9

Strength and Failure of Rocks

9·1. Failure

Failure, in a general sense, includes both fracture and flow.

As defined by Jaeger "the term fracture implies the appearance of distinct surfaces of separation in the body". Yield is used to describe the onset of plastic deformation with the resulting unrestricted plastic deformation defined as flow. Before discussing failure, it

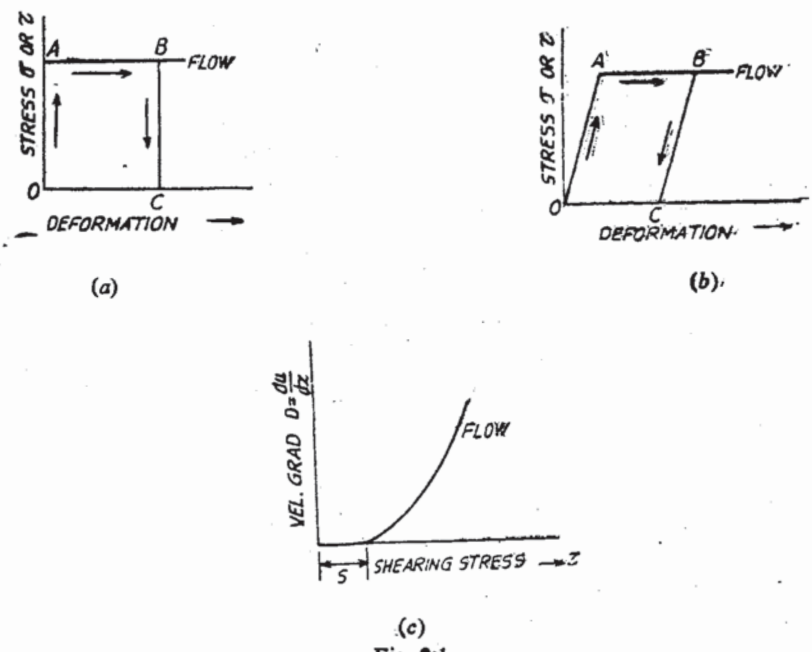


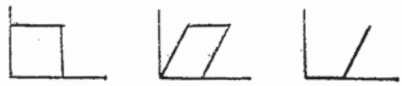
Fig. 9·1

is necessary to discuss about stress∼deformation behaviour of materials. Stress∼deformation behaviour of three distinct types of materials is shown in Fig. 9·1.

9·2. Idealy Plastic, Perfectly Plastic and Elastic Plastic Matelials

Upto a limiting stress σ_0, no deformation takes place and if σ_0 stress is reached, the material deforms continuously with no increment in stress then the material is known as "Ideally plastic". Its stress strain curve is shown in Fig. 9·1 (*a*).

In some materials, due to application and removal of stress greater than the failure or threshold stress, a permanent deformation results. Elastic effects are included in such a case. The loading and unloading are indicated by the path *OABC*, and a permanent deformation is given by abscissa of point *C*. The materials of such a type are "Perfectly plastic".



(*a*) (*b*) (*c*)

Fig. 9·1

If the total deformation is calculated from a sum of elastic and viscous deformations, the material is known as "Viscoelastic" [Fig. 9·1 (*c*)]. When the flow stress has been reached, the actual behaviour of the material with time is not specified. Failure alone is sufficiently important.

In some materials, the recoverable deformation which takes place prior to on-set of failure, is small in comparison to the deformation after reaching the limiting stress. The material possessing such a property is known as "rigid-plastic" [Fig. 9·1 (*a*)]. But if the deformation prior to the on-set of failure is such that it plays an important part in the stress-analysis, then they are known as "elastic-plastic" materials. The stress-strain curve of such materials is shown in Fig. 8·1 (*a*).

The above definitions have been given to have an idea about the different types of materials which behave in different manners on an application of load and prior to failure. Next, before discussing strength criteria of rocks, a brief idea about different failure theories will be necessary which explain failure criteria of different types of materials.

9·3. Types of Failure

Fracture and failure are also of different types which take place in different types of materials. Several types of fracture which commonly occur are described below.

(*a*) Rupture

Rupture occurs when a ductile material fails in tension. It is preceded by a plastic deformation causing "necking" and is usually known as a "Cup and Cone" fracture.

(*b*) Brittle fracture

Brittle fracture occurs in brittle materials due to tension as a tensile or cleavage fracture which occurs on a plane perpendicular to the direction of the tension.

(*c*) Shear fracture

Shear fracture occurs in case of brittle materials subjected to compression. The resulting failure planes are approximately in the direction of the greatest shear stress.

At an elevated temperature and pressure however, brittle material behave as ductile or plastic materials.

9·4. Yield Criteria of Failure Theories

Different theories have been given to explain the yield or failure criteria in different types of materials. They are also known as strength theories. A few of the important theories are being discussed.

9·4·1. Maximum-Stress Theory

This theory is also known as Rankin's theory. The theory states that the maximum principal stress in the material determines failure regardless of the magnitude and senses of the other two principal stresses. Thus, in a stressed body, yielding starts when the absolute value of the maximum stress reaches the yield point stress of the material in simple tension or compression.

The theory is contradicted in solid materials where three equal tensile or compressive stresses cannot produce a plastic but only an elastic deformation. The theory is best suited for considering the strength of non isotropic materials, specially layered materials, where there is a pronounced difference in strength properties in different directions. For example, a layered rock has almost no tensile strength or very little in the direction normal to the layers, and fails in tension by splitting along these layers.

9·4·2. The Maximum Elastic Strain Theory

This theory is also known as the St. Venant theory. The theory assumes that a ductile material begins to yield when the maximum strain equals the yield point strain in simple tension. This is true for compressive strains (shortening) also. In such a case, the minimum strain equals the yield point strain in simple compression. Mathematically, it can be described as

$$\left|\frac{\sigma_1}{E} - \frac{\nu}{E}(\sigma_2+\sigma_3)\right| = \frac{\sigma_y}{E} \text{ (tensile)} \qquad \ldots[9·1\ (a)]$$

or

$$\left|\frac{\sigma_3}{E} - \frac{\sigma}{E}(\sigma_1+\sigma_2)\right| = \frac{\sigma_y}{E} \text{ (comp.)} \qquad \ldots[9·1\ (b)]$$

where
σ_1 = major principal stress
σ_3 = minor principal stress
σ_2 = intermediate principal stress
E = modulus of elasticity
ν = Poisson's ratio
and σ_y = yield stress.

This theory is again contradicted by material behaviour under hydrostatic tensile or compressive stresses.

9·4·3. The Constant Elastic-Strain Energy Theory

In this theory, the quantity of strain energy per unit volume of the material is used as the basis for determining failure in the material.

Therefore as per this theory, failure will occur when strain energy for a given state of stress in a material reaches the value of energy stored at the yield point in simple tension. Mathematically, the criteria can be described as

$$\frac{(\sigma_y)^2}{2E} = \frac{1}{2E}(\sigma_1^2+\sigma_2^2+\sigma_3^2) - \frac{\nu}{E}(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1) \qquad \ldots(9{\cdot}2)$$

This theory does not explain the behaviour of the material under a hydrostatic stress. Because performance of materials, under a hydrostatically stressed condition, indicates that the elastic energy can have no significance as a limiting condition.

9·4·4. The Maximum Shear-Stress Theory

This is also known as Tresca theory. As per this theory yielding begins when the maximum shear stress in the material equals the maximum shear stress at the yield point in sample tension. Mathematically it can be written as

$$\frac{\sigma_y}{2} = \frac{\sigma_1 - \sigma_3}{2}$$

or

$$\sigma_y = \sigma_1 - \sigma_3 \qquad \ldots(9{\cdot}3)$$

It can also be written as

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \text{constant} \qquad \ldots(9{\cdot}4)$$

In uniaxial tension

$$\sigma_1 = \sigma_0$$

and

$$\sigma_2 = \sigma_3 = 0$$

$$\therefore \quad \tau_{max} = \frac{\sigma_0}{2}$$

and in uniaxial compression

$$\sigma_1 = \sigma_2 = 0$$

and

$$\sigma_3 = -\sigma_0$$

$$\therefore \quad \tau_{max} = -\left(\frac{\sigma_0}{2}\right)$$

∴ Yield criterion, as per this theory, is

$$\tau_{max} = \tfrac{1}{2}(\sigma_1 - \sigma_3) = \frac{\sigma_0}{2} \qquad \ldots(9{\cdot}5)$$

The slip lines (failure lines) which appear on set of plastic flow are inclined at an angle of 45° with respect to the directions of principal stress σ_1 and σ_3 which is coincident with the direction of maximum shearing stress.

This theory is applicable to explain failure criteria of some type of soils and rock materials.

9·4·5. The Constant Elastic Strain-Energy of Distoration Theory

This theory is also known as Von Mises or Maxwell theory of failure.

The theory states that plastic yielding begins when the strain energy of distortion S_D reaches a critical value given by equation (9·6).

$$S_D=\frac{1+\nu}{6E}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right] \quad \text{...(9·6)}$$

For a material in simple tension the yield point occurs at σ_y then, we have

$$\sigma_1=\sigma_y$$

and $\sigma_2=\sigma_3=0$

Putting these values in Eq. (9·6)

$$S_D=\frac{1+\nu}{3E}(\sigma_y)^2$$

Thus, the condition for yield to occur based on the theory is

$$2(\sigma_y)^2=(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2 \quad \text{...(9·7)}$$

It can also be proved that

$$\tau_{oct}=\frac{\sqrt{2}}{3}\sigma_y \quad \text{...(9·8)}$$

where τ_{oct} = Octahedral shearing stress. Equation (9·8) states that the octahedral shearing stress in the material at the time of on-set of a plastic limit is constant and it depends on the yield point of the material in simple tension or compression.

9·4·6. Mohr's Theory

This theory assumes slippage as a mode of failure and states that failure occurs due to both yielding and fracture. It provides a functional relationship between the normal and the shear stress given by Eq. (9·9)

$$\tau=f(\sigma) \quad \text{...(9·9)}$$

As per Mohr's theory failure depends upon the stresses on the slip planes which has got minimum strength. The failure takes place when the obliquity of the resultant stress exceeds a certain maximum value known as shear strength.

9·4·7. The Coulomb Theory

The Coulomb theory states that failure in the material takes place when the shear stress on the material along the failure plane exceeds the value of shear strength of the material given by equation

$$\tau = C + \sigma \tan \phi \qquad \ldots(9.10)$$

where C = cohesion

σ = normal stress

and ϕ = angle of internal friction.

The Coulomb equation, represented by equation (9·10), is a special case of Mohr's theory of strength in which the Mohr's envelope is a straight line which is inclined to the normal stress axis at an angle ϕ.

The use of the Coulomb Eq. 9·10 to represent the Mohr's envelope in the Mohr's diagram is known as the "Mohr-Coulomb" theory.

Although soil failure criteria are explained by Coulomb, and Mohr's theory, the failure criteria of some of the rock materials are explained by 'Mohr-Coulomb' theory.

9·5. Behaviour of Rock Materials

Rock in general, is neither homogeneous, isotropic, nor perfectly elastic. Its behaviour can range from that of a brittle material to that of a ductile material depending on the combination of pressure and temperature to which it is subjected. The limit of temperature and pressure in most engineering works is such that for harder rocks, their basic behaviour is that of a brittle material, or in a transition from brittle to ductile. With the softer rocks, ductile behaviour becomes more pronounced. In the field, on these intrinsic characteristics, a "macroscopic fabric" pattern is superimposed, *i.e.*, rocks range from a massive formation with a few joints to an accumulation of uncoherent, although interlocked, blocks or fragments. Further, rock in-situ is always in a state of stress and strain. At the same time, an anisotropic behaviour is found more frequently than an isotropic behaviour not only in the stratified rocks, where the planes of stratification and schistosity give rise to wide variation in strength in different directions, but also in harder igneous rocks.

The above characteristics and less quantitative information available on rock behaviour make it essential to critically review and objectively assess the adequacy of every phase of the investigation-design-construction process, if rock structures are to be permanently stable and adequate under their specified conditions of use. Although importance and the procedure of investigation work have already been discussed in earlier chapters, for proper design of a structure the strength criteria of rocks also are essential. We have seen that at ordinary temperature and pressure, most of the rocks behave like brittle materials. Therefore, first we shall discuss the failure criteria of brittle materials. The most important theory up-till-now which describes the strength criteria of brittle materials is Griffith's theory. The theory describes how fracture is initiated, how it propagates and how failure is caused in the material due to fracture.

9·6. Griffith's Theory of Fracture Initiation in the Rock Mass

Griffith postulated that a rock material contains a large number of randomly oriented zones of potential failure in the form of grain boundaries. The grain boundary contains a number of open flaws which are approximately elliptical in shape. Very high tensile stresses occur on the boundary of a suitably oriented elliptical opening even under compressive stress conditions. Griffith assumed that fracture initiates from the boundary of an open flow when the tensile stress at this boundary exceeds the local tensile strength of the material.

9·6·1. Stress Around Boundary of an Open Flow

For obtaining the stresses around the boundary of an open flaw let us take one grain boundary as shown in Fig. 9·2 which contains a number of open flaws. Considering only one flaw with the following simplifying assumptions.

(*i*) The flaw, which is of an elliptical shape, can be treated as a single ellipse in a semi-infinite elastic medium ;

(*ii*) Adjacent flaws do not interact ;

(*iii*) The material is assumed to be homogeneous ;

(*iv*) The ellipse and the stress system are taken to be two-dimensional.

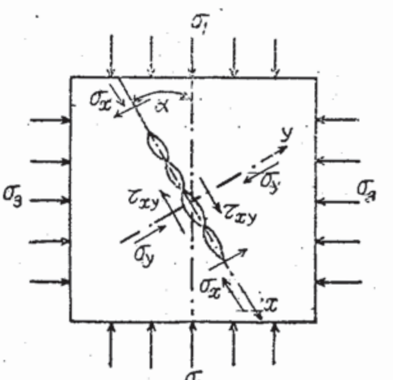


Fig. 9·2 (*a*). Stress acting around a crack in a rock mass.

As shown in figure, the flaw is inclined at an angle α to the major principal stress (σ_1) direction. σ_y and τ_{xy} are the normal and the shear stress respectively acting on the material surrounding the flaw. Their values are given by Eqs. 9·11 and 9·12.

$$\sigma_y=\tfrac{1}{2}(\sigma_1-\sigma_3)-\tfrac{1}{2}(\sigma_1-\sigma_3)\cos 2\alpha \quad ...(9·11)$$

and

$$\tau_{xy}=\tfrac{1}{2}(\sigma_1-\sigma_3)\sin 2\alpha \quad ...(9·12)$$

where σ_1 = major principal stress

and σ_3 = minor principal stress.

The parameters defining the ellipse have been shown in Fig. 9·2 (*b*).

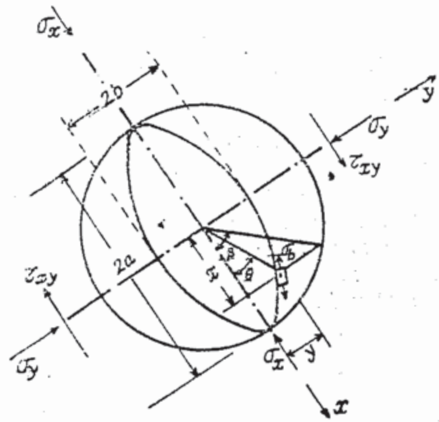


Fig. 9·2 (*b*). Equation of ellipse, $x=a\cos\beta$, $y=b\sin\beta$, $m=b/a$.
Stress around an elliptical flaw.

The tangential stress σ_b on the boundary of ellipse is given by the Eq. (9·13) derived by Inglis (1913), Denkhaus (1964).

$$\sigma_b=\frac{\sigma_y\,[m\,(m+2)\cos^2\beta-\sin^2\beta]+\sigma_x\,[(1+2m)\sin^2\beta-m^2\cos^2\alpha]-\tau_{xy}\,[2(1+m^2)\sin\beta\cos\beta]}{m^2\cos^2\beta+\sin^2\beta}\qquad ...(9{\cdot}13)$$

Since the flaws in rock are very flat in shape, the axis ratio m of the ellipse will be very small. Eq. 9·13 states that maximum tensile stress will occur near the tip of the flaw when $\beta=0$. When $\beta\to 0$, $\sin\beta\to\beta$ and $\cos\beta=1$. Putting these values in Eq. 9·13 and neglecting the terms of second order and higher in the numerator the following approximate relation is obtained

$$\sigma_b=\frac{2\,(\sigma_y m-\tau_{xy}\beta)}{m^2+\beta^2}\qquad ...(9{\cdot}14)$$

σ_b is the boundary stress near the tip of the flaw.

When $$\frac{d\sigma_b}{d\beta}=0$$

the tangential stress on the boundary will be maximum. Now differentiating Eq. 9·14 with respect to β and equating it to zero, we have

$$\frac{d\sigma_b}{d\beta}=\frac{-2\tau_{xy}}{m^2+\beta^2}-\frac{2\,(\sigma_y m-\tau_{xy}\beta)\,2\beta}{(m^2+\beta^2)^2}$$

$$=\frac{(-2\tau_{xy})\,(m^2+\beta^2)-2\,(\sigma_y m-\tau_{xy}\,\beta)\,2\beta}{(m^2+\beta^2)^2} \quad \ldots(9\cdot15)$$

Eq. (9·15) is to be equated with zero for

$$\frac{d\sigma_b}{d\beta}=0$$

$$\therefore \quad (-2\tau_{xy})\,(m^2+\beta^2)\,-2\,(\sigma_y m-\tau_{xy})\,2\beta=0$$

or $$(m^2+\beta^2)\,(-2\tau_{xy})=4\beta\,(\sigma_y m-\tau_{xy}\,\beta) \quad \ldots(9\cdot16)$$

or $$-2m^2\,\tau_{xy}-2\tau_{xy}\,\beta^2=4\beta\,\sigma_y m-4\tau_{xp}\,\beta^2$$

or $$-2m^2\,\tau_{xy}+2\tau_{xy}\,\beta^2-4\beta\,\sigma_y\,m=0$$

or $$-\frac{1}{\beta^2}+\frac{1}{m^2}-\frac{2\sigma_y}{\beta\,\tau_{xy}m}=0$$

or $$-\frac{1}{\beta^2}-\frac{2\sigma_y}{m\tau_{xy}}\cdot\frac{1}{\beta}+\frac{1}{m^2}=0$$

or $$\frac{1}{\beta}=\frac{\frac{2\sigma_y}{\sigma_{xy}.m}\pm\sqrt{\frac{4\sigma^2_y}{\tau^2_{xy}.m^2}+4\frac{1}{m^2}}}{-2}$$

$$=-\frac{1}{m\tau_{xy}}\left\{\sigma_y\pm\left(\sigma^2_y+\tau^2_{xy}\right)^{\frac{1}{2}}\right\} \quad \ldots(9\cdot17)$$

From Eq. (9·14), we have

$$2\,(\sigma_y m-\tau_{xy}\,\beta)=\sigma_b\,(m^2+\beta^2)$$

Putting these values in Eq. (9·16)

$$(m^2+\beta^2)\,(-2\tau_{xy})=2\beta\,\sigma_b\,(m^2+\beta^2)$$

or $$-\tau_{xy}=\beta\,\sigma_b$$

$$\therefore \quad \sigma_b=\frac{-\tau_{xy}}{\beta} \quad \ldots(9\cdot18)$$

Putting these values in Eq. (9·17)

$$-\sigma_b m=\frac{1}{m\,\tau_{xy}}\left\{\sigma_y\pm\left(\sigma_y{}^2+\tau^2_{xy}\right)^{\frac{1}{2}}\right\}$$

or $$m\,\sigma_b=\sigma_y\pm(\sigma^2_y+\tau^2_{xy})^{\frac{1}{2}} \quad \ldots(9\cdot19)$$

9·6·2. Equations Defining Fracture Initiation

Thus, the criteria for fracture to initiate at the boundary of elliptical flow is that when tangential stress σ_b reaches a value equal to the tensile strength of the material at that point. But local tensile strength of the material is difficult to measure Hence, the term $\sigma_b.m$

has to be expressed in the term of uniaxial tensile strength σ_t of the rock body in which the flaw exists. This is obtained when

$$\sigma_y = \sigma_t$$

and $$\tau_{xy} = 0$$

From Eq. (9·19)

$$\sigma_b m = 2\ \sigma_t \qquad \text{...(9·20)}$$

Putting this value in Eq. (9·19)

$$2\ \sigma_t = \sigma_y \pm (\sigma^2{}_y + \tau^2{}_{xy})^{1/2}$$

or $$2\sigma_t - \sigma_y = \pm(\sigma^2{}_y + \tau^2{}_{xy})^{1/2}$$

Squaring both sides,

$$4\sigma^2{}_t + \sigma^2{}_y - 4\sigma_t\sigma_y = \sigma^2{}_y + \tau^2{}_{xy}$$

or $$\tau^2{}_{xy} = 4\sigma_t(\sigma_t - \sigma_y) \qquad \text{...(9·21)}$$

The Eq. 9·21 is an equation of parabola in $\sigma_{xy} - \sigma_y$ plane and defines the relation between the shear and normal stresses at which fracture will initiate at the boundary of an elliptical flaw.

Since fracture initiates when tangential stress on the boundary of the flaw exceeds the total tensile strength of the material and the crack initiated will propagate in a direction which is normal to the

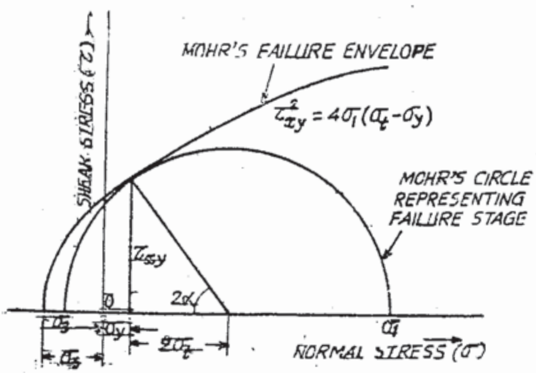


Fig. 9·3

boundary of the ellipse. The normal to the ellipse can be defined by the equation

$$\tan\theta = -\frac{dx}{dy}$$

where $$dx = -a \sin\beta \,.\, d\beta$$

$$dy = ma \cos\beta d\beta$$

$$\tan\theta = \frac{\tan\beta}{m}$$

Since β is very small $\tan\beta \rightarrow \beta$

$$\tan\theta = \frac{\beta}{m} \qquad \text{...(9·22)}$$

Since the inclination α of the elliptical flaw is such that the boundary stress σ_b is a maximum for any combination of the principal stress σ_1 and σ_3 then Eq. (9·21) is the equation of the failure envelope (Fig. 9·3) which will be tangential to the Mohr's Circle giving failure stages.

$$\therefore \qquad \tan 2\alpha = \frac{\tau_{xy}}{2\sigma_t}$$

Again since $\qquad \sigma_b m = 2\sigma_t$

$$= \frac{-m\tau_{xy}}{\beta}$$

$$\therefore \qquad \beta = \frac{-m\tau_{xy}}{2\sigma_t}$$

$$= -m \tan 2\alpha \qquad \text{...(9·23)}$$

From Eqs. (9·22) and (9·23)

$$\tan \theta = -\tan 2\alpha$$

or $\qquad \theta = -2\alpha$ or $(\pi - 2\alpha)$ $\qquad$ ...(9·24)

The condition is shown in Fig. 9·4.

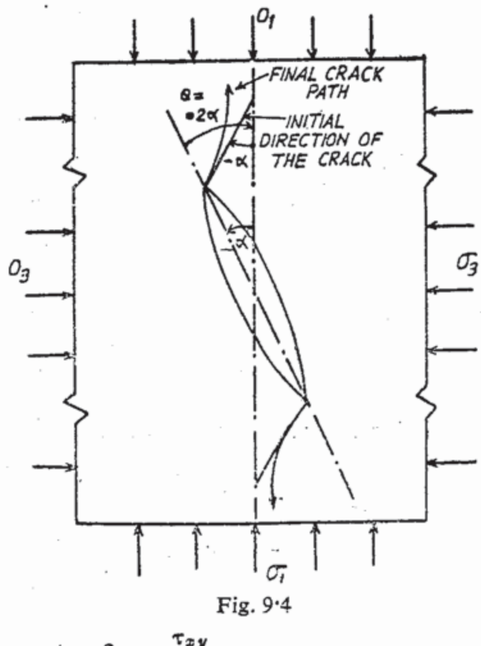


Fig. 9·4

Since $\qquad \tan 2\alpha = \dfrac{\tau_{xy}}{2\sigma_t}$,

therefore, till $\tau_{xy} > 0$, $\alpha > 0$ and so $\theta > 0$; therefore, the direction of the crack will try to reverse *i.e.*, negative to the value.

Hence, it will go out of direction at which it had initiated and this is the reason why all cracks existing in the rock mass do not propagate always in their direction of crack initiation. When $\tau_{xy}=0$, $\alpha=0$ and $\theta=0$ *i.e.*, in case of a uniaxial tensile stress to which the crack is perpendicular, a crack is initiated at the tip ($\beta=0$) of the elliptical flaw and propagates in the plane of initial flaw. This is the Griffith's theory. The pre-existing cracks or flaw is known as the "Griffith cracks." Griffith's theory states that macroscopic fractures start at pre-existing flaws (so called Griffith cracks) which enlarge and spread under the influence of applied stress. Thus, the brittle strength of a material can be calculated for any system of loading, if the shape and the size of Griffith cracks are known, of course, with certain material constants.

9·7. Relation between Grain Size and Strength of Rocks

According to the Griffith theory, fracture occurs when cracks of certain orientation and length (as discussed above) spread. For a tensile stress, fracture occurs when the stress exceeds the tensile strength T_0 given by Orowan (1949).

$$T_0=\left[\frac{2E.s}{\pi a}\right]^{\frac{1}{2}} \qquad ...(9·25)$$

where E=Young's modulus

s=specific surface energy of the material. It is the unit work, required to separate a crystal into two parts along a plane and depends on the atomic binding strength per unit of surface.

a=half length of the Griffith crack.

Orowan further states that at atmospheric pressure, the compressive strength C_0 is 8 times the tensile strength.

$$\therefore \qquad C_0=8\left[\frac{2E.s}{\pi a}\right]^{\frac{1}{2}} \qquad ...(9·26)$$

Since strength is inversely proportional to the square root of a crack length, the longest crack in the material determines its strength. Brace, (1961) on the basis of his extensive research with lime stone, concluded that the Griffith crack length is about the same size as the maximum grain diameter. This has been substantiated by Paterson who concluded that coarse-grained lime stone was found to be weaker than a fine-grained lime stone. Wood (1930) has reported that brittle strength of certain polycrystalline metals is inversely proportional to the squre root of mean grain diameter. Thus, in general, it can be concluded that strength of rock material depends upon its grain size.

9·8. Experimental Investigations of Rock Strength

Many investigators have investigated the mode of failure in rocks, testing the cylindrical rock samples in an unconfined compression or under a triaxial compression which is more realistic considering the rock existence in nature and the mode of application of stress in the rock mass. Griggs observed two types of failures—shear and tension. The shear surfaces gave the evidence of fracturing and powdering of the crystals. The slip planes, developed by shear failure were always at an angle with the direction of the axial load. The tension fracture occurred by splitting parallel to the direction of compression. No granulation was observed on the tensile fracture planes. The tension surfaces appear as a fresh break in the rock showing a clear separation of particles.

Griggs (1936) explained the tension or splitting phenomenon as the result of the formation of wedge shaped failure planes at the top and the bottom of the sample. If the confining pressures in the triaxial tests were of low magnitude, the wedges caused the sample to split because of the tension developed between the grains at the point of the wage. Careful observation, with a high speed movie film confirmed the hypothesis that the slip or shear surfaces appear prior to the tension fracture. Terzaghi also presented a similar theory for splitting failure, stating that tension between two grains is due to the wedging action of a third grain trying to push its way between them. Shear failure produces a surface which is covered by granulated particles of variable size. Failure by a combination of shear and tension is also there.

9·8·1. Failure in Rock

Failure in Rock is classified by Terzaghi as splitting. shear and pseudo-shear depending on the inclination of the failure planes. Splitting is recognised by cracks appearing parallel to the direction of the axial local which indicates that the bond between grains fails by tension. Shear failure occurs when grains and bonds alike are displaced along a glide plane. Pseudo-shear failure represents a combination of shear and tension fracture.

Mechanism of tension fracture has already been explained. A shear failure depends upon the shear strength of the material which is influenced by :

(*i*) cohesion

(*ii*) internal-friction and dilatancy.

Cohesion is defined as the inherent shear strength of a material in the absence of external stresses. Physically it is the resistance to particle separation without the influence of any external forces or pressures This resistance to separation consists of molecular bonding ionic attraction and particle interlocking. The value of cohesion is obtained by intercept of the Mohr's envelope at a zero normal stress.

The term dilatancy refers to a volume change which is due to one particle blocking the path of another as slip is initiated on a glide plane. Dilatancy is also referred as particle interferénce.

Dilatancy and internal friction cannot be evaluated separately, because they depend on each other. The non-metallic brittle materials consist of crystalline solids which may be considered as heterogeneous bodies and must have an interior surfaces. Each crystal may be considered as an independent cell or element. The frictional strength of the entire aggregate of crystals is developed when one crystal face is pushed against an adjacent crystal face.

9·8·2. Behaviour of Brittle Materials

A brittle material is defined as one which accepts only a limited amount of strain after yielding and before its rupture.

When the surrounding pressure is increased sufficiently high, a brittle material changes to a ductile one. This happens because as the confining pressure increases, splitting failure is prevented and shear fracture predominates. The transition, which occurs, is in progression from tension failure to pseudo-shear and finally to shear failure. The tension produced between the grains is gradually reduced by the superposition of hydrostatic pressures untill the sum of the forces between the grains is totally compressive. The tensile cracks are prevented in the same manner as prestressing prevents tensile failure in a structural concrete. If splitting is completely prevented (by a very high confining pressure) the samples fail by shear and produce the slip planes. Failure by pseudo-shear takes place when splitting is prevented partially.

9·8·3. Conclusion on Strength and Failure Criteria of Rocks

Following conclusion can be made finally on the strength and failure criteria of rocks.

(*i*) Rocks fail by splitting, shear or combination of these, known as pseudo-shear.

(*ii*) Failure of rock is either ductile or brittle and it depends upon the amount of confining pressure.

(*iii*) Shear failure occurs in a rock, if confining pressures are sufficiently high to prevent splitting.

(*iv*) The angle of slip for shear failure is predicted by the Mohr's criterion.

(*v*) Shear-strength of rock is a function of cohesion, internal friction and "fracture interference" also known as dilatancy.

(*vi*) Fracture interference is a unique function of a normal stress.

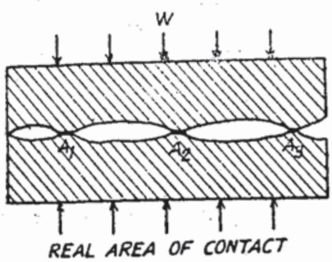
(*vii*) For a brittle failure in rock mass, the Griffith's theory is most suitable to explain the behaviour.

(*viii*) As per Griffith's theory, the strength of rock depends on pre-existing flaw in the rock mass, size of which depends on the grain size of the rock. Therefore, it is inferred that the strength of rock depends upon its grain size.

9·9. Strength Criteria of Jointed Rocks

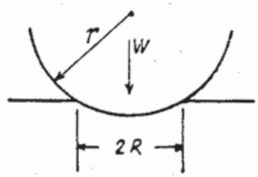
The behaviour of jointed rock in-situ depends upon the friction and shear on the joints and the deformation originating in the joints.

The systems of joints or cracks contain hills and valleys and the solid surfaces in contact are supported on the top of these irregularities as shown in Fig. 9·5. Therefore, the actual area of an intimate contact is very small compared to the total area. Therefore, the pressure applied is not acting on the whole area but at the points of



Area of contact $=\Sigma A_1+A_2+A_3+\ldots$

(*a*)



(*b*)

Fig. 9·5

contact only, and under these localised intense pressures at a point of contact, deformation occurs. Let us assume that the tip of a particular asperity is spherical of radius *r* as shown in Fig. 9·5 (*b*). Then as per Hertz's solution for an elastic deformation.

$$R=C_1W^{1/3} \qquad \ldots(9{\cdot}27)$$

where W = load on the asperity

R = the radius of the circle of contact

and C_1 = constant which is a function of r, the radius of tip of the asperity and elastic moduli of the material.

The area of contact A

$$=\pi R^2$$

$$=\pi C_1^2 W^{2/3} \qquad \ldots(9{\cdot}28)$$

Therefore, the average pressure P_a on the area of contact is given by

$$P_a=\frac{W}{A}$$

$$=\frac{W}{\pi C_1^2 W^{2/3}}$$

$$=\frac{W^{1/3}}{\pi C_1^2} \quad \ldots(9.29)$$

When the load is increased, the mean pressure increases and the material where the stress is maximum ceases to be elastic and yield occurs. This yield occurs at point P which is about 0·5 R below the circle of contact. When the load is increased further, the plastic zone grows rapidly and at a particular stage the average pressure P_a becomes approximately constant as shown in Fig. 9·6. At this point, the whole region of contact deforms permanently. After this stage,

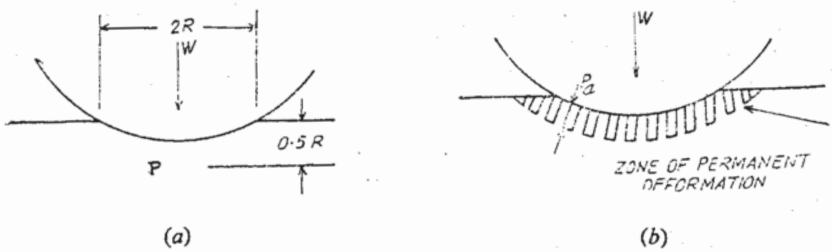


Fig. 9·6

although area A increases, the average pressure P_a remains constant at a value P and the real area of contact is almost proportional to the load. Therefore,

$$A=\frac{W}{P}$$

where P is known as mean flow pressure of the asperities.

9·9·1. Failure Propagation of Jointed Rocks

Bowden and Tabor (1954) with their experiments have concluded that with two surfaces in contact, a high pressure, which acts at the summits of the surface irregularities, causes a strong adhesion of the surfaces over the actual area of contact. When sliding takes place, work is required to shear these joints and also to plough out the softer of the materials by the harder asperities of the other surfaces.

Therefore $F_f = P_l + S_f$ ...(9·28)

where F_f = friction force required to slide the surfaces over one another.

P_l = ploughing force required to displace material in front of the asperities.

S_f = shearing force to shear junctions of the real area of contact.

For relatively smooth surfaces, the ploughing force P_l is smaller compared to S_f and may be neglected.

$$\therefore \quad F_f = S_f = As = W\frac{s}{p} = W\mu \quad \ldots(9.29)$$

where s=shear strength of the junction
μ=coeff. of friction
p=flow pressure of the material.

But, rock has got rough surfaces and hence, the plowing force term cannot be neglected. For a relative movement, there is a complex situation where both shearing of junction and ploughing are occurring simultaneously and junctions are continuously being made and sheared and elastic as well as plastic deformations are taking place. For a given condition, P_l can be expressed as

$$P_l = Kp \quad \ldots(9.30)$$

where K=a constant depending on the geometric characteristics of the surface and asperities

Eqs. 9·28, 9·29 and 9·30 are based on the use of a real area of contact which is a fraction of the gross area. If the forces F_f, P_l and W are taken as average stresses over the gross unit area, then we can write that

$$\tau = \tau_0 + \sigma\mu \quad \ldots(9.31)$$

where τ=shearing stress per gross unit area
τ_0=shearing stress depending on the geometric characteristics of the surface and asperities and flow pressure of the material. It is approximately constant and is independent of normal stress.
σ=normal stress on gross unit area
μ=coeff. or friction and is approximately dependent on "s" (the shear strength of the junction) and p (the flow pressure of the material).

Eq. (9·31) is of the same type as expressed by Mohr's failure criteria for the on-set of a permanent deformation.

Now, we shall see how fractures propagate when a load is applied on the jointed rock mass. In practical cases, there is a wide range in the size of the asperities which constitute the real area of contact.

In Fig. 9·7 (*a*) the small asperities are only deformed plastically due to a small load and the major part of the deformation is elastic. In parts *b* and *c* of the figure further stages in the deformation process are shown due to an increase in load. As the load is increased plastic deformation is more and more. The real area of contact, which is flattened tips of the asperities, is very less than the area over which macroscopic deformation has taken place. The permanent deformation of the asperities in fact provides the real area of contact

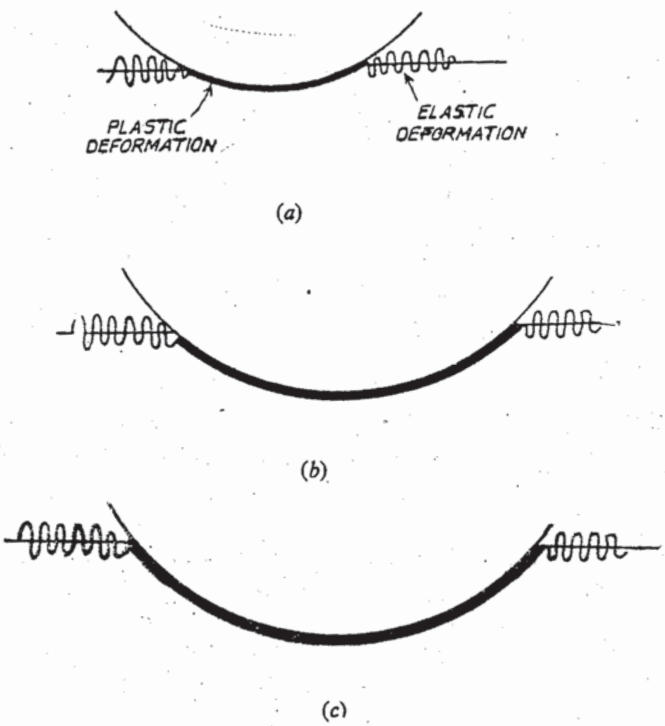


Fig. 9·7. Elastic to plastic deformation of a small asperity.

which supports the load while the stresses in the asperities are taken up by an elastic deformation of the underlying material as shown in Fig. 9·8. If the load will be reduced, then due to an elastic relaxation of the deformation the junctions will be progressively broken.

It has already been explained that the friction between the surfaces depends on the intimacy of contact, the flo v pressure, the shear strength of the junctions and the resistance to plowing. The strength of the junctions shown is higher if the rate of application of load, that is the speed of shearing, is very low. This happens, because as sliding begins, the material displaced by plowing accumulates in front of the asperities and causes the force to increase which resists the motion.

9·9·2. "Stick-slip" Process of Rupture

If one of the surfaces has a certain degree of elastic freedom, then the motion is not continuous but intermittent. The process of rupture has been described by as "stick-slip". The "stick" is described due to the higher static friction between the surfaces and the "slip" is due to a lower kinetic friction during the slipping process itself.

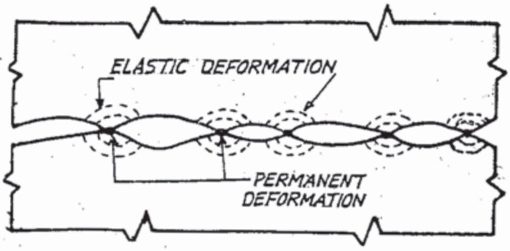


Fig. 9·8. Sequence of elestic and plastic deformation in a jointed rock mass.

If "slip" occurs, there is a sudden release of stresses and, therefore, energy also and that is why, brittle materials, such as rock, slide with jerks.

This "stick-slip" behaviour is observed in the field when it is found that after excavation in the rock mass, the "delayed fall-out" of the rock occurs.

9·9·3. Barton (1973) has suggested an empirical relation to describe the shear strength of any joint along the jointed surface.

$$\tau=\sigma_n \tan\left((JRC \log_{10} \frac{JCS}{\sigma_n}+\phi_b\right) \quad ...(9·32)$$

where τ=shear strength of the joint

σ_n=normal stress acting in the joint surface

JRC=joint roughness coefficient

JCS=joint wall compressive strength which can be determined by Schmidt hammer test and has the upper limit as σ_c for fresh surfaces

ϕ_b=base angle of friction.

9·9·4. Conclusion

On the basis of different investigations by different investigators, it has been concluded that the roughness also has a pronounced influence on the strength of a rock joint. The rougher the joint, the higher is the shear strength.