Decide if the following are Pythagorean triples.

a 3, 4, 6　b 4, 2, 5　c 3, 4, 5

d 9, 12, 15　e 5, 12, 13　f 2, 5, 6

g 9, 40, 41　h 10, 12, 20　i 4, 9, 12

6 Complete this table and answer the questions.

a	b	c	a^2	b^2	$a^2 + b^2$	c^2
3	4	5				
6	8	10				
8	15	17				

a Which two columns give equal results?

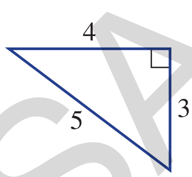
b What would be the value of c^2 if:

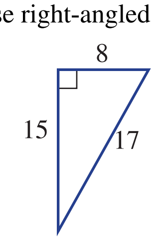
i $a^2 = 4$ and $b^2 = 9$?　ii $a^2 = 7$ and $b^2 = 13$?

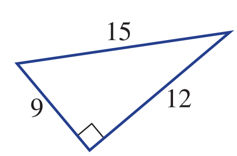
c What would be the value of $a^2 + b^2$ if:

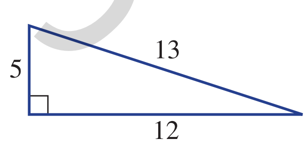
i $c^2 = 25$?　ii $c^2 = 110$?

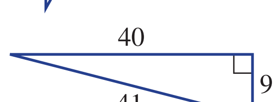
7 Check that $a^2 + b^2 = c^2$ for all these right-angled triangles.

a


b


c


d


e


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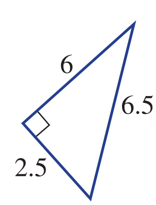
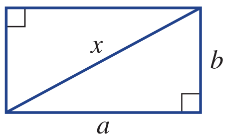


PROBLEM-SOLVING

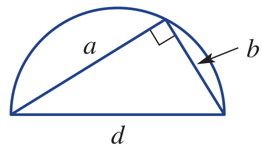
8, 9	8, 9	9, 10

8 Write down an equation using the pronumerals given these diagrams.

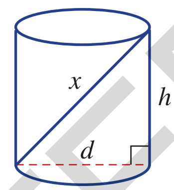
a



b



c



9 A cable connects the top of a 30 m mast to a point on the ground. The cable is 40 m long and connects to a point 20 m from the base of the mast.

a Using $c = 40$, decide if $a^2 + b^2 = c^2$.

b Do you think the triangle formed by the mast and the cable is right angled? Give a reason.

10 (3, 4, 5) and (5, 12, 13) are Pythagorean triples since $3^2 + 4^2 = 5^2$ and $5^2 + 12^2 = 13^2$.

a Find 10 more Pythagorean triples using whole numbers less than 100.

b Find the total number of Pythagorean triples with whole numbers of less than 100.

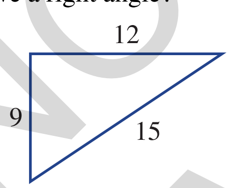
REASONING

11	11, 12	11–13

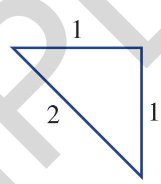
Example 24

11 If $a^2 + b^2 = c^2$, we know that the triangle must have a right angle. Which of these triangles must have a right angle?

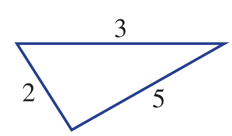
a



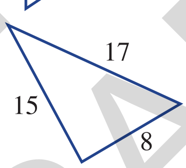
b



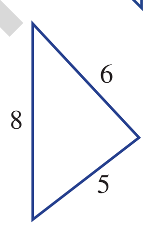
c



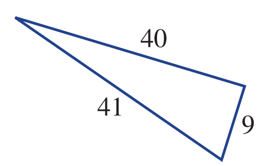
d



e



f



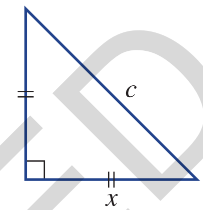
12 If $a^2 + b^2 = c^2$ is true, complete these statements.

a $c^2 - b^2 =$ __ **b** $c^2 - a^2 =$ __ **c** $c =$ _______

4K

REASONING

13 This triangle is isosceles. Write Pythagoras' theorem using the given pronumerals. Simplify if possible.



Pythagoras proof

—	—	14

ENRICHMENT

14 There are many ways to prove Pythagoras' theorem, both algebraically and geometrically.

a Here is an incomplete proof of the theorem that uses this illustrated geometric construction.

Area of inside square $= c^2$

Area of 4 outside triangles $= 4 \times \frac{1}{2} \times \text{base} \times \text{height}$

$= \_\_\_\_\_\_\_\_$

Total area of outside square $= (\_\_\_\_ + \_\_\_\_)^2$

$= a^2 + 2ab + b^2$

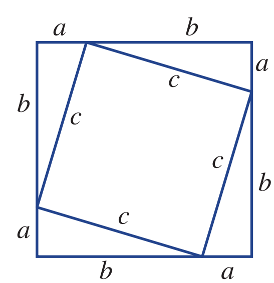
Area of inside square = Area (outside square) − Area of 4 triangles

$= \_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_$

Comparing results from the first and last steps gives

$c^2 = \_\_\_\_\_\_\_\_$

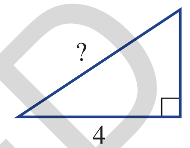


b Use the internet to search for other proofs of Pythagoras' theorem. See if you can explain and illustrate them.

4L Using Pythagoras' theorem EXTENDING

Interactive
Widgets
HOTsheets
Walkthroughs

From our understanding of equations, it may be possible to solve the equation to find an unknown. This is also the case for equations derived from Pythagoras' theorem, where, if two of the side lengths of a right-angled triangle are known, then the third can be found.



So if $c^2 = 3^2 + 4^2$ then $c^2 = 25$ and $c = 5$.

We also notice that if $c^2 = 25$ then $c = \sqrt{25} = 5$ (if $c > 0$).

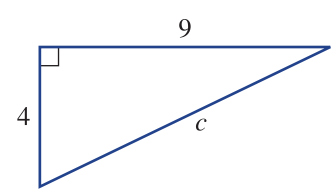
This application of Pythagoras' theorem has wide range of applications wherever right-angled triangles can be drawn.

Note that a number using a $\sqrt{\ }$ sign may not always result in a whole number. For example, $\sqrt{3}$ and $\sqrt{24}$ are not whole numbers and neither can be written as a fraction. These types of numbers are called surds and are a special group of numbers (irrational numbers) that are often approximated using rounded decimals.

The exterior of the Australian centre for the Moving Image is created from many right-angled triangles.

Let's start: Correct layout

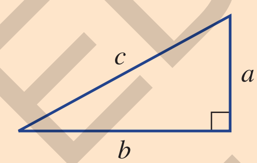
Three students who are trying to find the value of c in this triangle using Pythagoras' theorem write their solutions on a board. There are only very minor differences between each solution and the answer is written rounded to two decimal places. Which student has all the steps written correctly? Give reasons why the other two solutions are not laid out correctly.



Student 1	Student 2	Student 3
$c^2 = a^2 + b^2$ $= 4^2 + 9^2$ $= 97$ $= \sqrt{97}$ $= 9.85$	$c^2 = a^2 + b^2$ $= 4^2 + 9^2$ $= 97$ $\therefore c = \sqrt{97}$ $= 9.85$	$c = a^2 + b^2$ $= 4^2 + 9^2$ $= 97$ $= \sqrt{97}$ $= 9.85$

Key ideas

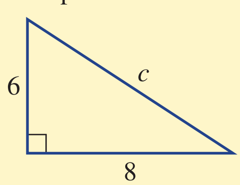
- **Surds** are numbers that have a $\sqrt{\ }$ sign when written in simplest form.
  - They are not a whole number and cannot be written as a fraction.
  - Written as a decimal, the decimal places would continue forever with no repeated pattern (just like the number pi). Surds are therefore classified as irrational numbers.
  - $\sqrt{2}, \sqrt{5}, 2\sqrt{3}$ and $\sqrt{90}$ are all examples of surds.
- Using Pythagoras' theorem.
  If $c^2 = a^2 + b^2$ then $c = \sqrt{a^2 + b^2}$.
- Note
  - $\sqrt{a^2 + b^2} \neq a + b$, for example, $\sqrt{3^2 + 4^2} \neq 3 + 4$
  - If $c^2 = k$ then $c = \sqrt{k}$ if $c \geq 0$.



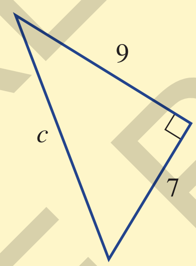
Example 25 Finding the length of the hypotenuse

Find the length of the hypotenuse for these right-angled triangles. Round the answer for part **b** to two decimal places.

a



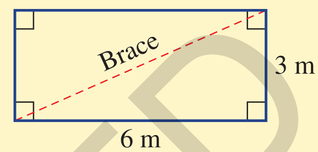
b



SOLUTION	EXPLANATION
a $c^2 = a^2 + b^2$ $= 6^2 + 8^2$ $= 100$	Write the equation for Pythagoras' theorem and substitute the values for the shorter sides.
$\therefore c = \sqrt{100}$ $= 10$	Find c by taking the square root.
b $c^2 = a^2 + b^2$ $= 7^2 + 9^2$ $= 130$	First calculate the value of $7^2 + 9^2$.
$\therefore c = \sqrt{130}$ $= 11.40$ (to 2 d.p.)	$\sqrt{130}$ is a surd, so round the answer as required.

Example 26 Applying Pythagoras' theorem

A rectangular wall is to be strengthened by a diagonal brace. The wall is 6 m wide and 3 m high. Find the length of brace required correct to the nearest cm.



SOLUTION

$c^2 = a^2 + b^2$

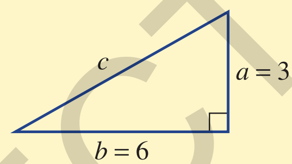
$= 3^2 + 6^2$

$= 45$

$\therefore c = \sqrt{45}$

$= 6.71$ m or 671 cm (to nearest cm)

EXPLANATION



Exercise 4L

1–3	3	—

UNDERSTANDING

1 Decide if these numbers written with a $\sqrt{\ }$ sign simplify to a whole number. Answer Yes or No.

a $\sqrt{9}$ b $\sqrt{11}$ c $\sqrt{20}$ d $\sqrt{121}$

2 Round these surds correct to two decimal places using a calculator.

a $\sqrt{10}$ b $\sqrt{26}$ c $\sqrt{65}$ d $\sqrt{230}$

3 Copy and complete this working out.

a $c^2 = a^2 + b^2$
$= 5^2 + 12^2$
$= \_\_\_\_$
$\therefore c = \sqrt{\_\_\_\_}$
$= \_\_\_\_$

b $c^2 = \_\_\_\_$
$= 9^2 + 40^2$
$= \_\_\_\_$
$\therefore c = \sqrt{\_\_\_\_}$
$= \_\_\_\_$

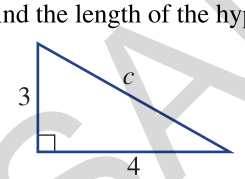
c $\_\_\_\_ = \_\_\_\_$
$= 9^2 + 12^2$
$= \_\_\_\_$
$\therefore c = \sqrt{\_\_\_\_}$
$= \_\_\_\_$

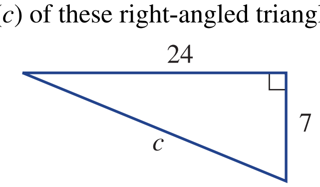
4–5(½)	4–5(½)	4–5(½)

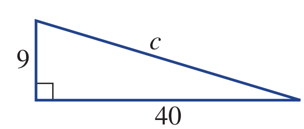
FLUENCY

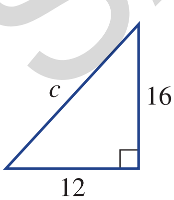
Example 25a

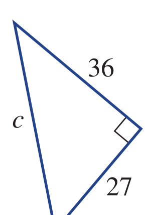
4 Find the length of the hypotenuse (c) of these right-angled triangles.

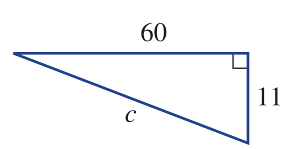
a


b


c


d


e


f


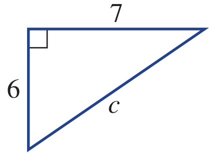
4L

Example 25b

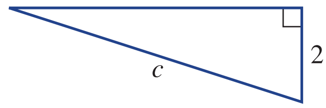
5 Find the length of the hypotenuse (c) of these right-angled triangles correct to two decimal places.

FLUENCY

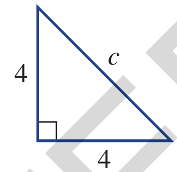
a



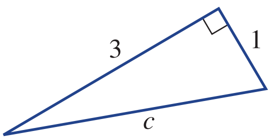
b



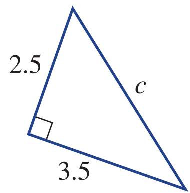
c



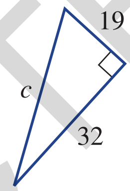
d



e



f

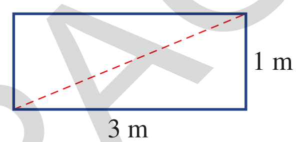


6, 7	7–9	7–9

PROBLEM-SOLVING

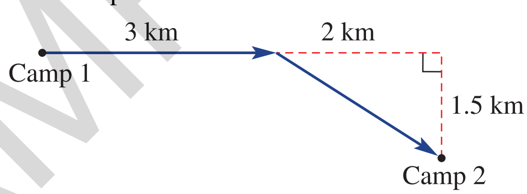
Example 26

6 A rectangular board is to be cut along one of its diagonals. The board is 1 m wide and 3 m high. What will be the length of the cut, correct to the nearest cm?

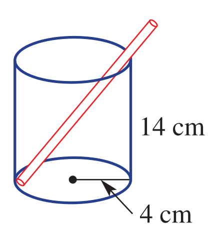


7 The size of a television screen is determined by its diagonal length. Find the size of a television screen that is 1.2 m wide and 70 cm high. Round the answer to the nearest cm.

8 Here is a diagram showing the path of a bushwalker from Camp 1 to Camp 2. Find the total distance calculated to one decimal place.



9 A 20 cm straw sits in a cylindrical glass as shown. What length of straw sticks above the top of the glass? Round the answer to two decimal places.



REASONING

10	10	10, 11

10 Explain the error in each set of working.

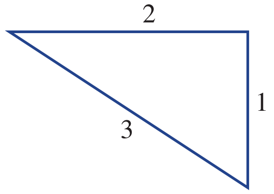
a $c^2 = 2^2 + 3^2$
$\therefore c = 2 + 3$
$= 5$

b $c^2 = 3^2 + 4^2$
$= 7^2$
$= 49$
$\therefore c = 7$

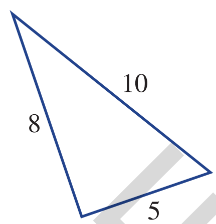
c $c^2 = 2^2 + 5^2$
$= 4 + 25$
$= 29$
$= \sqrt{29}$

11 Prove that these are not right-angled triangles.

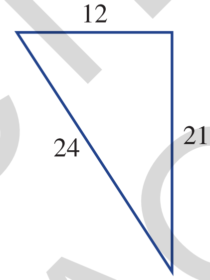
a



b



c



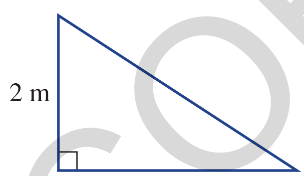
Perimeter and Pythagoras

ENRICHMENT

—	—	12

12 Find the perimeter of these shapes correct to two decimal places.

a



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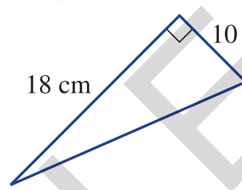

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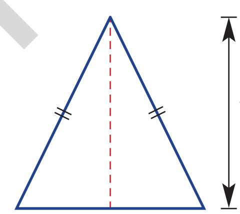

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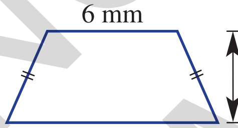
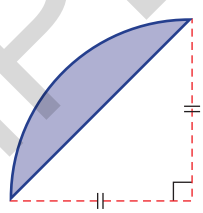


e



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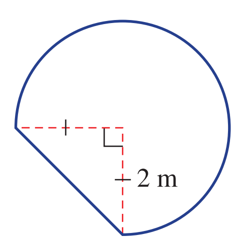


4M Finding the length of a shorter side EXTENDING

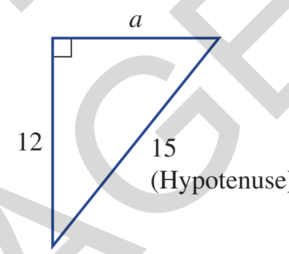
Interactive
Widgets
HOTsheets
Walkthroughs

We know that if we are given the two shorter sides of a right-angled triangle we can use Pythagoras' theorem to find the length of the hypotenuse. Generalising further, we can say that if given *any* two sides of a right-angled triangle we can use Pythagoras' theorem to find the length of the third side.

Let's start: What's the setting out?

The triangle shown has a hypotenuse length of 15 and one of the shorter sides is of length 12. Here is the setting out to find the length of the unknown side a.

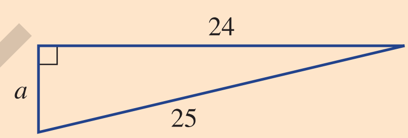
Fill in the missing gaps and explain what is happening at each step.



$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + \underline{\quad}^2 &= \underline{\quad}^2 \\ a^2 + \underline{\quad} &= \underline{\quad} \\ a^2 &= \underline{\quad} \text{ (Subtract } \underline{\quad} \text{ from both sides)} \\ \therefore a &= \sqrt{\underline{\quad}} \\ &= \underline{\quad} \end{aligned}$$

Key ideas

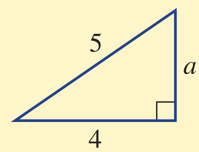
- Pythagoras' theorem can be used to find the length of the shorter sides of a right-angled triangle if the length of the hypotenuse and another side are known.
- Use subtraction to make the unknown the subject of the equation.
  For example:



$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 24^2 &= 25^2 \\ a^2 + 576 &= 625 \\ a^2 &= 49 \text{ (Subtract 576 from both sides.)} \\ \therefore a &= \sqrt{49} \\ &= 7 \end{aligned}$$

Example 27 Finding the length of a shorter

Find the value of a in this right-angled triangle.



SOLUTION

$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 4^2 &= 5^2 \\ a^2 + 16 &= 25 \\ a^2 &= 9 \\ \therefore a &= \sqrt{9} \\ &= 3 \end{aligned}$$

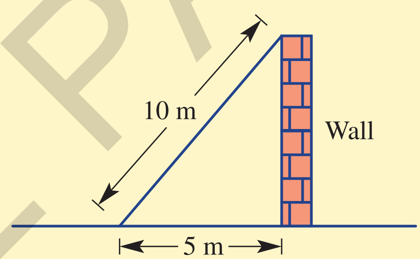
EXPLANATION

Write the equation using Pythagoras' theorem and substitute the known values.

Subtract 16 from both sides.

Example 28 Applying Pythagoras to find a shorter side

A 10 m steel brace holds up a concrete wall. The bottom of the brace is 5 m from the base of the wall. Find the height of the concrete wall correct to two decimal places.



SOLUTION

Let a metres be the height of the wall.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 5^2 &= 10^2 \\ a^2 + 25 &= 100 \\ a^2 &= 75 \\ \therefore a &= \sqrt{75} \\ &= 8.66 \text{ (to 2 d.p.)} \end{aligned}$$

The height of the wall is 8.66 metres.

EXPLANATION

Choose a letter (pronumeral) for the unknown height.

Substitute into Pythagoras' theorem.

Subtract 25 from both sides.

$\sqrt{75}$ is the exact answer.

Round as required.

Answer a worded problem using a full sentence.

Exercise 4M

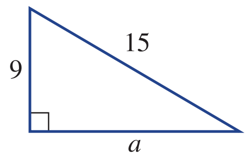
1(½), 2	2	—

UNDERSTANDING

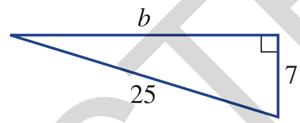
1 Find the value of a in these equations. (Assume a is a positive number.)

a $a^2 = 16$ b $a^2 = 49$ c $a^2 + 16 = 25$
d $a^2 + 9 = 25$ e $a^2 + 36 = 100$ f $a^2 + 441 = 841$
g $10 + a^2 = 19$ h $6 + a^2 = 31$ i $25 + a^2 = 650$

2 Copy and complete the missing steps.

a


$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 9^2 &= \_\_\_\_ \\ a^2 + \_\_\_\_ &= 225 \\ a^2 &= \_\_\_\_ \\ \therefore a &= \sqrt{\_\_\_\_} \\ &= \_\_\_\_ \end{aligned}$$

b


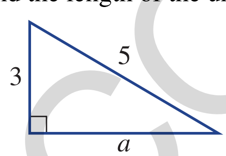
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 7^2 + b^2 &= \_\_\_\_ \\ \_\_\_\_ + b^2 &= \_\_\_\_ \\ b^2 &= 576 \\ \therefore b &= \sqrt{\_\_\_\_} \\ &= \_\_\_\_ \end{aligned}$$

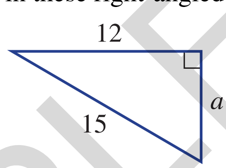
3–4(½)	3–4(½)	3–4(½)

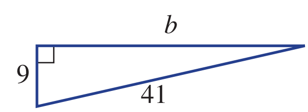
FLUENCY

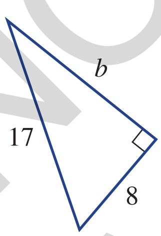
Example 27

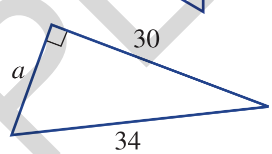
3 Find the length of the unknown side in these right-angled triangles.

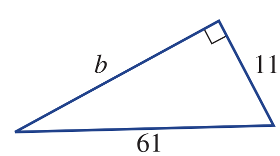
a


b


c


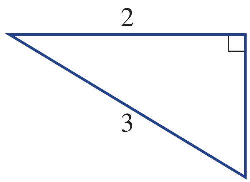
d


e


f


FLUENCY

4 Find the length of the unknown side in these right-angled triangles, giving the answer correct to two decimal places.

a


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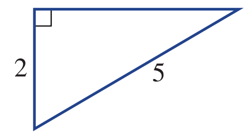
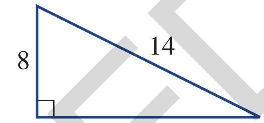
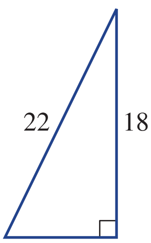
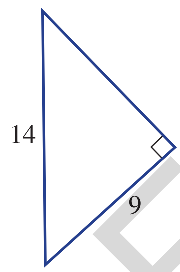
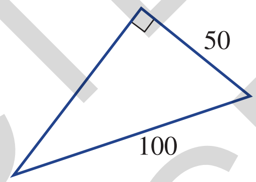


c


d


e


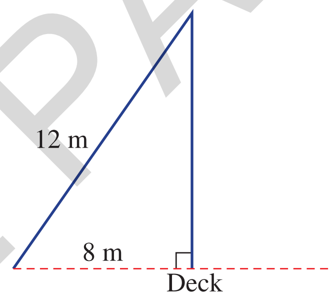
f


5, 6	5–7	6–8

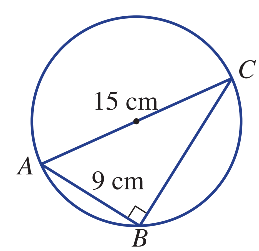
PROBLEM-SOLVING

Example 28

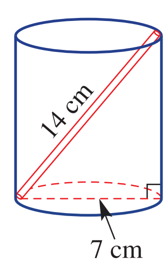
5 A yacht's mast is supported by a 12 m cable attached to its top. On the deck of the yacht, the cable is 8 m from the base of the mast. How tall is the mast? Round the answer to two decimal places.



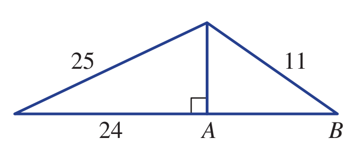
6 A circle's diameter AC is 15 cm and the chord AB is 9 cm. Angle ABC is 90°. Find the length of the chord BC.



7 A 14 cm drinking straw just fits into a can as shown. The diameter of the can is 7 cm. Find the height of the can correct to two decimal places.



8 Find the length AB is this diagram. Round to two decimal places.



4M

9	9, 10	10, 11

REASONING

9 Describe what is wrong with the second line of working in each step.

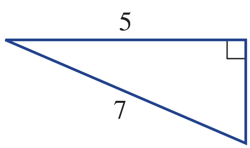
a $a^2 + 10 = 24$
$a^2 = 34$

b $a^2 = 25$
$= 5$

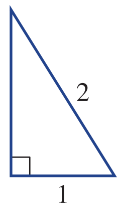
c $a^2 + 25 = 36$
$a + 5 = 6$

10 The number $\sqrt{11}$ is an example of a surd that is written as an exact value. Find the surd that describes the exact lengths of the unknown sides of these triangles.

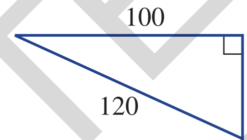
a



b

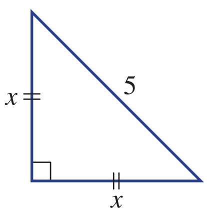


c



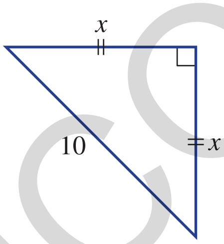
11 Show how Pythagoras' theorem can be used to find the unknown length in these isosceles triangles. Complete the solution for part **a** and then try the others. Round to two decimal places.

a

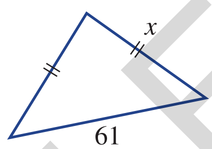


$a^2 + b^2 = c^2$
$x^2 + x^2 = 5^2$
$2x^2 = 25$
$x^2 = \_\_\_\_$
$\therefore x = \sqrt{\_\_\_\_}$

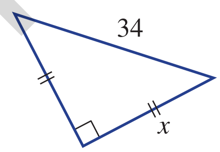
b



c



d



Pythagorean families

—	—	12

ENRICHMENT

12 (3, 4, 5) is called a Pythagorean triple because the numbers 3, 4 and 5 satisfy Pythagoras' theorem ($3^2 + 4^2 = 5^2$).

a Explain why (6, 8, 10) is also a Pythagorean triple.
b Explain why (6, 8, 10) is considered to be in the same family as (3, 4, 5).
c List 3 other Pythagorean triples in the same family as (3, 4, 5) and (6, 8, 10).
d Find another triple not in the same family as (3, 4, 5), but has all 3 numbers less than 20.
e List 5 triples that are each the smallest triple of 5 different families.

Investigation

GMT and travel

As discussed in Section 4J, the world is divided into 24 time zones, which are determined loosely by each 15° meridian of longitude. World time is based on the time at a place called Greenwich near London, United Kingdom. This time is called Coordinated Universal Time (UTC) or Greenwich Mean Time (GMT). Places east of Greenwich are ahead in time and places west of Greenwich are behind.

In Australia, the Western Standard Time is 2 hours behind Eastern Standard Time and Central Standard Time is $\frac{1}{2}$ hour behind Eastern Standard Time. Use the world time zone map on pages 232–233 to answer these questions and to investigate how the time zones affect the time when we travel.

East and west

1 Name five countries that are:
 a ahead of GMT
 b behind GMT

Noon in Greenwich

2 When it is noon in Greenwich, what is the time in these places?
 a Sydney
 b Perth
 c Darwin
 d Washington, DC
 e Auckland
 f France
 g Johannesburg
 h Japan

2 p.m. EST

3 When it is 2 p.m. Eastern Standard Time (EST) on Wednesday, find the time and day in these places.
 a Perth
 b Adelaide
 c London
 d western Canada
 e China
 f United Kingdom
 g Alaska
 h South America

Adjusting your watch

4 Do you adjust your watch forwards or backwards when you are travelling to these places?
 a India
 b New Zealand

5 In what direction should you adjust your watch if you are flying over the Pacific Ocean?

Flight travel

6 You fly from Perth to Brisbane on a 4 hour flight that departed at noon. What is the time in Brisbane when you arrive?

7 You fly from Melbourne to Edinburgh on a 22 hour flight that departed at 6 a.m. What is the time in Edinburgh when you arrive?

8 You fly from Sydney to Los Angeles on a 13 hour flight that departed at 7:30 p.m. What is the time in Los Angeles when you arrive?

9 Copy and complete the following table.

Departing	Arriving	Departure time	Flight time (hours)	Arrival time
Brisbane	Broome	7 a.m.	3.5	
Melbourne	London	1 p.m.	23	
Hobart	Adelaide		1.5	4 p.m.
London	Tokyo		12	11 p.m.
New York	Sydney		15	3 a.m.
Beijing	Vancouver	3:45 p.m.		7:15 p.m.

10 Investigate how daylight saving alters the time in some time zones and why. How does this affect flight travel? Give examples.

Pythagorean triples and spreadsheets

Pythagorean triples (or triads) can be grouped into families. The triad (3, 4, 5) is the base triad for the family of triads ($3k$, $4k$, $5k$). Here are some triads in this same family.

k	1	2	3
Triad	(3, 4, 5) (base triad)	(6, 8, 10)	(9, 12, 15)

1 Write down three more triads in the family ($3k, 4k, 5k$).

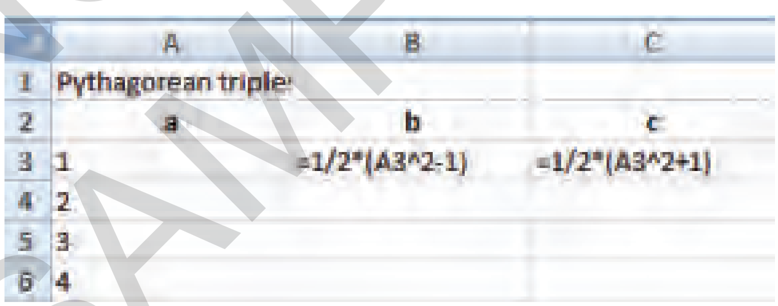
2 Write down three triads in the family ($7k, 24k, 25k$).

3 If ($3k, 4k, 5k$) and ($7k, 24k, 25k$) are two triad families, can you find three more families that have whole numbers less than 100.

4 Pythagoras discovered that if the smaller number in a base triad is a then the other two numbers in the triad are given by the rules:

$$\frac{1}{2}(a^2 + 1) \quad \text{and} \quad \frac{1}{2}(a^2 - 1)$$

Set up a spreadsheet to search for all the families of triads of whole numbers less than 200. Here is how a spreadsheet might be set up.

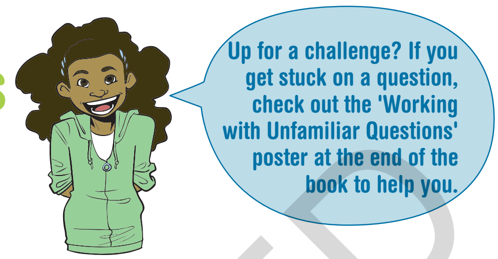


	A	B	C
1	Pythagorean triple:		
2	a	b	c
3	1	=1/2*(A3^2-1)	=1/2*(A3^2+1)
4	2		
5	3		
6	4		

Fill down far enough so that c is a maximum of 200.

5 List all the base triads using whole numbers less than 200. How many are there?

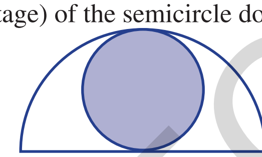
Problems and challenges



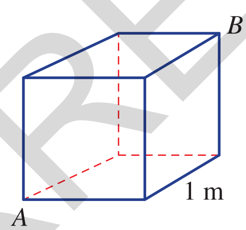
1 A cube has capacity 1 L. What are its dimensions in cm correct to one decimal place?

2 A fish tank is 60 cm long, 30 cm wide, 40 cm high and contains 70 L of water. Rocks with a volume of 3000 cm^3 are placed into the tank. Will the tank overflow?

3 What proportion (fraction or percentage) of the semicircle does the full circle occupy?

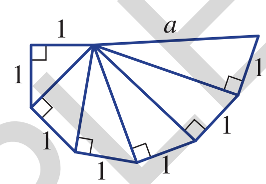


4 What is the distance AB in this cube? (Pythagoras' theorem is required.)

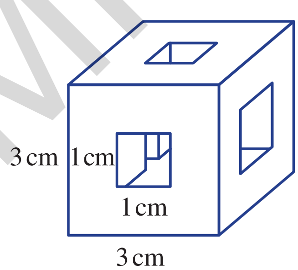


5 By what number do you multiply the radius of a circle to double its area?

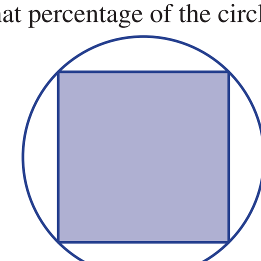
6 Find the exact value (as a surd) of a in this diagram. (Pythagoras' theorem is required.)



7 A cube of side length 3 cm has its core removed in all directions as shown. Find its total surface area both inside and out.



8 A square just fits inside a circle. What percentage of the circle is occupied by the square?



Chapter summary

Measurement

Length

Perimeter

4 cm, 10 cm

$P = 2 \times 10 + 2 \times 4$
$= 28$ cm

Units

1 km = 1000 m
1 m = 100 cm
1 cm = 10 mm

Circumference

$C = 2\pi r$ or πd
$= 2 \times \pi \times 3$
$= 6\pi$ m²

3 m

Area

Triangle

3 cm, 6 cm

$A = \frac{1}{2}bh$
$= \frac{1}{2} \times 6 \times 3$
$= 9$ cm²

Units

$\times 1000^2$ $\times 100^2$ $\times 10^2$
km² m² cm² mm²
$\div 1000^2$ $\div 100^2$ $\div 10^2$

1 ha = 10 000 m²

Quadrilaterals

Square $A = l^2$
Rectangle $A = lw$
Parallelogram $A = bh$
Rhombus $A = \frac{1}{2}xy$
Kite $A = \frac{1}{2}xy$
Trapezium $A = \frac{1}{2}(a + b)h$

Circle

$A = \pi r^2$
$= \pi \times 7^2$
$= 49\pi$ cm²

7 cm

Sectors (Ext)

$A = \frac{\theta}{360} \times \pi r^2$
$= \frac{280}{360} \times \pi \times 2^2$
$= 9.77$ m²

2 m, 280°

Surface area (Ext)

Net

3 m, 1 m, 2 m

SA = 2×(1×2)+2×(1×3)+2×(2×3)
= 4 + 6 + 12
= 22 m²

Time

1 min = 60 s
1 h = 60 min
0311 is 03:11 a.m.
2049 is 08:49 p.m.

Volume

Units

km³, m³, cm³, mm³

$\times 1000$ $\times 1000$ $\times 1000$
ML kL L mL
$\div 1000$ $\div 1000$ $\div 1000$

1 mL = 1 cm³
1 m³ = 1000 L

Rectangular prism

$V = lwh$
$= 10 \times 20 \times 30$
$= 6000$ cm³
$= 6$ L

30 cm, 20 cm, 10 cm

Prism and cylinders

$V = Ah$
$= \frac{1}{2} \times 3 \times 1 \times 2$
$= 3$ m³

2 m, 3 m, 1 m

$V = \pi r^2 h$
$= \pi \times 2^2 \times 6$
$= 75.40$ cm³

2 cm, 6 cm

Pythagoras' theorem (Ext)

Theorem

a, c, b

$a^2 + b^2 = c^2$

Finding c

$c^2 = 5^2 + 7^2$
$= 74$
$\therefore c = \sqrt{74}$

7, 5, c

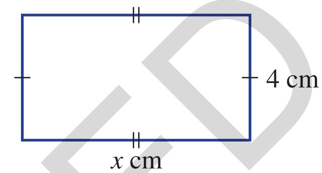
Finding a shorter side

$a^2 + 1^2 = 2^2$
$a^2 + 1 = 4$
$a^2 = 3$
$a = \sqrt{3}$

a, 1, 2

Multiple-choice questions

4A **1** The perimeter of this rectangle is 20 cm. The unknown value x is:

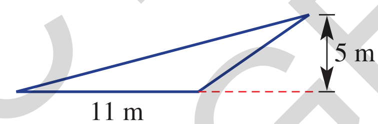


A 4 **B** 16 **C** 5
D 10 **E** 6

4B **2** A wheel has a diameter of 2 m. Its circumference and area (in that order) are given by:

A π, π^2 **B** $2\pi, \pi$ **C** $4\pi, 4\pi$
D 2, 1 **E** 4, 4

4C **3** The area of this triangle is:

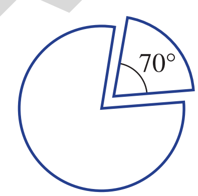


A 27.5 m^2 **B** 55 m **C** 55 m^2
D 110 m **E** 16 m^2

4E **4** Using $\pi = 3.14$, the area of a circular oil slick with radius 100 m is:

A 7850 m^2 **B** 314 m^2 **C** 31 400 m^2 **D** 78.5 m^2 **E** 628 m^2

4F Ext **5** A sector of a circle is removed as shown. The fraction of the circle remaining is:



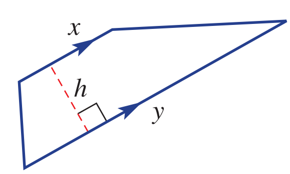
A 290 **B** $\frac{29}{36}$ **C** $\frac{7}{36}$

D $\frac{7}{180}$ **E** $\frac{3}{4}$

4G Ext **6** A cube has a surface area of 24 cm^2. The length of its sides is:

A 12 cm **B** 8 cm **C** 6 cm
D 4 cm **E** 2 cm

4D Ext **7** The rule for the area of the trapezium shown is:



A $\frac{1}{2}xh$ **B** $\frac{1}{2}(x+y)$ **C** $\frac{1}{2}xy$

D πxy^2 **E** $\frac{1}{2}(x+y)h$

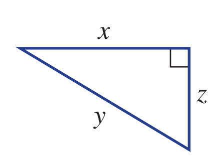
4H **8** The volume of a rectangular prism is 48 cm^3. If its width is 4 cm and height 3 cm, its length would be:

A 3 cm **B** 4 cm **C** 2 cm **D** 12 cm **E** 96 cm

4I **9** A cylinder has radius 7 cm and height 10 cm. Using $\pi = \frac{22}{7}$, its volume would be:

A 1540 cm^2 **B** 440 cm^3 **C** 440 L **D** 1540 cm^3 **E** 220 cm^3

4K **10** The rule for Pythagoras' theorem for this triangle would be:



A $a^2 - b^2 = c^2$ **B** $x^2 + y^2 = z^2$ **C** $z^2 + y^2 = x^2$
D $x^2 + z^2 = y^2$ **E** $y = \sqrt{x^2 - z^2}$

Chapter review

Short-answer questions

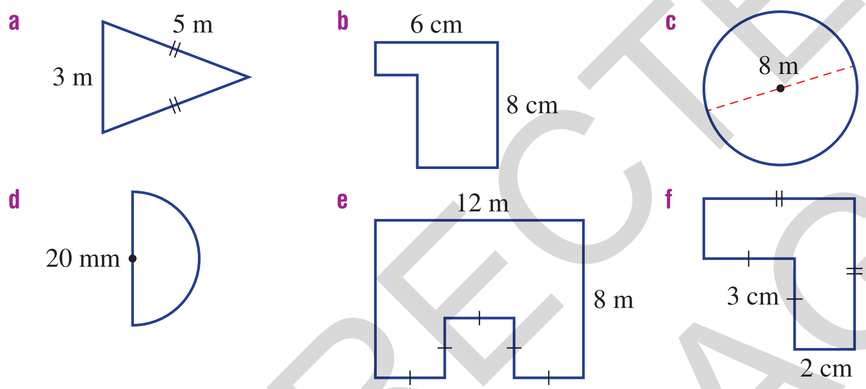
4A

1 Convert these measurements to the units given in the brackets.

a 2 m (mm) **b** 50 000 cm (km) **c** 3 cm^2 (mm^2) **d** 4000 cm^2 (m^2)
e 0.01 km^2 (m^2) **f** 350 mm^2 (cm^2) **g** 400 cm^3 (L) **h** 0.2 m^3 (L)

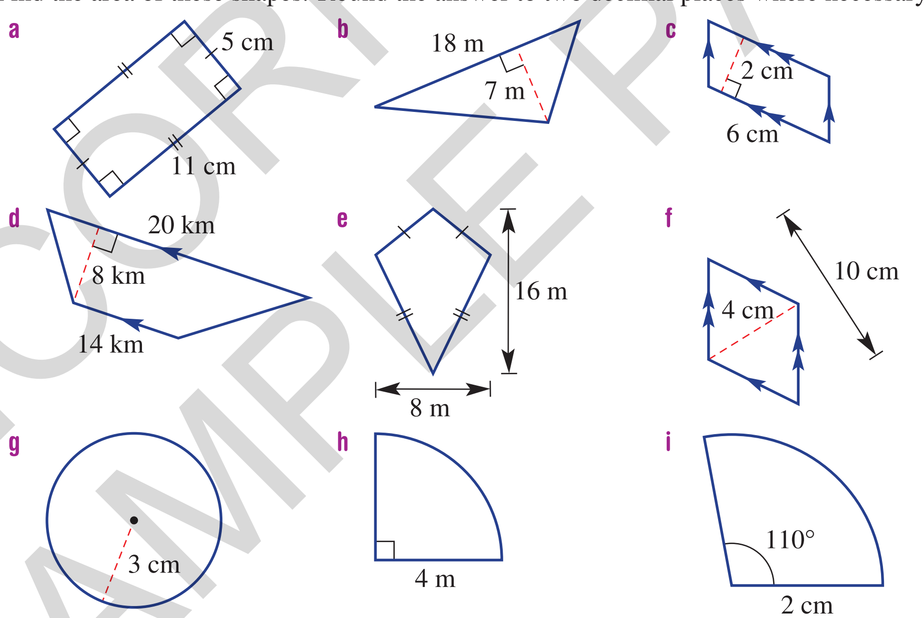
4A/B

2 Find the perimeter/circumference of these shapes. Round the answer to two decimal places where necessary.



4C/D/E

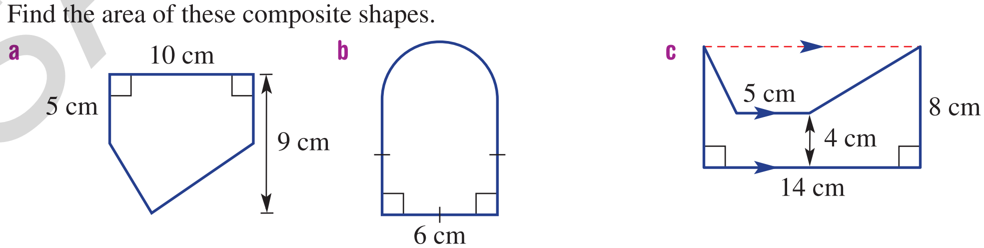
3 Find the area of these shapes. Round the answer to two decimal places where necessary.



4F

Ext

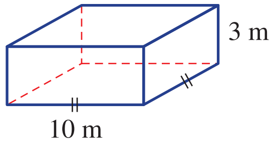
4 Find the area of these composite shapes.



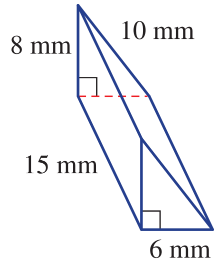
4G Ext

5 Find the surface area of each prism.

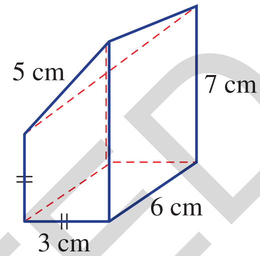
a



b



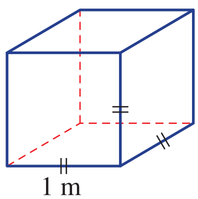
c



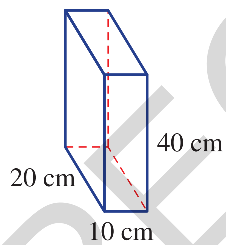
4H/I

6 Find the volume of each prism, giving your answer in litres. Remember 1 L = 1000 cm^3 and 1 m^3 = 1000 L.

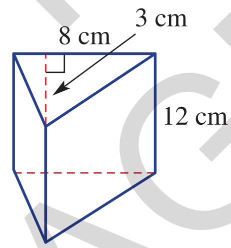
a



b



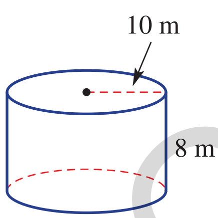
c



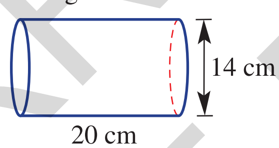
4I

7 Find the volume of these cylinders, rounding the answer to two decimal places.

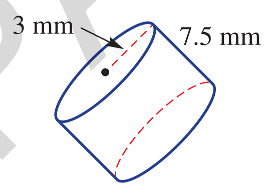
a



b



c



4J

8 An oven is heated from 23°C to 310°C in 18 minutes and 37 seconds. It then cools by 239°C in 1 hour, 20 minutes and 41 seconds.

a Give the temperature:

i increase **ii** decrease

b What is the total time taken to heat and cool the oven?

c How much longer does it take for the oven to cool down than heat up?

4J

9 **a** What is the time difference between 4:20 a.m. and 2:37 p.m.?

b Write 2145 hours in a.m./p.m. time.

c Write 11:31 p.m. in 24-hour time.

4J

10 When it is 4:30 p.m. in Western Australia, state the time in each of these places.

a New South Wales **b** Adelaide **c** United Kingdom

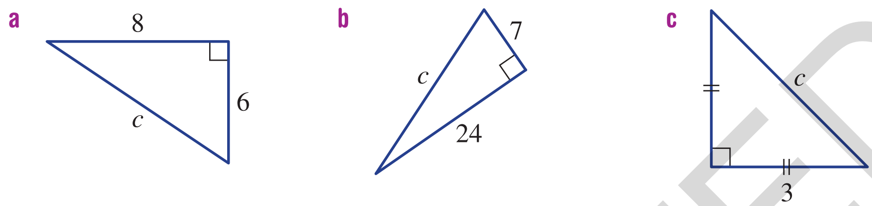
d China **e** Finland **f** South Korea

g Russia (eastern mainland) **h** New Zealand

Chapter review

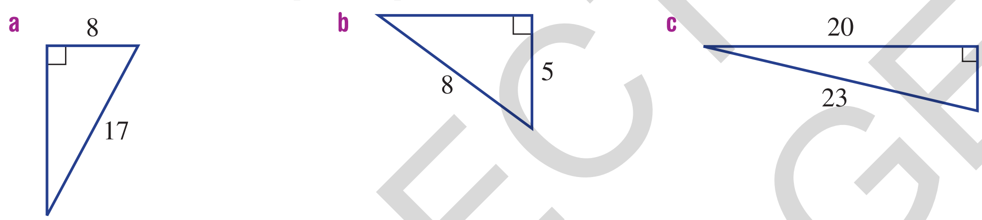
4L Ext

11 Use Pythagoras' theorem to find the length of the hypotenuse in these right-angled triangles. Round the answer to two decimal places in part **c**.



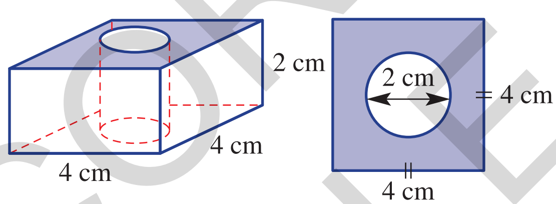
4M Ext

12 Use Pythagoras' theorem to find the unknown length in these right-angled triangles. Round the answer to two decimal places in parts **b** and **c**.



Extended-response questions

1 A company makes square nuts for bolts to use in building construction and steel structures. Each nut starts out as a solid steel square prism. A cylinder of diameter 2 cm is bored through its centre to make a hole. The nut and its top view are shown here.



The company is interested in how much paint is required to paint the nuts. The inside surface of the hole is not to be painted. Round all answers to two decimal places where necessary.

- **a** Find the area of the top face of the nut.
- **b** Find the total outside surface area of the nut.
- **c** If the company makes 10 000 nuts, how many square metres of surface needs to be painted?

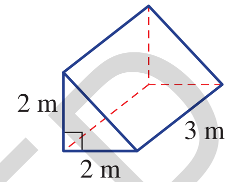
The company is also interested in the volume of steel used to make the nuts.

- **d** Find the volume of steel removed from each nut to make the hole.
- **e** Find the volume of steel in each nut.
- **f** Assuming that the steel removed to make the hole can be melted and reused, how many nuts can be made from 1 L of steel?

2 A simple triangular shelter has a base width of 2 m, a height of 2 m and a length of 3 m.

a Use Pythagoras' theorem to find the hypotenuse length of one of the ends of the tent. Round the answer to one decimal place.

b All the faces of the shelter including the floor are covered with canvas material. What area of canvas is needed to make the shelter? Round the answer to the nearest whole number of square metres.



c Every edge of the shelter is to be sealed with a special tape. What length of tape is required? Round to the nearest whole number of metres.

d The shelter tag says that is occupies 10 000 L of space. Show working to decide if this is true or false. What is the difference?