Chapter 6

Angles, triangles and polygons

This chapter will show you how to

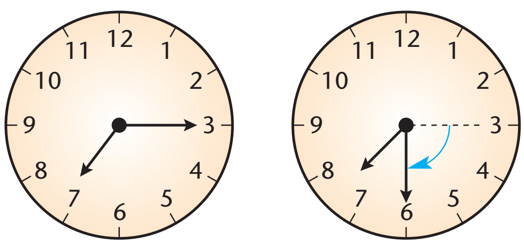
- ✔ describe a turn, recognise and measure angles of different types
- ✔ discover angle properties
- ✔ use bearings to describe directions
- ✔ investigate angle properties of triangles, quadrilaterals and other polygons
- ✔ understand line and rotational symmetry

6.1 Angles

Turning

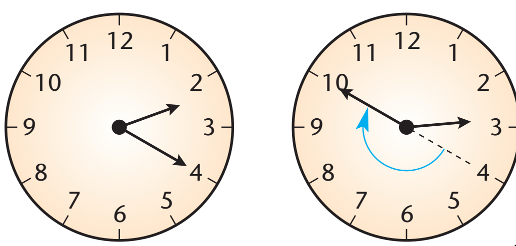
If you turn all the way round in one direction, back to your starting position, you make a **full turn** .

The minute hand of a clock moves a **quarter turn** from 3 to 6.



Minute hand makes $\frac{1}{4}$ turn.
Time taken: $\frac{1}{4}$ hour.

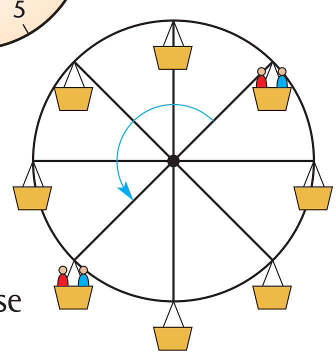
The minute hand of a clock moves a **half turn** from 4 to 10.



Minute hand makes $\frac{1}{2}$ turn.
Time taken: $\frac{1}{2}$ hour.

The hands of a clock turn **clockwise**.

This fairground ride turns **anti-clockwise** .



$\frac{1}{2}$ turn anti-clockwise

EXERCISE 6A

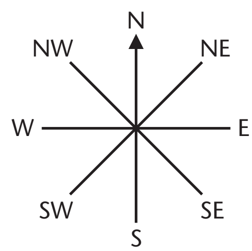
1 Describe the turn the minute hand of a clock makes between these times.

(a) 3 am and 3.30 am **(b)** 6.45 pm and 7 pm

(c) 2215 and 2300 **(d)** 0540 and 0710

Look at the clock examples on page 142.

2 Here is a diagram of a compass.



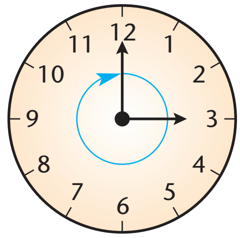
You are given a starting direction and a description of a turn. What is the finishing direction in each case?

	Starting direction	Description of turn
(a)	N	$\frac{1}{4}$ turn clockwise
(b)	SE	$\frac{1}{4}$ turn anti-clockwise
(c)	SW	$\frac{1}{2}$ turn anti-clockwise
(d)	E	$\frac{3}{4}$ turn anti-clockwise
(e)	NE	$\frac{1}{4}$ turn clockwise
(f)	W	$\frac{1}{2}$ turn clockwise
(g)	NW	$\frac{3}{4}$ turn clockwise
(h)	S	$\frac{1}{4}$ turn anti-clockwise

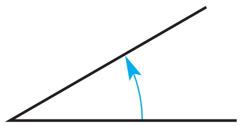
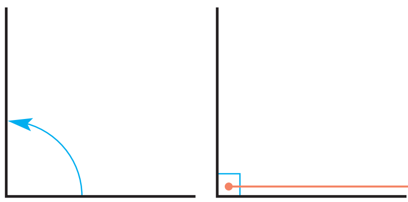
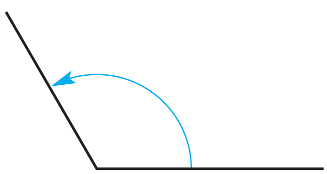
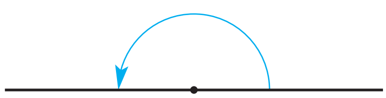
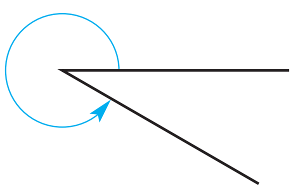
Describing angles

An angle is a measure of turn. Angles are usually measured in degrees.
A complete circle (or full turn) is 360°.

The minute hand of a clock turns through 360° between 1400 (2 pm) and 1500 (3 pm).

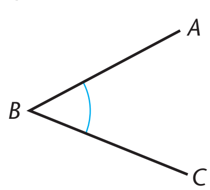


You will need to recognise the following types of angles.

acute		An angle between 0° and 90°, less than a $\frac{1}{4}$ turn.
right angle		An angle of 90°, a $\frac{1}{4}$ turn. A right angle is usually marked with this symbol.
obtuse		An angle between 90° and 180°, more than $\frac{1}{4}$ turn but less than $\frac{1}{2}$ turn.
straight line		An angle of 180°, a $\frac{1}{2}$ turn.
reflex		An angle between 180° and 360°, more than $\frac{1}{2}$ turn but less than a full turn.

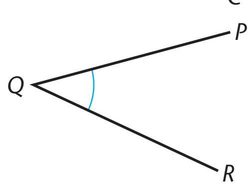
You can describe angles in three different ways.

- 'Trace' the angle using capital letters. Write a 'hat' symbol over the middle letter: $A\hat{B}C$



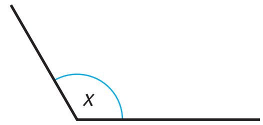
AB, BC, PQ and QR are called **line segments**.

- Use an angle sign or write the word 'angle': $\angle PQR$ or angle PQR



The letter on the point of the angle always goes in the middle.

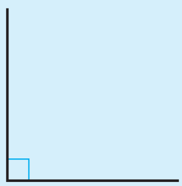
- Use a single letter.



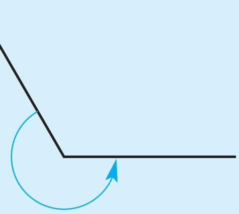
EXAMPLE 1

State whether these angles are acute, right angle, obtuse or reflex.

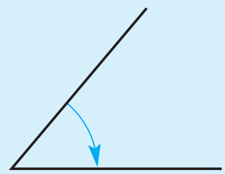
(a)



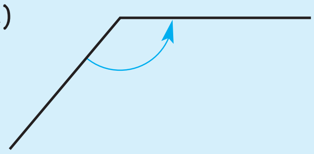
(b)



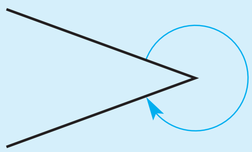
(c)



(d)



(e)



(a) right angle — The 'box' symbol shows the angle is 90°.

(b) reflex — More than a half turn.

(c) acute — Less than 90°.

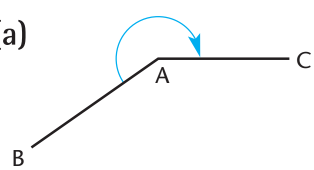
(d) obtuse — More than 90° but less than 180°.

(e) reflex — Almost a full turn.

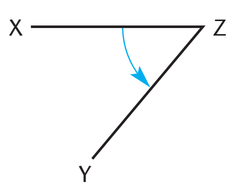
EXERCISE 6B

1 State whether these angles are acute, right angle, obtuse or reflex.

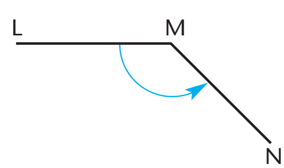
(a)



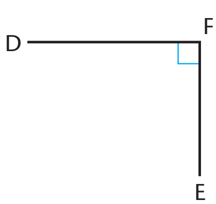
(b)



(c)



(d)



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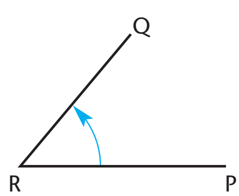
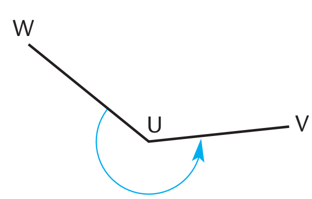


(f)



2 Describe each of the angles in question 1 using three letters.

For example
∠GHK $G\hat{H}K$ angle GHK

Measuring angles

You use a **protractor** to measure angles accurately.

Follow these instructions carefully.

1 Estimate the angle first, so you don't mistake an angle of 30°, say, for an angle of 150°.

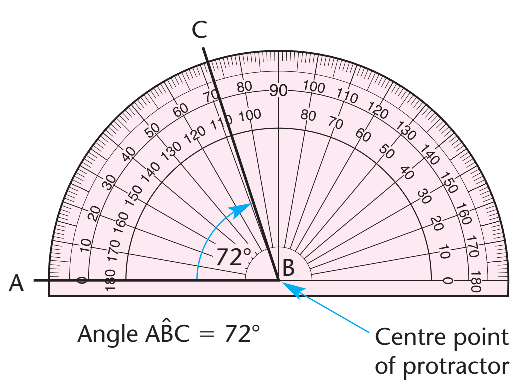
2 Put the centre point of the protractor exactly on top of the point of the angle.

3 Place one of the 0° lines of the protractor directly on top of one of the angle 'arms'.
If the line isn't long enough, draw it longer so that it reaches beyond the edge of the protractor.

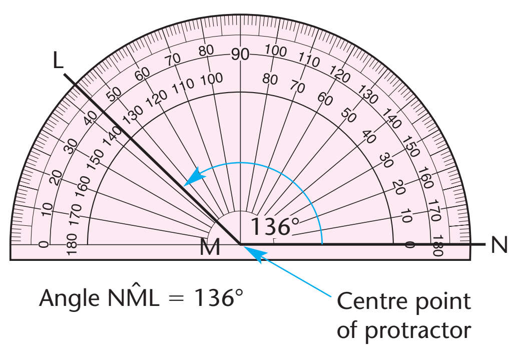
4 Measure from the 0°, following the scale round the edge of the protractor.
If you are measuring from the *left-hand* 0°, use the *outside* scale.
If you are measuring from the *right-hand* 0°, use the *inside* scale.

5 On the correct scale, read the size of the angle in degrees, where the other 'arm' cuts the edge of the protractor.

Use your estimate to help you choose the correct scale.



Measuring from the left-hand 0°.



Measuring from the right-hand 0°.

6 To measure a reflex angle (an angle that is bigger than 180°), measure the acute or obtuse angle, and subtract this value from 360°.

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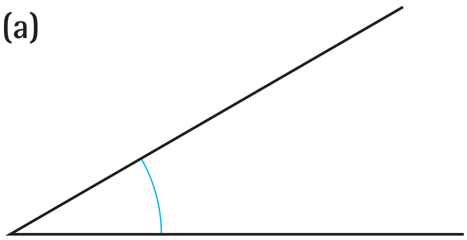
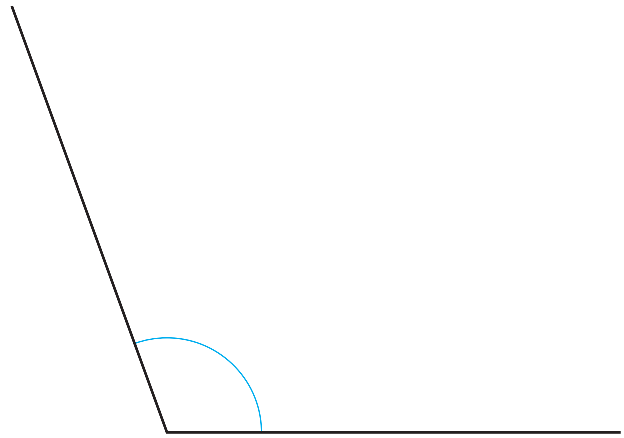
EXERCISE 6C

1 Measure these angles using a protractor.

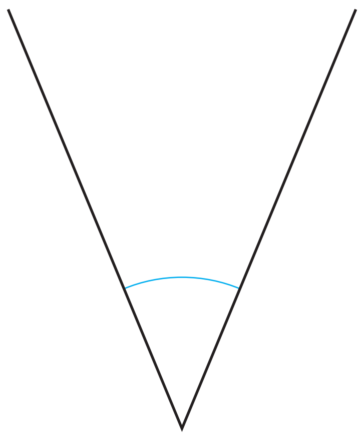
(a)



(b)



(c)



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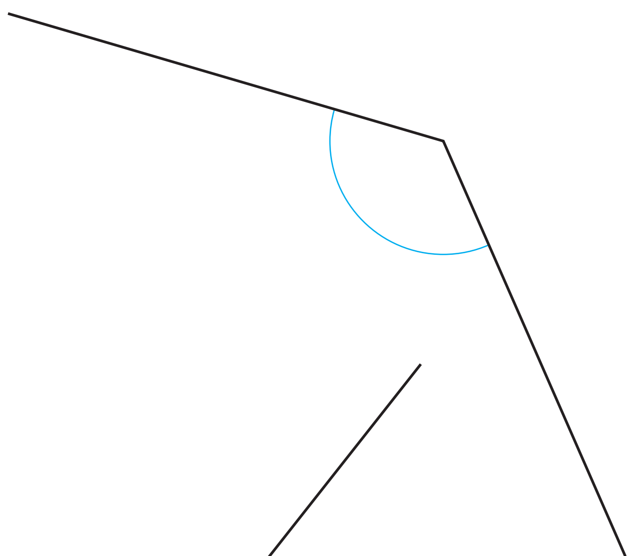
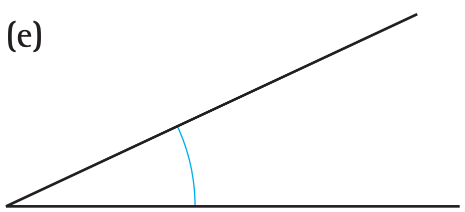
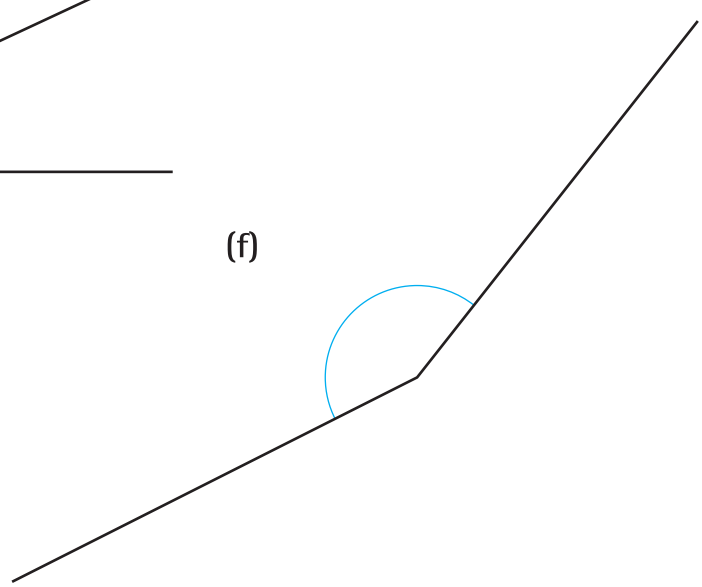


(e)



(f)



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2 Draw these angles using a protractor. Label the angle in each case.

(a) angle PQR = 54°

(b) angle STU = 148°

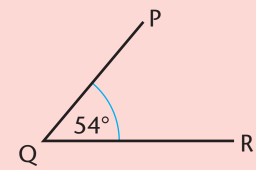
(c) angle MLN = 66°

(d) angle ZXY = 157°

(e) angle DFE = 42°

(f) angle HIJ = 104°

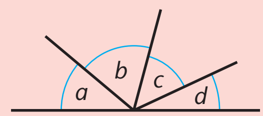
For example



Angle properties

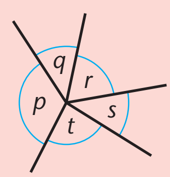
You need to know these angle facts.

Angles on a straight line add up to 180°.



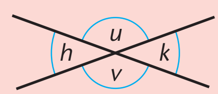
These angles lie on a straight line, so $a + b + c + d = 180°$.

Angles around a point add up to 360°.

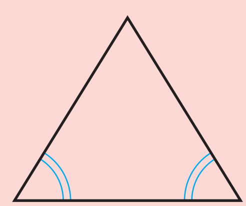


These angles make a full turn, so $p + q + r + s + t = 360°$.

Vertically opposite angles are equal.

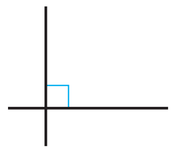


In this diagram $h = k$ and $u = v$.



Equal angles are shown by matching arcs.

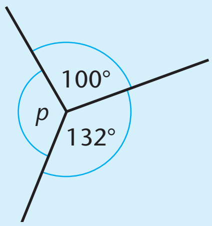
Perpendicular lines intersect at 90° and are marked with the right angle symbol.



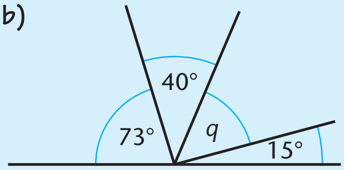
EXAMPLE 2

Calculate the size of the angles marked with letters.

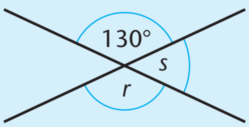
(a)



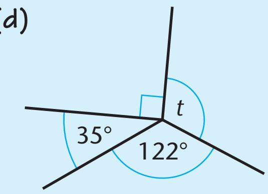
(b)



(c)



(d)



(a) $p + 100° + 132 = 360°$ (angles around a point)

$p = 360° - 100° - 132°$

$p = 128°$

(b) $q + 73° + 40° + 15° = 180°$ (angles on a straight line)

$q = 180° - 73° - 40° - 15°$

$q = 52°$

(c) $r = 130°$ (vertically opposite)

$s + 130° = 180°$ (angles on a straight line)

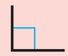
$s = 180° - 130° = 50°$

(d) $t + 122° + 35° + 90° = 360°$ (angles around a point)

$t = 360° - 122° - 35° - 90°$

$t = 113°$

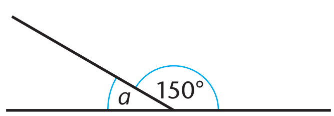
Solve this equation to find *p*.
Use Chapter 5 to help you.

 means 90°.

EXERCISE 6D

Calculate the size of the angles marked with letters.

1



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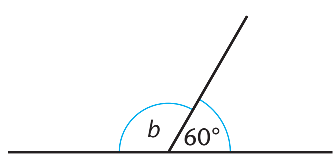

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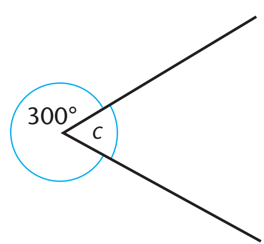

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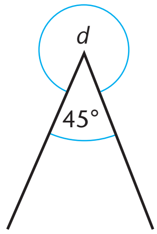

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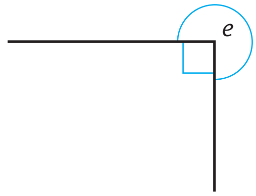

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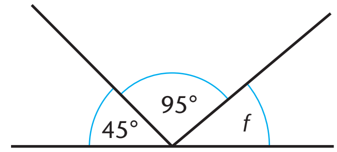

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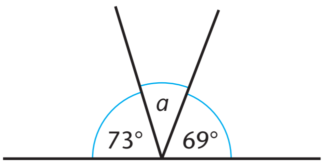

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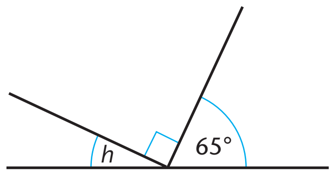

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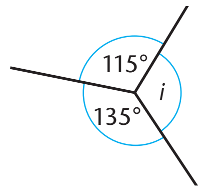

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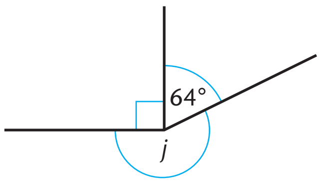

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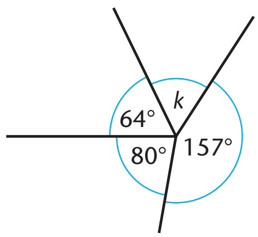

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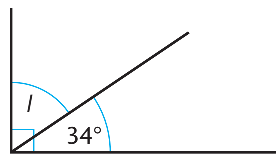

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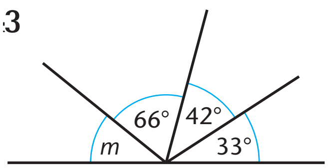

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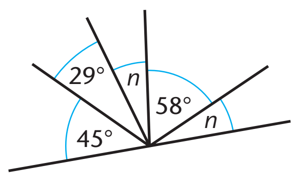

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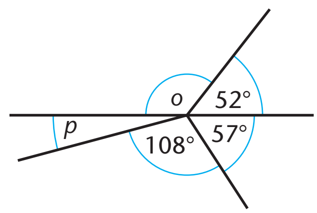

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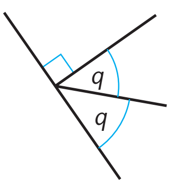

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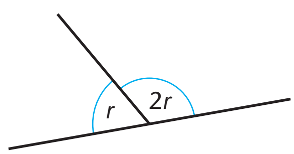
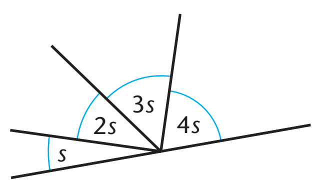
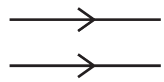


18



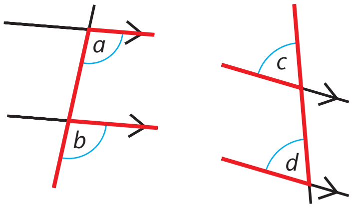
Angles in parallel lines

Parallel lines are the same distance apart all along their length. You can use arrows to show lines are parallel.



A straight line that crosses a pair of parallel lines is called a **transversal**.

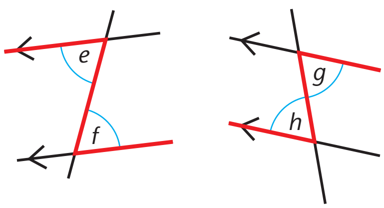
A transversal creates pairs of equal angles.



$a = b$ $c = d$

a and b }
c and d } are **corresponding** angles

The lines make an F shape.



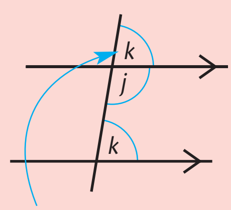
$e = f$ $g = h$

e and f }
g and h } are **alternate** angles.

The lines make a Z shape.

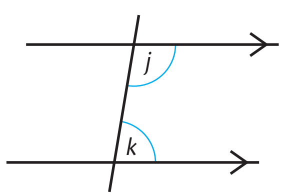
In this diagram the two angles are not equal, j is obtuse and k is acute.

The two angles lie on the *inside* of a pair of parallel lines. They are called **co-interior** angles or allied angles. Co-interior angles add up to 180°.



This angle $= k$ (corresponding angles)
$j + k = 180°$ (angles on a straight line).

$j + k = 180°$

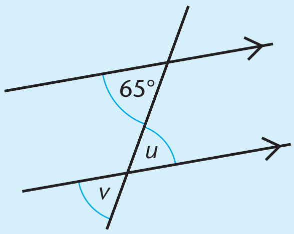


Corresponding angles are equal.
Alternate angles are equal.
Co-interior angles add up to 180°.

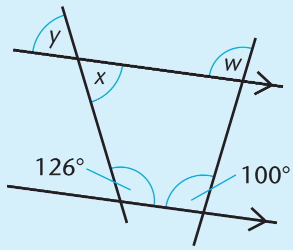
EXAMPLE 3

Calculate the size of the angles marked with letters.

(a)



(b)



(a) Method A

$u = 65°$ (alternate)

$v = 65°$ (vertically opposite)

Method B

$v = 65°$ (corresponding)

$u = 65°$ (vertically opposite)

(b) $w = 100°$ (corresponding)

$x + 126° = 180°$ (co-interior angles)

$x = 180° - 126°$

$x = 54°$

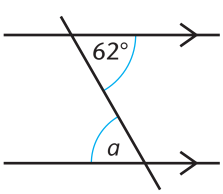
$y = 54°$ (vertically opposite)

You could use method A or method B.

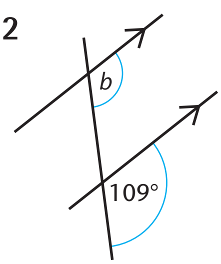
EXERCISE 6E

Calculate the size of the angles marked with letters.

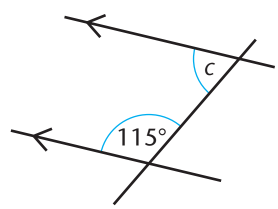
1



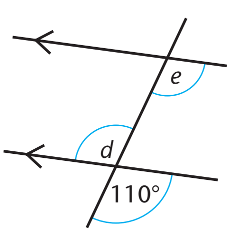
2



3



4



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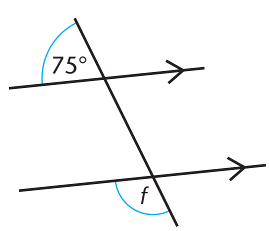
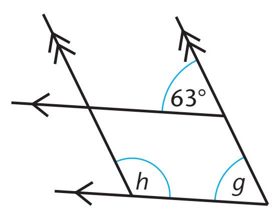


6



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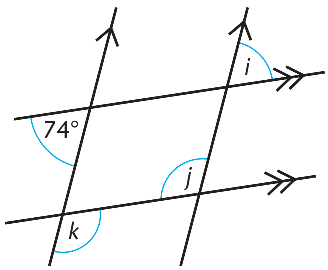

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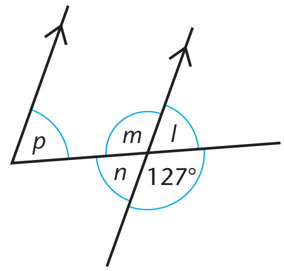
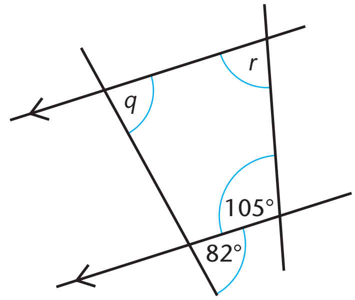


9

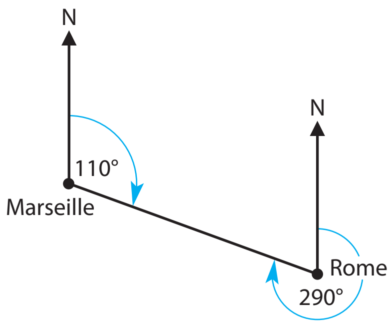


6.2 Three-figure bearings

You can use compass points to describe a direction, but in Mathematics we use three-figure **bearings**.

A three-figure bearing gives a direction in degrees.

It is an angle between 0° and 360°. It is always measured *from the north* in a *clockwise* direction.

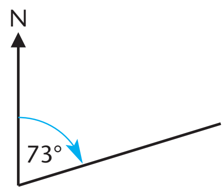


The diagram shows the cities of Marseille and Rome.
The bearing of Rome from Marseille is 110°.
The bearing of Marseille from Rome is 290°.

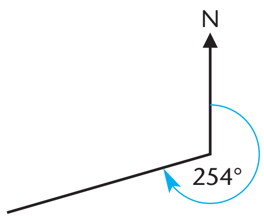
A bearing must always be written with *three figures*.

A bearing of 073°

You write 073° because the bearing must have three figures.



A bearing of 254°



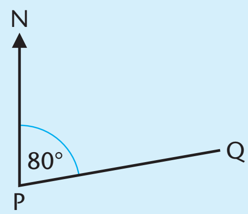
You need to be able to measure bearings accurately using a protractor.

Your answer needs to be within 1° of the correct value.

EXAMPLE 4

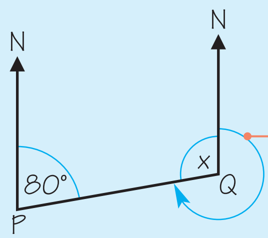
(a) Write down the bearing of Q from P.

(b) Work out the bearing of P from Q.



(a) 080°

(b)



The two north lines are parallel, so
$80° + x = 180°$ (co-interior angles)
$80° - 80° + x = 180° - 80°$
$x = 100°$
Bearing of P from Q $+ x = 360°$ (angles around a point)
Bearing of P from Q $+ 100° = 360°$
Bearing of P from Q $= 260°$.

If you stand at P and face north, you need to turn through 80° to face Q.

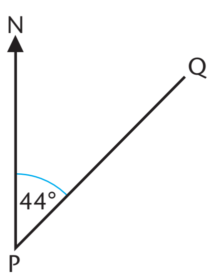
Draw in the north line at Q.

This angle is the bearing of P from Q.

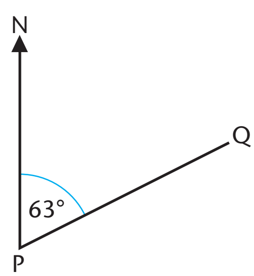
EXERCISE 6F

1 For each diagram, write down the bearing of *Q* from *P*.

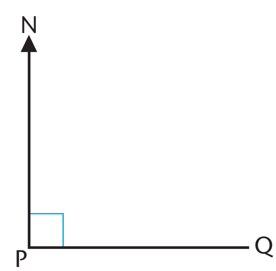
(a)



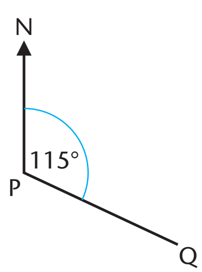
(b)



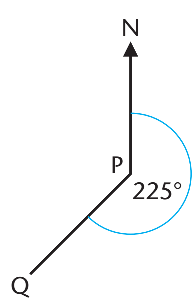
(c)



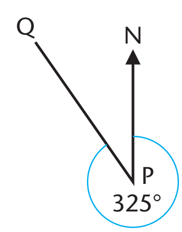
(d)



(e)

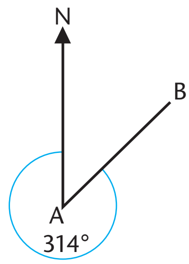


(f)

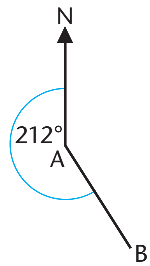


2 For each diagram work out the bearing of *B* from *A*.

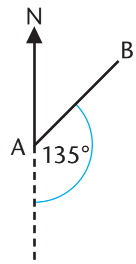
(a)



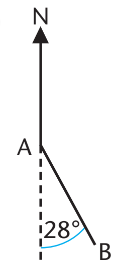
(b)



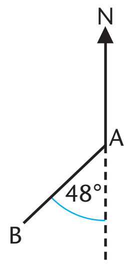
(c)



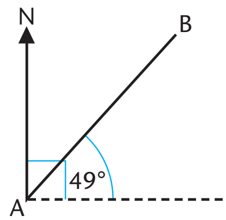
(d)



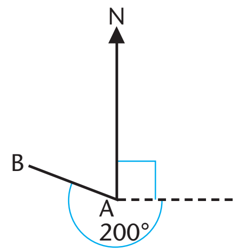
(e)



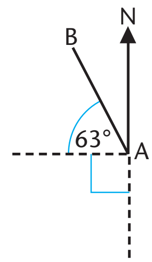
(f)



(g)



(h)

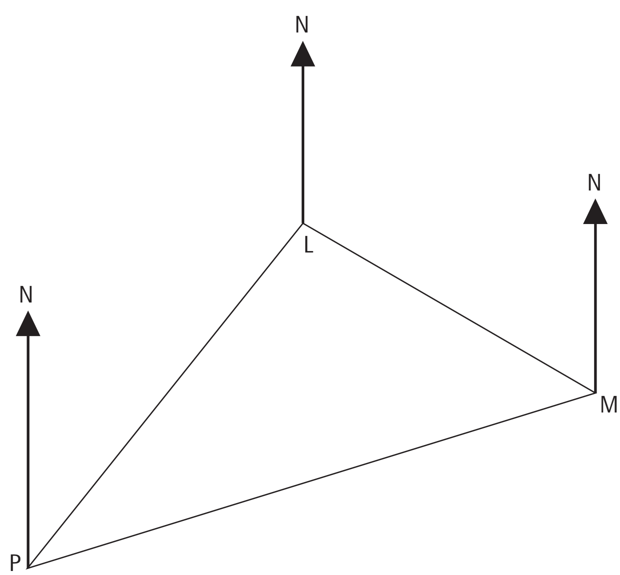


3 Draw accurate diagrams to show these three-figure bearings.

(a) 036° **(b)** 145° **(c)** 230° **(d)** 308°

(e) 074° **(f)** 256° **(g)** 348° **(h)** 115°

4 The diagram shows the peaks, *P*, *L* and *M*, of three mountains in the Himalayas.

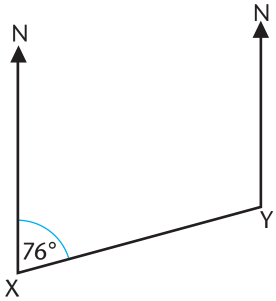


Use your protractor to find these bearings.

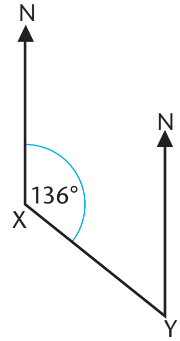
(a) *L* from *P* **(b)** *M* from *P* **(c)** *M* from *L*

5 In each diagram work out the bearing of *X* from *Y*.

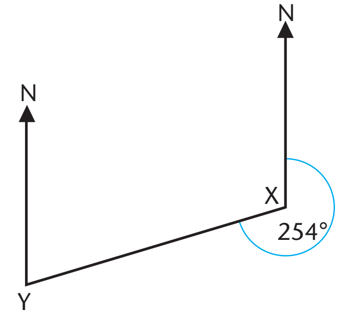
(a)



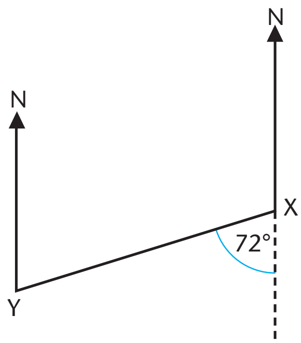
(b)



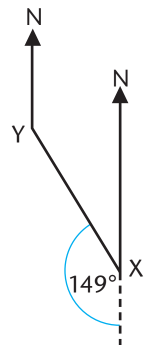
(c)



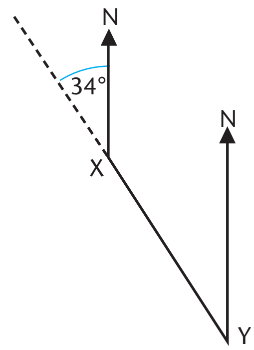
(d)



(e)



(f)



6 In each case

- Sketch a diagram to show the bearing of *D* from *C*
- Work out the bearing of *C* from *D*

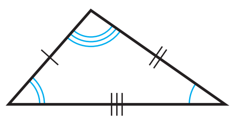
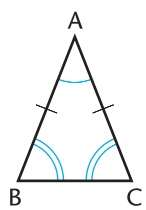
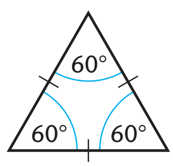
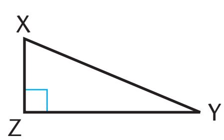
Bearing of *D* from *C*

(a) 037° **(b)** 205° **(c)** 167° **(d)** 296°

6.3 Triangles

Types of triangle

There are four types of triangle. They can be described by their properties.

Triangle	Picture	Properties
scalene		The three sides are different lengths. The three angles are different sizes.
isosceles		Two equal sides. $AB = AC$ Two equal angles ('base angles') angle ABC = angle ACB.
equilateral		All three sides are equal in length. All angles 60°.
right-angled		One of the angles is a right angle (90°). Angle $XZY = 90°$.

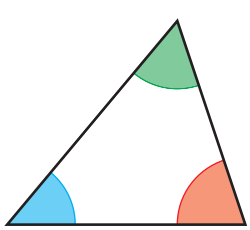
The marks across the sides of the triangles show which sides are equal and which sides are different.

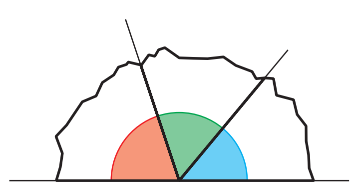
Angles in a triangle

1 Draw a triangle on a piece of paper.

2 Mark each angle with a different letter or shade them different colours.

3 Tear off each corner and place them next to each other on a straight line.



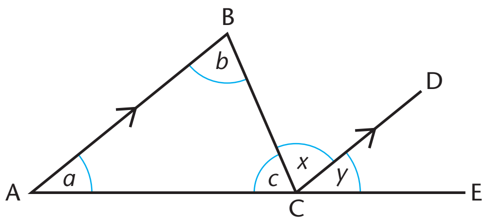


You will see that the three angles fit exactly onto the straight line.
So the three angles add up to 180°.

You can prove this for all triangles using facts about alternate and corresponding angles.

You are not *proving* this result, you are simply showing that it is true for the triangle you drew.

See page 151.



For the triangle ABC:

1 Extend side *AC* to point *E*.
2 From *C* draw a line CD parallel to *AB*.

Let $\angle BCD = x$ and $\angle DCE = y$.

$x = b$ (alternate angles)
$y = a$ (corresponding angles)
$c + x + y = 180°$ (angles on a straight line at point *C*)

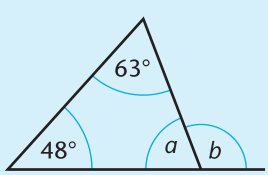
which means that $c + b + a = 180°$.

The sum of the angles of a triangle is 180°.

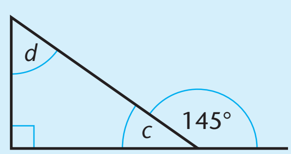
EXAMPLE 5

Calculate the size of the angles marked with letters.

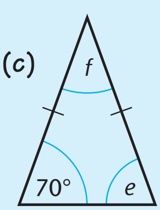
(a)



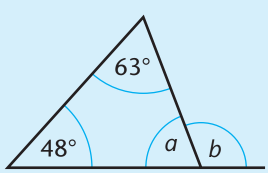
(b)



(c)



(a)

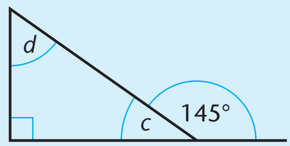


$a = 180° - 63° - 48°$
(angle sum of △)
$a = 69°$
$b = 180° - 69°$
(angles on a straight line)
$b = 111°$

Continued...

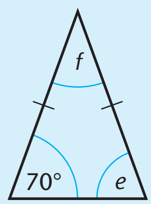
△ means 'triangle'

(b)



$c = 180° - 145°$
(angles on a straight line)
$c = 35°$
$d = 180° - 90° - 35°$
(angle sum of △)
$d = 55°$

(c)

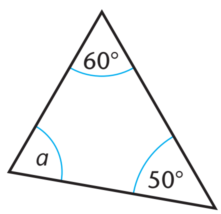


$e = 70°$ (base angles isosceles △)
$f = 180° - 70° - 70°$
(angle sum of △)
$f = 40°$

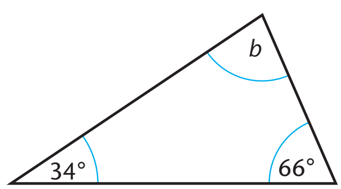
EXERCISE 6G

Calculate the size of the angles marked with letters.

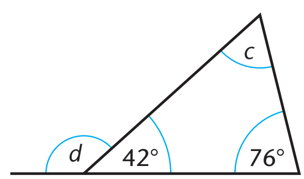
1



2



3



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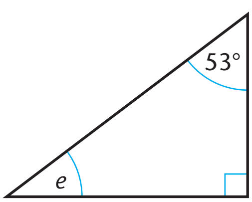

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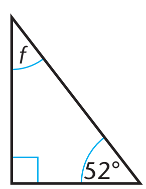

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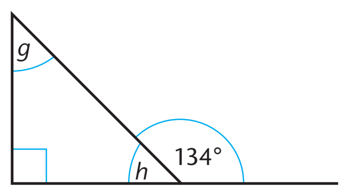

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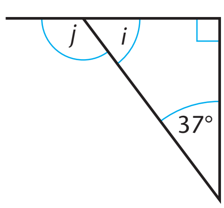

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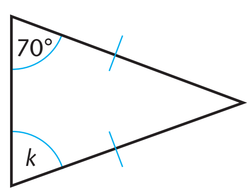

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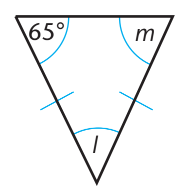

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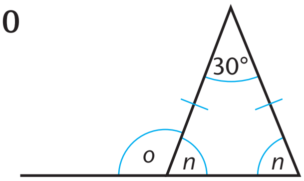

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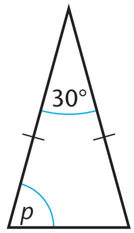

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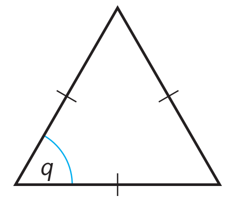

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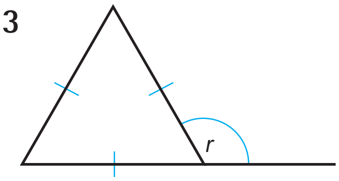

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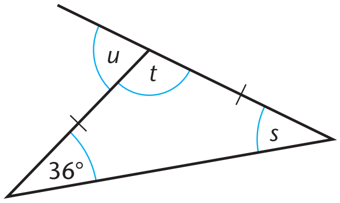

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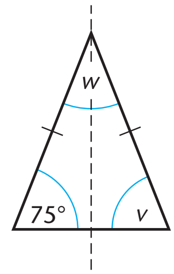

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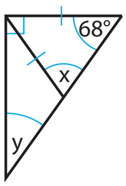

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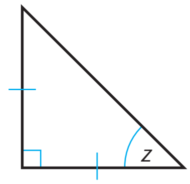

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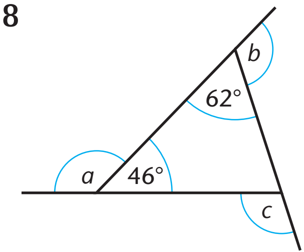
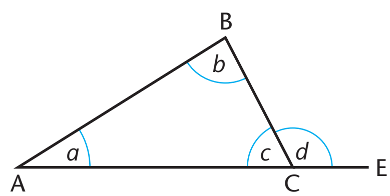


Interior and exterior angles in a triangle

The angles inside a triangle are called **interior** angles.

An **exterior** angle is formed by extending one of the sides of the triangle.

Angle *BCE* in this diagram is an exterior angle.



Let $BCE = d$

then $c + d = 180°$ (angles on a straight line)

but $c + b + a = 180°$ (angle sum of nABC)

which means that $d = a + b$

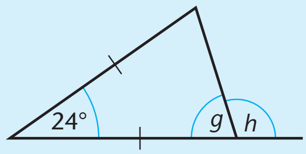
In a triangle, the exterior angle is equal to the sum of the two opposite interior angles.

Sometimes you need to calculate the size of 'missing' angles before you can calculate the ones you want.
You can label the extra angles with letters.

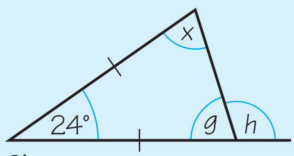
This helps make your explanations clear.

EXAMPLE 6

Calculate angles g and h.



Let the third angle of the triangle $= x$.



$g + x + 24° = 180°$ (angle sum of △)

$g + x = 180° - 24°$

$g + x = 156°$

$g = x$ (base angles isosceles △)

so both g and $x = \frac{1}{2}$ of $156°$ (or $156° \div 2$)

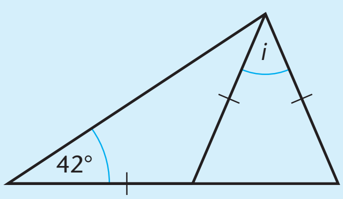
$156 \div 2 = 78.$

$g = 78°$

$h = 180° - 78°$ (angles on a straight line)

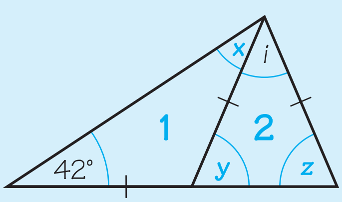
EXAMPLE 7

Calculate angle i.



Label the unknown angles x, y and z.

Label the two triangles 1 and 2.



$x = 42°$

(base angles isosceles △ 1)

$y = 42° + 42°$

(exterior angle of △ 1)

$y = 84°$

$z = 84°$

(base angles isosceles △ 2)

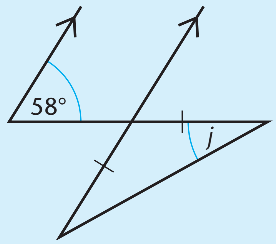
$i = 180° - 84° - 84°$

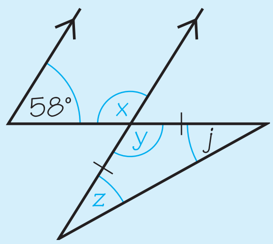
(angle sum △ 2)

$i = 12°$

EXAMPLE 8

Calculate angle j.





$x = 180° - 58°$
(co-interior angles, parallel lines)
$x = 122°$
$y = 122°$ (vertically opposite)
$z + j = 180° - 122°$ (angle sum of △)
$z + j = 58°$
$z = j$ (base angles isosceles △)
so both z and $j = \frac{1}{2}$ of 58° (or 58° ÷ 2)
$j = 29°$

Label the 'missing' angles x, y and z.

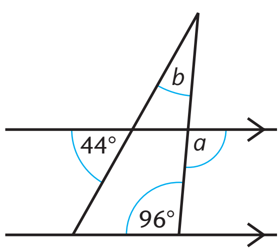
58 ÷ 2 = 29.

EXERCISE 6H

Copy the diagram and label any 'missing' angles.

Calculate the size of the angles marked with letters.

1



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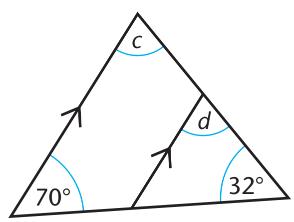

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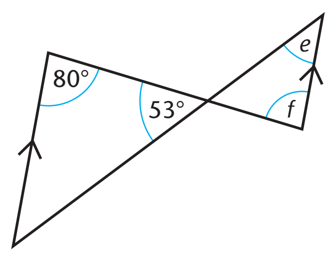

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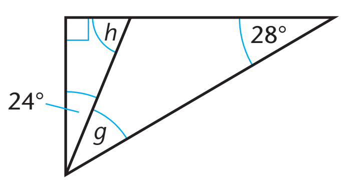

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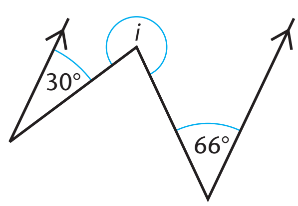

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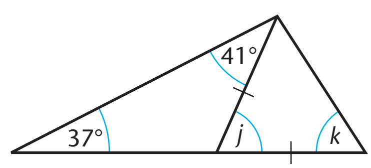

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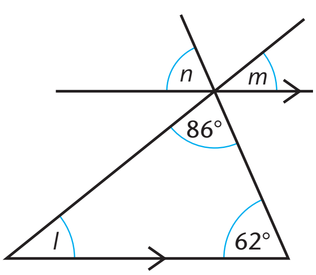

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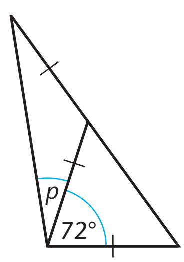

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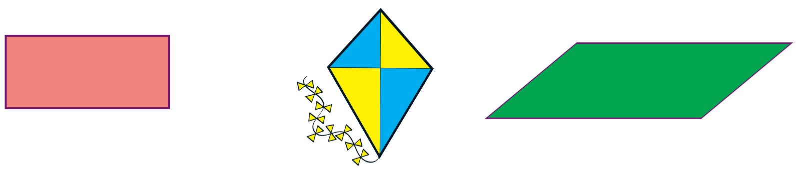
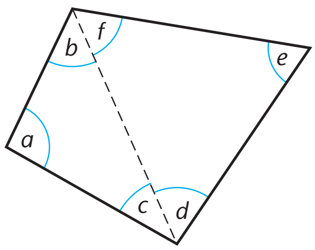
6.4 Quadrilaterals and other polygons

Quadrilaterals



A **quadrilateral** is a 2-dimensional 4-sided shape.

This quadrilateral can be divided into 2 triangles:



You can divide *any* quadrilateral into 2 triangles.

In each triangle, the angles add up to 180°,

so $a + b + c = 180°$

and $d + e + f = 180°$

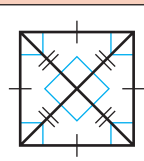
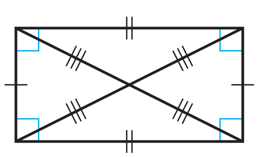
The 6 angles from the 2 triangles add up to 360°.

$a + b + c + d + e + f = 360°$

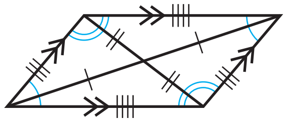
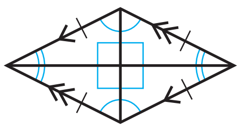
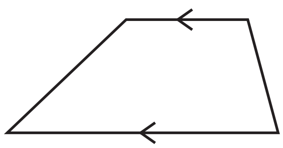
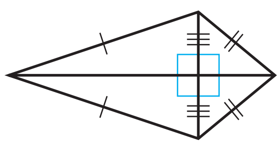
a, $(b + f)$, e, $c + d$ are the angles of the quadrilateral.

In any quadrilateral the sum of the interior angles is 360°.

Some quadrilaterals have special names and properties.

Quadrilateral	Picture	Properties
square		Four equal sides. All angles 90°. Diagonals **bisect** each other at 90°.
rectangle		Two pairs of equal sides. All angles 90°. **Diagonals** bisect each other.

Bisect means 'cut in half'.

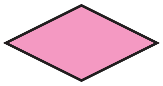
Quadrilateral	Picture	Properties
parallelogram		Two pairs of equal and parallel sides. Opposite angles equal. Diagonals **bisect** each other.
rhombus		Four equal sides. Two pairs of parallel sides. Opposite angles equal. **Diagonals** bisect each other at 90°.
trapezium		One pair of parallel sides.
kite		Two pairs of **adjacent** sides equal. One pair of opposite angles equal. One diagonal bisects the other at 90°.

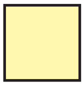
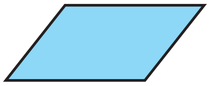
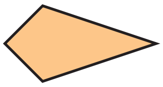
Lines with equal arrows are parallel to each other.

Adjacent means 'next to'.

EXERCISE 6I

1 Write down the name of each of these quadrilaterals.

(a)  (b)  (c) 

(d)  (e)  (f) 

2 Write down the names of all the quadrilaterals with these properties:

(a) diagonals which cross at 90°

(b) all sides are equal in length

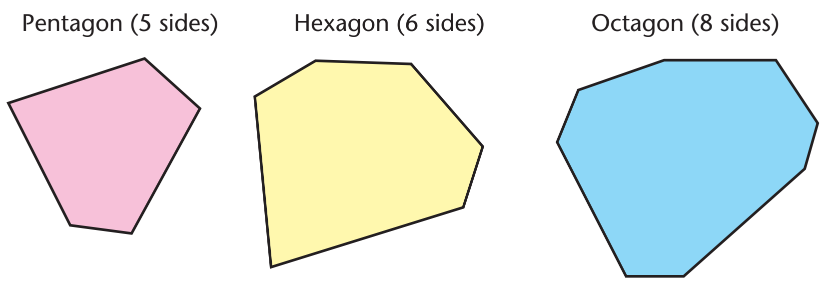
(c) only one pair of parallel sides

(d) two pairs of equal angles but not all angles equal
(e) all angles are equal
(f) two pairs of opposite sides are parallel
(g) only one diagonal bisected by the other diagonal
(h) two pairs of equal sides but not all sides equal
(i) the diagonals bisect each other
(j) at least one pair of opposite sides are parallel
(k) diagonals equal in length
(l) at least two pairs of adjacent sides equal.

Polygons

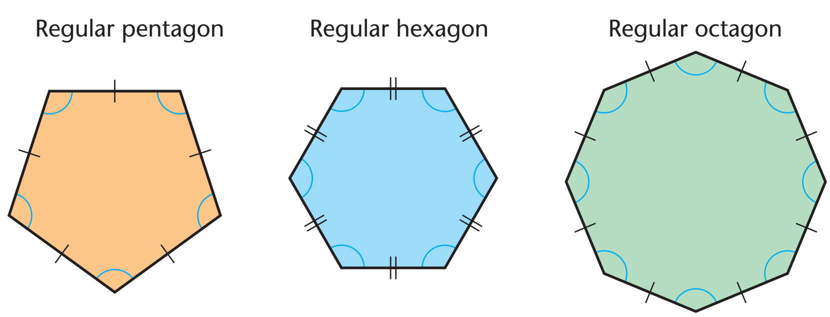
A **polygon** is a 2-dimensional shape with many sides and angles.

Here are some of the most common ones.



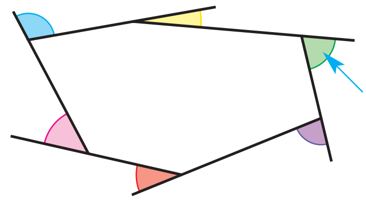
Polygon means 'many angled'.

A polygon with all of its sides the same length and all of its angles equal is called **regular**.



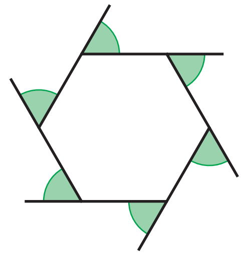
The sum of the exterior angles of any polygon is 360°.

You can explain this result like this.



If you 'walk round' the sides of this hexagon until you get back to where you started, you complete a full turn or 360°.

The exterior angles represent your 'turn' at each corner, so they must add up to 360°.

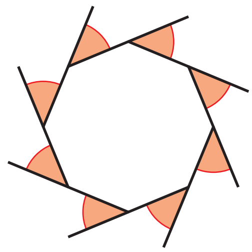


On a regular hexagon, all 6 exterior angles are the same size.

$\frac{360°}{6} = \frac{360°}{\text{number of sides}}$

For a regular hexagon:

$$\text{exterior angle} = \frac{360°}{6} = 60°$$



For a regular octagon

$$\text{exterior angle} = \frac{360°}{8} = 45°$$

A regular octagon has 8 equal exterior angles.

For a regular polygon,

$$\text{exterior angle} = \frac{360°}{\text{number of sides}}$$

You can use this equation to work out the number of sides in a regular polygon.

For a regular polygon,

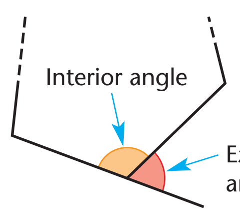
$$\text{number of sides} = \frac{360°}{\text{exterior angle}}$$

In a regular polygon with exterior angles of 18°:

$$\text{number of sides} = \frac{360°}{18°} = 20$$

Once you know the exterior angle, you can calculate the interior angle.

At each **vertex**, the exterior and interior angles lie next to each other (adjacent) on a straight line.



In a polygon, each pair of interior and exterior angles adds up to 180°.

So, for any polygon

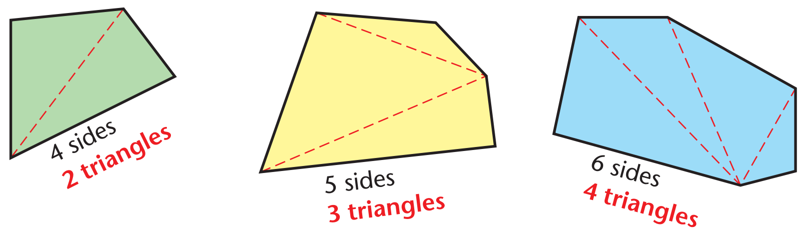
interior angle = 180° − exterior angle

For example,

- regular hexagon interior angle = 180° − 60° = 120°
- regular octagon interior angle = 180° − 45° = 135°

When a polygon is *not* regular the interior angles could all be different sizes. You can find the *sum* of the interior angles.

These diagrams show how you can divide any polygon into triangles.



You can see that the number of triangles is always 2 less than the number of sides of the polygon.

The sum of the interior angles for each polygon is given by the formula

$(n - 2) \times 180°$ where n is the number of sides of the polygon.

Name of polygon	Number of sides	Number of triangles	Sum of interior angles
Triangle	3	1	1 × 180° = 180°
Quadrilateral	4	2	2 × 180° = 360°
Pentagon	5	3	3 × 180° = 540°
Hexagon	6	4	4 × 180° = 720°
Heptagon	7	5	5 × 180° = 900°
Octagon	8	6	6 × 180° = 1080°
Nonagon	9	7	7 × 180° = 1260°
Decagon	10	8	8 × 180° = 1440°

In a regular polygon all the exterior angles are equal. So all the interior angles are equal.

Draw dotted lines from the **vertices**.

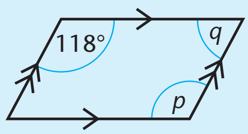
You should learn this formula.

EXAMPLE 9

Calculate the size of the angles marked with letters in each of these diagrams.

> You could use the fact that the sum of the angles of a quadrilateral is 360° to calculate *q*.

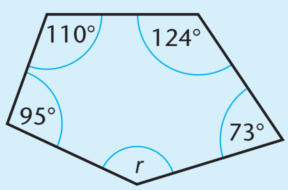
(a)



$p = 118°$ (opposite angles of parallelogram are equal

$q = 62°$ (p and q are co-interior angles, so $p + q = 180°$)

(b)



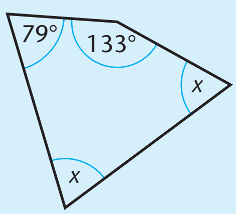
> Sum of the interior angles of a pentagon (5 sides) = 3 × 180° = 540°.

$r = 540° -$ (the sum of the other 4 angles)

$r = 540 - (95° + 110° + 124° + 73°)$

$r = 138°$

(c)



> Sum of interior angles of quadrilateral = 360°.

$x + x + 133° + 79° = 360°$

$2x = 360° - 133° - 79°$

$2x = 148°$

$x = 74°$

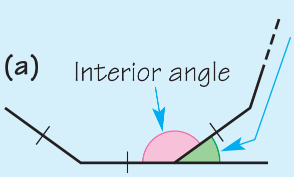
EXAMPLE 10

(a) Calculate the size of the exterior and interior angles of a regular polygon with 20 sides.

(b) How many sides has a regular polygon with interior angle 168°?

> Exterior angle of regular polygon $= \dfrac{360°}{\text{number of sides}}$

(a)



Exterior angle $= \dfrac{360°}{20} = 18°$

Interior angle $= 180° -$ exterior angle

$= 180° - 18°$

$= 162°$

(b) Exterior angle $= 180° -$ interior angle

$= 180° - 168°$

$= 12°$

Number of sides $= \dfrac{360}{12°} = 30$

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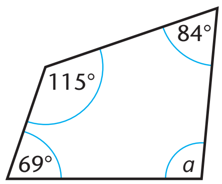
EXERCISE 6J

Calculate the size of the angles marked with letters.

1


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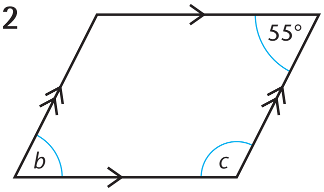

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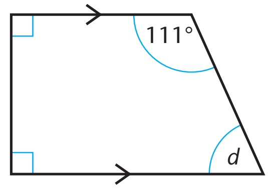

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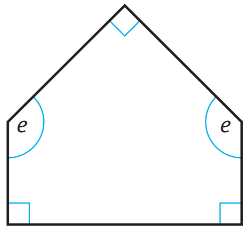

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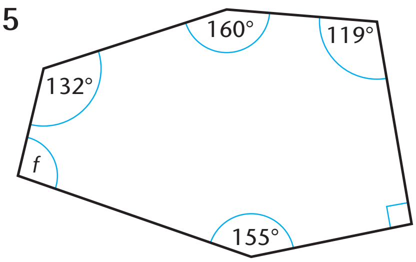
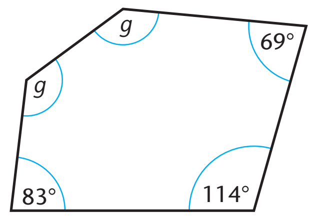


6


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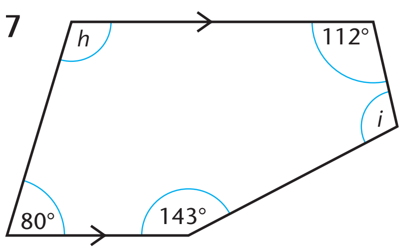
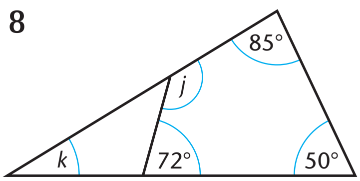


8


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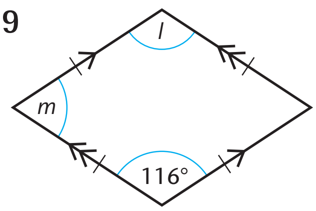

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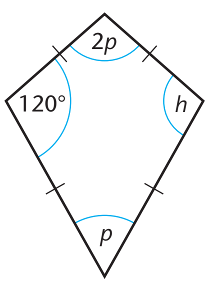

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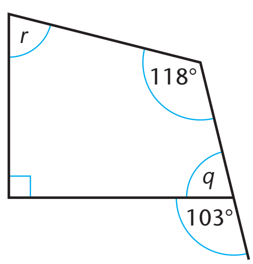

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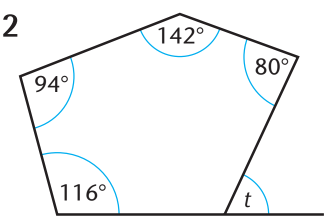
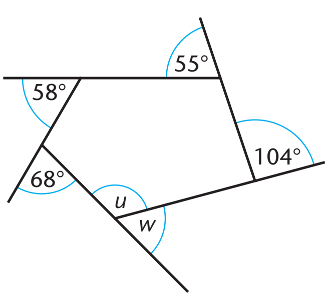
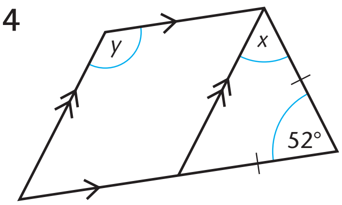


13


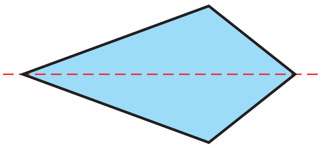
14


15 Calculate the interior angle of a regular polygon with 18 sides.

16 Can a regular polygon have an interior angle of 130°? Explain your answer.

6.6 Symmetry

Line symmetry

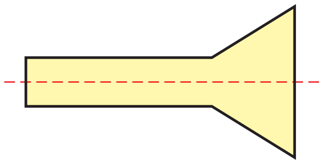


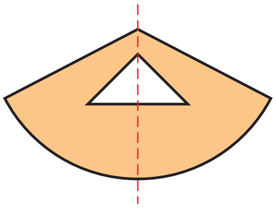
A kite is **symmetrical**.

If you fold it along the dashed line, one half fits exactly onto the other.

The dashed line is called a **line of symmetry**.

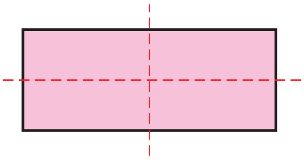
A line of symmetry divides a shape into two halves. One half is the mirror image of the other.





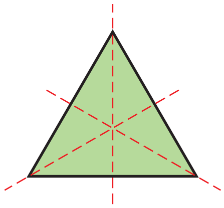
Some shapes have more than one line of symmetry.

Rectangle ... 2 lines of symmetry

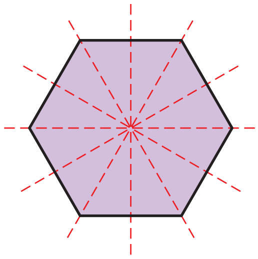


You can draw lines of symmetry with solid lines.

Equilateral triangle ... 3 lines of symmetry

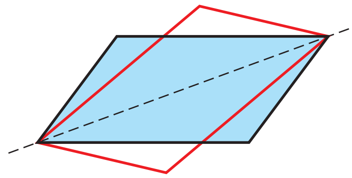


Regular hexagon ... 6 lines of symmetry



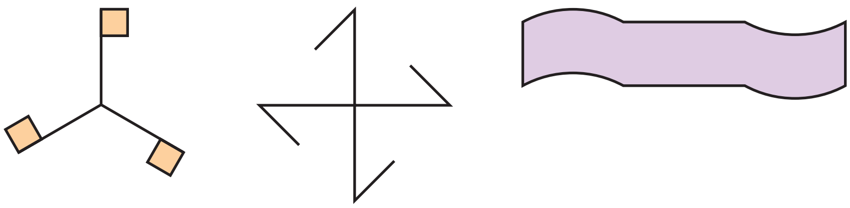
Some shapes have no lines of symmetry.

A parallelogram has no lines of symmetry.



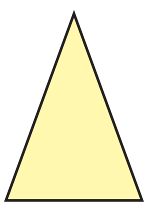
The dashed line is *not* a line of symmetry. If you reflect the parallelogram in it you get the red parallelogram.

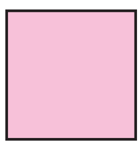
These shapes have no lines of symmetry.

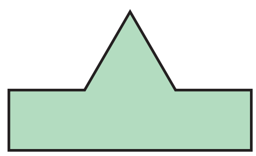


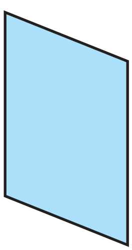
EXERCISE 6K

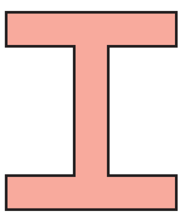
Copy these shapes and draw in all the lines of symmetry (if any).

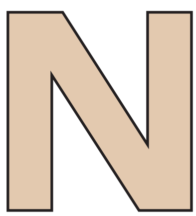
1 

2 

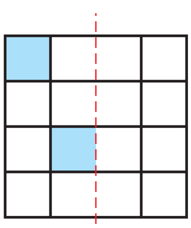
3 

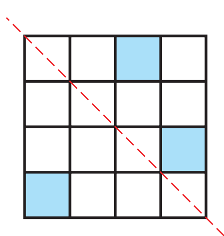
4 

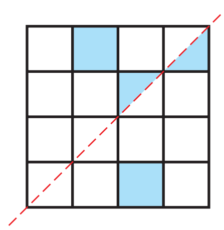
5 

6 

7 Copy and complete these grids so that they are symmetrical about the dashed line.

(a) 

(b) 

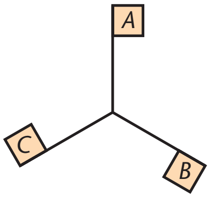
(c) 

Rotational symmetry

Look again at the shapes before Exercise 6K.
They have no lines of symmetry but they have **rotational symmetry.**

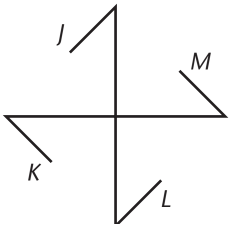
You can turn them and they will fit exactly into their original shape again.

The **order** of rotational symmetry is the number of times a shape looks the same during one full turn.



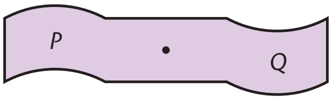
If *A* turns to any of the positions *A*, *B* or *C* the shape will look exactly the same.

Rotational symmetry of order 3.



If *J* turns to *J*, *K*, *L* or *M* the shape will look exactly the same.

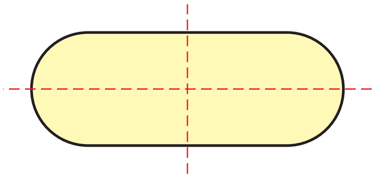
Rotational symmetry of order 4.



If *P* turns to positions *P* or *Q* the shape will look exactly the same.

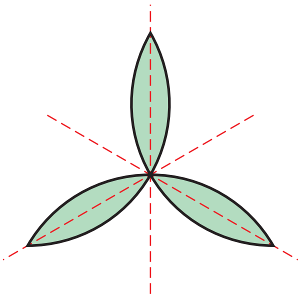
Rotational symmetry of order 2.

Some shapes have line symmetry *and* rotational symmetry.



2 lines of symmetry.

Rotational symmetry of order 2.

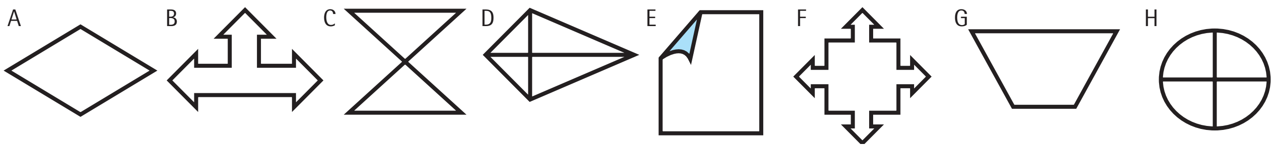


3 lines of symmetry.

Rotational symmetry of order 3.

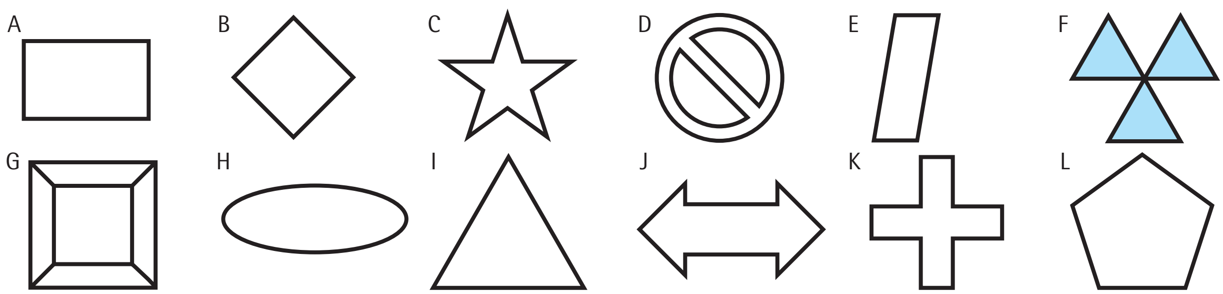
EXERCISE 6L

1 Which of these shapes has rotational symmetry.

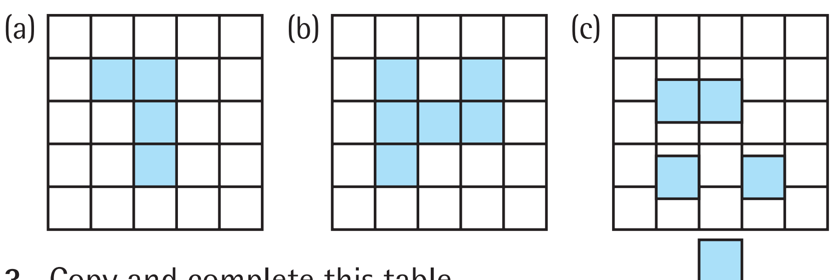


2 Write down the letters of the shapes that have rotational symmetry of order

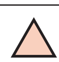
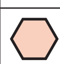
(a) 2 (b) 3 (c) 4 (d) 5



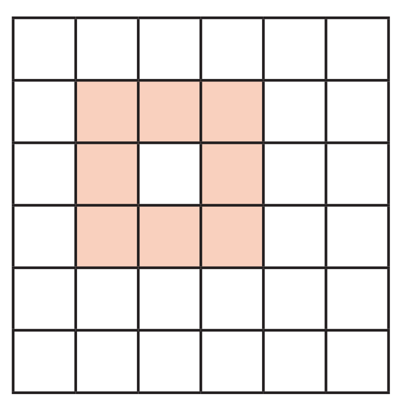
2 Shade one more square in each grid to make shapes with rotational symmetry of order 2



3 Copy and complete this table

Name of shape		Order of rotational symmetry
		3
Square		
Pentagon		5
		

4 Eight squares have been shaded in this grid to make a shape with rotational symmetry of order 4. Find other ways of shading eight squares to make shapes with rotational symmetry of order 4.



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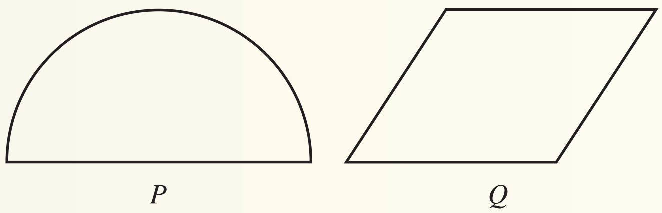
EXAMINATION QUESTIONS

1



(a) Write down the number of lines of symmetry of shape

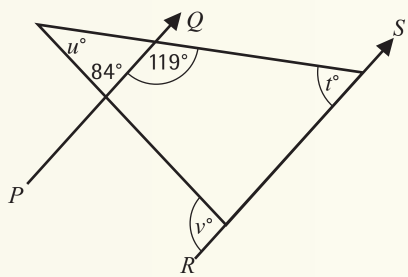
(i) p, [1]

(ii) q. [1]

(b) Write down the order of rotational symmetry of shape q. [1]

(CIE, Paper 1, Jun 2000)

2



In the diagram the lines PQ and RS are parallel.
Calculate the values of t, u and v. [3]

(CIE Paper 1, Jun 2000)

3

MATHEMATICS

(a) In the word above

(i) Which letters have two lines of symmetry? [2]

(ii) Which letter has rotational symmetry but no line symmetry? [1]

(CIE Paper 1, Nov 2000)

4

Ch6 EQ4

Insert drawing Nov 2000) Paper1 Q13 – find in folder and renumber as question 4 ????

(CIE Paper 1, Nov 2000)

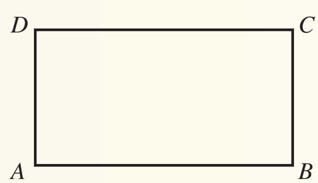
5

Ch6 EQ5

Insert drawing Paper 3 Q6 Jun 2001)– find in folder and renumber as question 5 ???

(CIE Paper 3, Jun 2001)

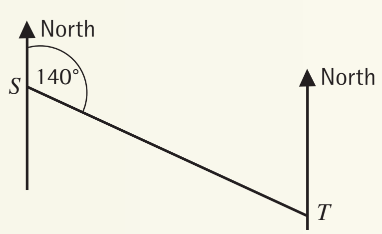
6



(Copy and) draw accurately the lines of symmetry of the rectangle *ABCD*. [1]

(CIE Paper 3, Nov 20001)

7



Samira (S) and Tamara (T) walk towards each other.
Samira walks on a bearing of 140°.
Find the bearing on which Tamara walks. [2]

(CIE Paper 1, Nov 2001)

---

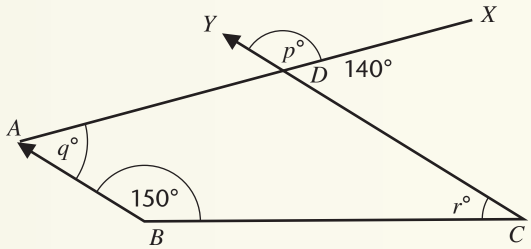
8 **(a)** Write down the name of the special quadrilateral which has rotational symmetry of order 2 but no line symmetry. [1]

(b) Draw, on a grid, a quadrilateral which has exactly one line of symmetry but no rotational symmetry.
Draw the line of symmetry on your diagram. [2]

(CIE Paper 1, Jun 2002)

---

9 **(a)**



In the diagram, *AX* and *CY* are straight lines which intersect at *d*.
BA and *CY* are parallel.
Angle *CDX* = 40° and angle *ABC* = 150°.

(i) Find p, q and r. [3]

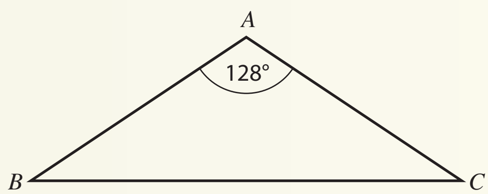
(ii) What is the name of the special quadrilateral *ABCD*? [1]

(b) (i) A nonagon is a polygon with nine sides.
Calculate the size of an interior angle of a **regular** nonagon. [3]

(ii) Each angle of another regular polygon is 150°.
Calculate the number of sides of this polygon. [3]

(CIE Paper 3, Jun 2002)

10



In the triangle *ABC*, *AB* = *BC*.

(a) What is the special name of this triangle? [1]

(b) Angle *BAC* = 128°. Work out angle *ABC* [2]

(CIE Paper 1, Nov 2002)

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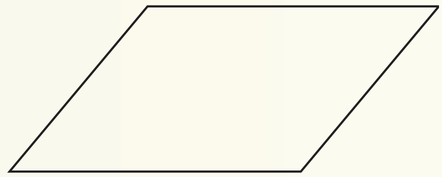
11 Find the size of one of the ten interior angles of a regular decagon. [3]

(CIE Paper 1, Nov 2003)

12



For the shape shown, write down
(a) the number of lines of symmetry, [1]
(b) the order of rotational symmetry. [1]

(CIE Paper 1, Jun 2004)

---

13 **(a)** (i) What is the special name given to a five-sided polygon? [1]
(ii) Calculate the total sum of the interior angles of a regular five-sided polygon. [2]
(iii) Calculate the size of one interior angle of a regular five-sided polygon. [1]

(CIE Paper 3, Jun 2004)

---

14 Reflex Right Acute Obtuse

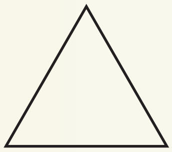
Use one of the above terms to describe each of the angles give.
(a) 100°,
(b) 200°.

(CIE Paper 1, Nov 2004)

---

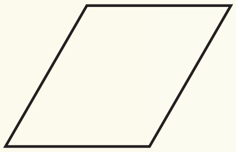
15 Write down the order of rotational symmetry of each of the following shapes.

(a)



Equilateral triangle

(b)



Rhombus

(CIE Paper 1, Nov 2004)
