

CHAPTER - I

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CONCEPTUAL FRAMEWORK

1.0 INTRODUCTION

The evolution of Mathematics might be seen as an ever-increasing series of abstractions or alternatively an expansion of subject matter as it is learnt from the **Wikipedia (2015a)** about Mathematics. The first abstraction was probably that of numbers: the realization that a collection of two apples and a collection of two oranges (for example) have something in common, namely quantity of their members. In addition to recognizing how to count physical objects, prehistoric peoples also recognized how to count abstract quantities, like time – days, seasons, years. Elementary arithmetic (addition, subtraction, multiplication and division) naturally followed. These ways many other areas and aspects of the Mathematics were identified and taken important place in real life world over the period of time. Hence, then it is realized and valued as how essential is to be acquainted with Mathematics and to make as indispensable part of Education.

Looking to the nature of Mathematics as according to the **National Council of Educational Research and Training (NCERT) (2010)** that Mathematics reveals hidden patterns that help us to understand the world around us. Much more than arithmetic and geometry, Mathematics today is a diversified discipline, which deals with data, measurement, observations, deduction & proof, mathematical models, natural phenomena, human behaviour and social systems. Thus, Mathematics has occupied key part of an education system of any country.

According to the **Ramanujam (2012)**, as stated in a report on ‘Mathematics education in India– An overview’ that like other modern nations, India is also concerned in building a mathematically literate society and hope for strong mathematical elite that can shape the knowledge economy of the 21st century. At the same time, mathematical proficiency is universally considered hard to achieve. Post-independence in India, the **Government of India (GOI) (1953)** through the report on **Secondary Education Commission (SEC) (1952)** significantly emphasised the need

for Mathematics as a compulsory subject in the schools with the consideration that Mathematics is used throughout the world as an essential tool in many fields, including natural science, engineering, medicine, and the social sciences.

As Mathematics was seen to be an essential part of any curriculum from early with different perspectives is referred to as Mathematics Education and Mathematics Education is means a way of practicing teaching and learning of Mathematics. As mentioned on **Wikipedia (2015b)** about Mathematics Education, at different times in different cultures and countries, Mathematics education has attempted to achieve a variety of various objectives which are in general stated as: (a) The teaching and learning of basic numeracy skills to all pupils; (b) The teaching of practical Mathematics (arithmetic, elementary algebra, plane and solid geometry, trigonometry) to most pupils to equip them to follow a trade or craft; (c) The teaching of abstract Mathematical concepts (like set and function) at an early age; (d) The teaching of selected areas of Mathematics (such as Euclidean geometry) as an example of an axiomatic system and a model of deductive reasoning; (e) The teaching of selected areas of Mathematics (such as calculus) as an example of the intellectual achievements of the modern world; (f) The teaching of advanced Mathematics to those pupils who wish to follow a career in Science, Technology, Engineering and Mathematics (STEM) fields; and (g) The teaching of Heuristics and other problem-solving strategies to solve non-routine problems. With such objectives, School Mathematics Education in general has been designed to teach and make students to be competent with logical abilities, abstract thinking and real life problem solver.

With the arrival of the 21st Century, drastic change propagated in aims and goals with the demands to laid more focus towards the higher level objectives underlying with Mathematics subject too like constructivist approach, innovative methodologies, ICT integration etc. in order to enhance the criteria for understanding, problem solving, decision making, critical, creative, logical and analytical thinking. Moreover, exploration of subject with the constructivism is now more appreciated which means an approach by which children discover and construct their knowledge, rather than it being simply given and taken uncritically. In mathematics, for example, this means that children's ability to come up with a formula is more important than being able to correctly use well known formulae. And hence, novelty approach, constructivist

approach or the innovative approach is always expected from the teachers in terms to reduce the boredom or the mechanical way of teaching-learning and moreover to sustain the interest of the learners with ease understanding and higher thinking.

Flowing with such aforesaid thoughts, the researcher made a search on the means for how to enhance and encourage for the Mathematics Education as well how to make it simple in terms to heighten the interest, understanding and learning of the learners. Then, the researcher had found and further investigated through many research reviews about the new phenomenon constituted with the concepts of *constructivism* and *comprehension*, that is the “**S.O.L.O. Taxonomy**” where **S.O.L.O. (SOLO)** stands for the “**Structure of Observed Learning Outcomes**”. This taxonomy is developed to study the *outcomes of academic teaching-learning* which is explained in detail in section on ‘SOLO Taxonomy’.

From the several reviews of the literatures (mentioned in next Chapter-2) on the SOLO taxonomy, it was derived that SOLO is a hierarchical model that is suitable for measuring learning outcomes of different subjects, levels and for all the lengths of assignments. Several researchers who have applied SOLO Taxonomy, credit its comprehensiveness in application and its objective criteria provided for measuring students’ cognitive attainment. They did try to make use of SOLO’s advantages & applied it to different subjects such as Mathematics, Biology and language studies. In order to achieve the improved quality of examinations; SOLO taxonomy is being used in many countries. In Australia, it is being practiced very successfully in various examinations. The researcher of the present study had not come across any study on the application of SOLO Taxonomy in India and hence, researcher has opted this research work to study such application in the context of Indian Education as well in the educational settings. Ahead, more explanations about Mathematics and SOLO taxonomy have been given.

1.1 MATHEMATICS EDUCATION IN INDIA: AN OVERVIEW

In India since the ancient time period the practice of education was well established with the core components of Arithmetic and Astronomy for the study as it is learnt from **Wikipedia (2015a)** about Mathematics. Geometry was taught because it was required for the construction of sacrificial altars and ‘havan kund’s of various shapes

and sizes. Perhaps interest in Mathematics was rudimentary-means ‘the different kinds of numbers, various shapes and sufficient astronomy to help to determine the dates of religious rituals’. With such features, Mathematics gradually made vital place in many areas of our today’s life and hence, Mathematics being so important today to know and to learn.

Earlier numbers and shapes as the components of Astrology as well Astronomy were comprised as the areas of Mathematics. Then, over the time periods other areas based on the structures, logic, patterns and computation were identified and classified into the terms as Algebra, Geometry, Measurement, Analysis, Statistic, Calculus etc. as the later areas of the Mathematics. And thus, more classifications were took place time to time. General classification shown in a figure-1.1 is representing the basic areas of Mathematics discipline and further classification of the same.

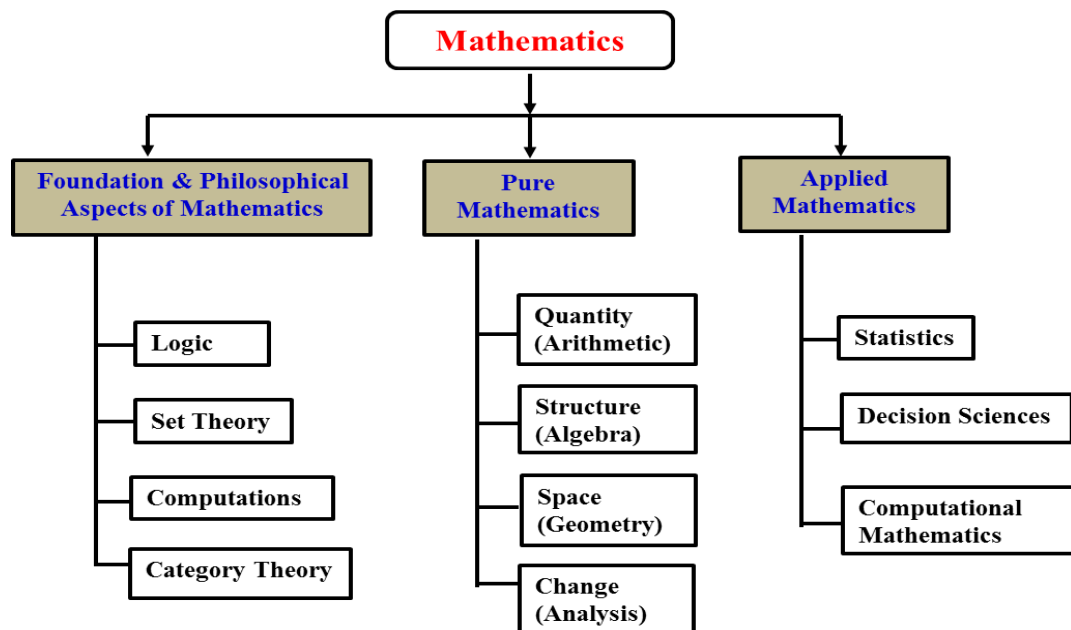


Figure – 1.1: Areas and further classification of Mathematics as a discipline
 (Source: Wikipedia – <https://en.wikipedia.org/wiki/Mathematics>)

Looking to such nature and aspects of Mathematics, it could be basically classified with various areas like Quantity (Arithmetic), Structure (Algebra), Shapes (Geometry) and Change (Analysis). Then again this discipline is re-categorised with three major fields as (i) Foundation and Philosophical aspects of Mathematics, (ii) Pure Mathematics and (iii) Applied Mathematics. And further these fields are comprised with sub-areas and applications of Mathematics which are shown in the above figure.

These sub-areas are logically connected with numerous aspects of our daily life and hence are covered in school or college education to learn.

Moreover, **NCERT (2010)** stated and described the Mathematics as a discipline in a practical matter as; Mathematics is a science of patterns and order. Its domains are numbers, chance, form, algorithm and change. In ancient India, Mathematics was the subject of shapes & sizes used to design various structures. Whereas now, Mathematics is considered as discipline of reasoning as it relies on logic rather than on observation. It's standard of truth; simulations and even experimentation are as means of discovering the truth.

Though, the areas of Mathematics are comprised in the curriculums spiralled into levels of primary education to tertiary education and also keep changing time to time in terms to update at any level of Mathematics Education as per the trend, demand or necessities. As **Ramanujam (2012)** mentioned that in response to global curricular processes in India too there has been considerable curricular acceleration in school Mathematics. For instance, calculus which was only taught in college three decades ago is taught now at the higher secondary level. On the other hand projective geometry has almost entirely disappeared from the school. At the undergraduate level, the core curriculum remains much the same, though the influence of computer science and other modern disciplines can be seen in the course mix on offer.

Also it is important how to impart education of such discipline like Mathematics at any level of education. Majorly it is concerned with the teaching or teaching methods. The Teaching methods used in any particular context are largely determined by the objectives that the relevant educational system is trying to achieve. It is derived from the **Wikipedia (2015b)** about Mathematics Education is that in conventional or traditional manner, following approaches or methods are generally practised in one or other ways for Teaching Mathematics:

- ❖ **Conventional Approach:** The gradual and systematic guiding through the hierarchy of Mathematical notions, ideas and techniques. Starts with Arithmetic and is followed by Euclidean geometry & Elementary Algebra taught concurrently.

- ❖ **Classical Education:** The teaching of Mathematics, part of the classical education curriculum of the Middle Ages, which was typically based on Euclid's *Elements* taught as a paradigm of deductive reasoning.
- ❖ **Rote Learning:** The teaching of Mathematical results, definitions and concepts by repetition and memorisation typically without meaning or supported by Mathematical reasoning. A derisory term is *drill and kills*. In traditional education, rote learning is used to teach multiplication tables, definitions, formulas, and other aspects of Mathematics.
- ❖ **Exercises:** The reinforcement of Mathematical skills by completing large numbers of exercises of a similar type.
- ❖ **Problem Solving:** Problem solving is used as a means to build new Mathematical knowledge, typically by building on students' prior understandings. The problems can range from simple word problems to problems from International Mathematics Competitions such as the International Mathematical Olympiad.
- ❖ **New Math:** A method of teaching Mathematics which focuses on abstract concepts such as Set theory, Functions and Bases other than ten (logarithm). In the new approach, the important thing is to understand what you're doing, rather than to get the right answer."
- ❖ **Historical Method:** Teaching the development of Mathematics within a historical, social and cultural context. Provides more human interest than a conventional approach.
- ❖ **Relational Approach:** Uses class topics to solve everyday problems and relates the topic to current events. This approach focuses on the many uses of Mathematics and helps students understand why they need to know it as well as helping them to apply math to real world situations outside of the classroom.
- ❖ **Recreational Mathematics:** Mathematical problems that are fun can motivate students to learn Mathematics and can increase enjoyment of Mathematics.
- ❖ **Computer-based Mathematics:** An approach based around use of Mathematical software as the primary tool of computation.

These are the approaches are practised based on the Mathematical topics, but all these approaches are conventional methods which are now needed to shift over to active and constructivist methods to make teaching an effective technique to enhance the learning of Mathematics as also emphasised by the policy on education of India.

Further, more on policy aspects on education is focused in next section where more is on Mathematics Education merely on School Mathematics Education, as present research study is based on the secondary school Mathematics education.

1.2 SCHOOL MATHEMATICS EDUCATION: A POLITY PERSPECTIVE

The Right of Children to Free and Compulsory Education Act (abbreviated as Right to Education or RTE Act) came into force in India as on April 1, 2010. It guarantees 8 years of elementary education to every child in the age group 6-14 in an age appropriate classroom in the vicinity of his/her neighborhood. This implies the right of every Indian child to quality mathematics education as well.

The **National Policy of Education (NPE) (1986)** mentioned with reference to inculcate the scientific attitude among the children, more importance has been given to the Science and specially for Mathematics education as: “Mathematics should be visualized as the vehicle to train a child to think, reason, analyze and to articulate logically. Apart from being a specific subject, it should be treated as a concomitant to any subject involving analysis and reasoning”. With this major thoughts, some more aims or goals; challenges and recommendation about the School Mathematics Education derived from the policies are paragraphed here.

1.2.1 Aims Of School Mathematics Education

The School Mathematics Education is as the important foundational beginning of the life-long learning and hence, the aims are framed to achieve the desired goals made for the targeted learners. Several aims framed for the policies are highlighted here.

The **National Council of Teachers of Mathematics (NCTM) (1989)** produced the *Curriculum and Evaluation Standards for School Mathematics* (subsequently known as the *Standards*) were a philosophical as well as a curricular document used to judge the quality of a Mathematics curriculum or methods of evaluation were oriented toward “five general goals for all students as that: (1) they learn to value Mathematics; (2) they become confident in their ability to do Mathematics; (3) they become Mathematical problem solvers; (4) they learn to communicate Mathematically; and (5) they learn to reason Mathematically”. Another goal was to “create a coherent vision of what it means to be Mathematically literate” in a rapidly changing world,

and to “create a set of standards to guide the revision of the school Mathematics curriculum”. Philosophically focused on the new goals for students and society which was included as (1) Mathematically literate workers; (2) lifelong learning; (3) opportunity for all and (4) an informed electorate. These goals are also of the concern with the present scenario.

According to **NCERT (2006)**, the main goal of Mathematics Education in schools is the Mathematisation of the child’s thinking and visualised as the school Mathematics should takes place in a situation where: (1) Children learn to enjoy Mathematics, (2) Children learn important Mathematics, (3) Mathematics is a part of children’s life experience which they talk about, (4) Children pose and solve meaningful problems, (5) Children use abstractions to perceive relationships and structure, (6) Children understand the basic structure of Mathematics and (7) Teachers expect to engage every child in class.

To achieve these goals are not easy task as it consist of equivalent challenges or hurdles too that should be addressed is the major concern. Though, the major challenges and simultaneously key recommendation to deal with such challenges derived from the policies are stated in a next sub-section.

1.2.2 Challenges For School Mathematics Education

As the **National Knowledge Commission (NKC) (2007)**, also has given stress on Science and Mathematics education at school level to address the problems which are drawn as findings well established as: (i) Science and Mathematics teaching and research has deteriorated in India; (ii) Fewer students are attracted to a career in Science or Mathematics, compared to other professional subjects and (iii) Availability of good teachers and absence of modern pedagogy are key limiting factors in the ability of schools and Universities to make Science and Mathematics exciting.

Generalised challenges are elaborated in the position paper by **NCERT (2006)** in a manner that schooling is a legal right, and Mathematics being a compulsory subject of study. Access to quality Mathematics education is every child’s right. Keeping in mind the Indian reality where few children have access to expensive material. Thus efforts are going on for Mathematics education that should be affordable to every

child and at the same time enjoyable. Any analysis of Mathematics education in schools will identify a range of issues as problematic. The understanding of these issues are structured around the following four problems which deem to be the core areas of concern: (1) A sense of fear and failure regarding Mathematics among a majority of children; (2) A curriculum that disappoints both a talented minority as well as the non-participating majority at the same time; (3) Crude methods of assessment that encourage perception of Mathematics as mechanical computation; and (4) Lack of teacher preparation and support in the teaching of Mathematics. To meet with these challenges, some recommendations are also provided in the policies are briefed in next paragraph.

1.2.3 Recommendations For School Mathematics Education

Recommendations formulated strategically and stated in the **NCERT (2006)** to address the challenges derived by the critical analysis on school Mathematics education are outlined here as: (a) Shifting the focus of Mathematics education from achieving ‘narrow’ goals to ‘higher’ goals; (b) Engaging every student with a sense of success, while at the same time offering conceptual challenges to the emerging Mathematician; (c) Changing modes of assessment to examine students’ Mathematization abilities rather than procedural knowledge and (d) Enriching teachers with a variety of Mathematical resources. Again it’s a challenge for the school Mathematics practitioners those are working at the fields or ground levels to fulfil as well to deal with the stated challenges and recommendations in efforts to maintain with the stated aims of the policies. Such more aspects are focused precisely for the secondary school Mathematics education in the next section.

1.3 FOCUSING ON SECONDARY SCHOOL MATHEMATICS EDUCATION

Secondary School Mathematics Education is means of practising teaching and learning of Mathematics at secondary level of school education. For universalization of access to and improvement of quality at the secondary and higher secondary stage, a scheme under the title **Rashtriya Madhyamik Shiksha Abhiyan (RMSA)** has implemented. As **RMSA (2009)** enlightened the importance of secondary school education as well as of Mathematics Education. Some of the objectives and structural aspects are presented below.

1.3.1 Significance Of The Secondary School Mathematics Education

According to the **RMSA (2009)**, Secondary Education is a crucial stage in the educational hierarchy as it prepares the students for higher education and also for the world of work. Classes IX and X constitute the secondary stage (and classes VIII-X in some states). The normal age group of the children in secondary classes is 14-16 (13-16). The population of the age group 14-18 was 8.55 crore in 2001 as per census data. The estimated population of this age group as on 1.3.2005 was 9.48 crore, which is likely to increase to 9.69 crore as on 1.3.2007 i.e., at the beginning of the 11th Five Year Plan. This is likely to stabilize at around 9.70 crore in 2011.

According to the report of **WorldBank (2009)** on ‘Secondary Education In India: Universalizing Opportunity’, mentioned about the Actual and Projected Demand for Secondary Education for 1990–2020 that at the lower secondary level (grades 9 and 10), the Gross Enrollment Rate (GER) is 52 percent, while at the senior secondary level (grade 11 and 12) it is just 28 percent and for a combined GER is of 40 percent (2005). Projections suggest an increase in absolute demand for secondary education between 2007/08 and 2017/18 of around 17 million students per year, with total enrollment growing from 40 to 57 million students. The number of students finishing upper primary education has been increasing at over five per cent per year since 2001; this is projected to continue through 2014 with increased elementary enrolments linked to Sarva Shiksha Abhyihan (SSA), the Government of India’s massive centrally sponsored scheme for elementary education. Also, stated in a report that India’s impressive as well sustained economic growth has increased household and labour market demand for secondary and higher education. Secondary education’s contribution to economic growth demonstrated high social benefits (particularly for girls) and support of democratic citizenship reinforce the need for increased public support at this level, particularly in light of the very large inequalities in access to secondary education, by income, gender, social group and geography. Following is the graphical presentation (figure-1.2) showing the Actual and Projected Demand for Secondary Education for 1990–2020 in India.

Further, the **RMSA (2009)** stated as the rigor of the secondary and higher secondary stage, enables Indian students to compete successfully for education and for jobs globally. Therefore, it is absolutely essential to strengthen this stage by providing

greater access and also by improving quality in a significant way. It is also necessary that besides general education up to secondary level, opportunities for improvement of vocational knowledge and skill should be provided at the higher secondary level to enable some students to be employable.

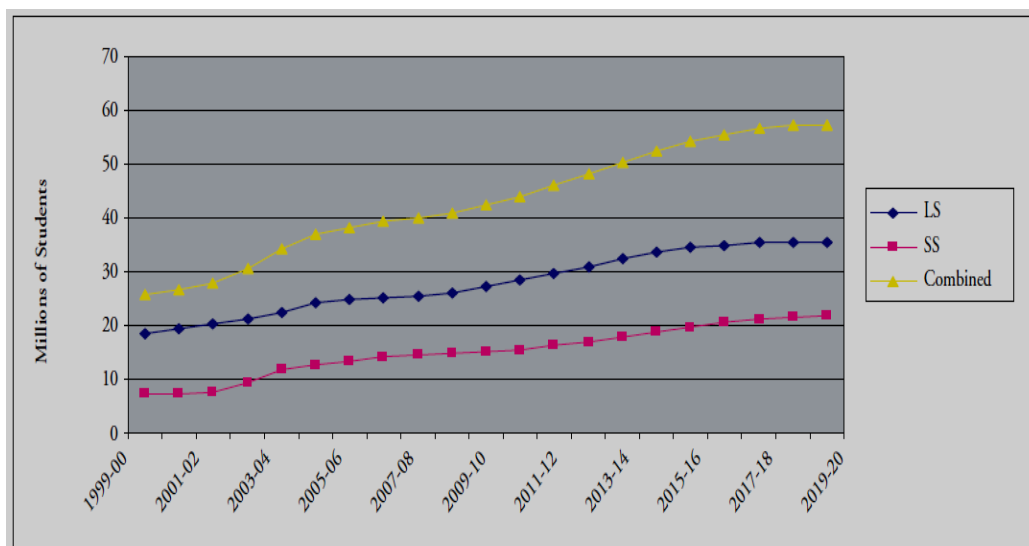


Figure-1.2: Actual and Projected Demand for Secondary Education, 1990–2020

(LS: Lower Secondary, Grades 9–10; SS: Senior Secondary, Grades 11–12.

Source: *Selected Education Statistics, 2004-05 and author's calculations*)

Looking to such flow of the students to the secondary school level as well as to the importance of learning at this level directs to have more focus in terms to improve the quality of education as well need to have major concern for the quality of Science and Mathematics education. Also, it is now to concern about to identify the actual learning components of Mathematics at school or higher level and reforms are needed in the context of the applicability as well employability of Mathematics learning. Next subsection is enlightening the key objectives of Mathematics education at the secondary school level.

1.3.2 Objectives Of The Secondary School Mathematics Education

The key objectives derived for Secondary School Mathematics stated in a report by **RMSA (2009)** viewing with the concern as the secondary school Mathematics curriculum continues the development of the learning of Mathematics in the primary school. To enable students to cope confidently with the Mathematics needed in their future studies, workplaces or daily life in a technological and information-rich society, the curriculum aims at developing students: (i) the ability to conceptualize, inquire,

reason and communicate Mathematically, and to use Mathematics to formulate and solve problems in daily life as well as in Mathematical contexts; (ii) the ability to manipulate numbers, symbols and other Mathematical objects; (iii) the number sense, symbol sense, spatial sense and a sense of measurement as well as the capability in appreciating structures and patterns; (iv) a positive attitude towards Mathematics and the capability in appreciating the aesthetic nature and cultural aspect of Mathematics. These key-objectives have been considered to imbibe with the curriculum. Here in the next sub-section, have a look on the curricular features of the Mathematics education at secondary school level.

1.3.3 Curricular Features Of The Secondary School Mathematics Education

The curricular aspects for the secondary school Mathematics education drafted by The **National Curriculum Framework for School Education (NCFSE)** by **NCERT (2000)** as at the secondary stage, the student begins to perceive the structure of Mathematics. For this, the notions of argumentation and proof become central to curriculum. Mathematical terminology is highly stylised, self-conscious and rigorous. Algebra, introduced earlier, is developed at some length at this stage. Proofs in geometry and trigonometry show the usefulness of algebraic machinery.

A substantial part of the secondary Mathematics curriculum can be devoted to consolidation. This can be and needs to be done in many ways as: (i) the student need to integrate the many techniques of Mathematics that has learnt into a problem solving ability. E.g. a need for posing problems to students which involve more than one content area: algebra and trigonometry, geometry and mensuration, and so on. (ii) Mathematics is used in the physical and social sciences, and making the connections explicit can inspire students immensely. (iii) Mathematical modelling, data analysis and interpretation, taught at this stage, can consolidate a high level of literacy. E.g. consider an environment related project, where the student has to set up a simple linear approximation and model a phenomenon, solve it, visualise the solution, and deduce a property of the modelled system.

The consolidated learning from such an activity builds a responsible citizen, who can later intuitively analyse information available in the media and contribute to democratic decision making.

At the secondary stage, a special emphasis on experimentation and exploration is given. As per the **NCERT (2006)**, Mathematics laboratories are a recent phenomenon, which hopefully will expand considerably in future. Activities in practical Mathematics help students immensely in visualisation. Looking to these directives, it is essential to bring changes in teaching-learning process and deep thinking is required that how and where to change it. For such quality-changes it needs to work on the problems and challenges that are responsible for the non-attainment of teaching and learning up-to the marks. Some of the insights about the actual problems or challenges with Mathematics teaching-learning are explored in the next section.

1.4 MATHEMATICS TEACHING AND LEARNING: THE CONCERNS

According to **Tella (2008)**, all decisions taken are based on such questions as what and how these questions are best answered by converting every statement to Mathematical statement before solution is sought. Acquiring such skills of problem solving directs to emphasis on the quality teaching – learning process. In the classroom context, quality of teaching–learning can be fruitfully improved by encouraging major two components: (i) Learners’ Learning, (ii) Teachers’ Teaching. As following sections have the brief descriptions basically about the Mathematics Learning and Mathematics Teaching and the problems as well challenges encountered with both.

1.4.1 Mathematics Learning: The Problems

In Mathematics, concepts are structured in spiral and hierarchical manner. According to **NCERT (2010)**, many concepts in Mathematics are needed to be learnt sequentially only. That means only after mastering arithmetic than algebra is learnt, and only when one can factor polynomials, and than is able to understand trigonometry and so on. Thus, since each theme is built on another results in a ‘Tall Shape’. This makes it difficult for children as someone who finds one stage difficult finds it hard to catch up later. In such matter it’s really need to redesign the aims and objectives accordingly to deal with the tall shape learning. Also it needs to stress on Mathematics teaching to inculcate deep understanding among the learners.

According to **Sensarma (2007)**, Mathematics has occupied a central place in curriculum since antiquity but Mathematical pedagogy has not. Mathematics has been

accepted as a compulsory subject at the Secondary Level with the hope that it will inculcate some minimum basic skills to every future citizen of the country to ensure their future happy life. But at present none of the consumer, the dispenser or the producer of knowledge of Mathematics education is satisfied with students' achievement in Mathematics. So ultimately, our education system directly or indirectly is responsible for maiming our future generation by not providing them with proper Mathematical knowledge at the Secondary Level. Moreover, **Piaget's (1973)** revolutionary finding state that: 'Every normal child is capable of learning Mathematics' has put greater responsibility on dispensers of Mathematical knowledge and producers of knowledge of Mathematics education, which they cannot escape by passing the buck to the poor Mathematical ability of the students.

According to **Education Initiatives (EI) (2010)**, understanding of Mathematics in primary classes is largely limited to 'procedural or rote-based learning' and in fact falling averages as we move from the primary to the elementary classes to so on indicate an increase in the level of incomprehension for children. Mostly during the Mathematics learning, the students at the initial steps of the logical explanations trying to understand and grasp but slowly the gap is created between the explanations transmitted by teacher and received by students which lead to the poor understanding on part of students and they develop a fear of the subject-"Mathsphobia". And this way somehow mechanical teaching-learning / rote-learning get practiced generally. Such phenomenon is regularised normally in terms to get time being academic success which keeps aside/ignore the actual aim/objective set by the **Education Commission (1964-66)** by **NCERT (1970)** that "In the teaching of Mathematics emphasis should be more on the understanding of basic principles than on the mechanical teaching of Mathematical computations".

Regarding to Mathematics learning, an important consequence of directing our attention towards the assessment of complex outcomes should be that to change the focus of assessment from quantity to quality. As per the **Killen & Hattingh, (2004)**, our focus should be changed from asking 'How many objective questions can the learner answer?' or 'Which particular skills can the learner demonstrate?' to the asking 'How well does the learner understand important concepts, theories and principles?' and 'How expertly can the learner integrate a range of skills into a

complex performance?' Descriptions of the difference between high-quality and low-quality achievement of complex outcomes should be in words rather than numbers can provide criteria by which to judge the quality of students' learning. From this quality perspective, 'understanding (rather than memorisation), creativity (rather than reproduction), diversity (rather than conformity), initiative (rather than compliance) and challenge (rather than blind acceptance) should become the yardsticks by which we need to measure, describe and report student learning.

Thus, it points to notice the neo-Piagetian approach to cognitive development because it provides a feasible explanation of learning across a wide range of situations (school, university, different subjects, and so on). Using this approach, learning can be described in terms of three characteristics: (1) the mode of cognitive functioning, (2) the forms of knowledge that are developed and (3) the ways that learners structure their knowledge. For such learning approaches, then accordingly changes need to bring in teaching of Mathematics also. But there are also many problems with teaching of Mathematics which are explored in next section.

1.4.2 Mathematics Teaching: The Problems

Quality teaching can be explained as 'teaching that expands learning for all students'. Teaching of Mathematics is a complex task as it is not only concerned with the calculation or computational phenomenon but also concerned with to inculcate deep understanding, applicability and leading towards the higher order thinking. The quality and effectiveness of this teaching process depends on many factors like content delivery, instructional strategies, methods, pedagogical skills of a teacher, connecting the subject with the world and the most important is the environment created by a teacher for appropriate learning and motivations for the students. All this factors constituted to fulfill the essential goals and objectives of the teaching Mathematics. These goal and objectives for the teaching of Mathematics were remained changed with time to time as well as according to the demands and needs for the learning which are overviewed in the next paragraph.

The **National Policy of Education (NPE) (1986)**, which was the major landmark document after the Kothari commission, and its subsequent revisions also emphasised Mathematics but the focus in these was to develop the capability of using

Mathematics in daily life and in applications of other areas. The understanding of Mathematics teaching for improving its everyday application and the capability to handle Mathematical aspects in other subjects of study were the core concerns in National Curriculum Framework for School Education (NCFSE) given by NCERT (2000), which also emphasised the need to develop capability of doing Mathematical calculations.

The **National Curriculum Framework (NCF) (2005)** made a break from this and emphasised developing the capability to abstract, use and understand logical forms, grasp ideas and discover, create as well as appreciate patterns. The idea of Mathematisation and giving learners the space to discover the way Mathematics functions was an important change in the **NCF (2005)** formulation. It also urged focus on developing concepts and learners' own ways of solving problems and building new algorithms rather than remembering short cuts and efficient ways to calculate. And such objectives could be fulfilled by practicing it with constructivist method of teaching.

According to **Killen & Hattingh (2004)**, Educators have long accepted that 'learning is not only adding something to our knowledge' but learning is a process of developing understanding by integrating new knowledge into the learner's world of sense and meaning. Therefore, we need to have appropriate ways to describe whatever it is that we want students to understand and appropriate ways to measure the understanding, so that we will know when our teaching has been successful. Teaching practices are central to understanding what makes for effective teaching. As **Peterson (1988)** has derived a list of effective teaching practices included: (1) A focus on meaning and understanding Mathematics and on the learning task; (2) Encouragement of students' autonomy, independence, self-direction and persistence in learning; (3) Teaching of higher-level cognitive processes and strategies.

To improve students' Mathematical knowledge, different researches have been done and are being done in different areas, like - content, method, evaluation, etc. But classroom teaching is the heart of the formal system of education, where researchers need to rethink or rework. Though, innovations in different areas of education must

have the direct or indirect reflections in classroom interactions or classroom teaching and hence during the teaching process, a teacher has to become very crucial and vital.

Also, giving the good understanding about the subject and its concepts is the most preferable phenomenon to develop the interest, motivation, thinking of the learners in the subject. To get a more complete picture of how students could learn and why they respond in particular ways to the questions we ask them, we need to consider or focus how they structure their thinking. Perhaps this helps to detect the misconceptions taking place during the teaching and learning. Thus, understanding needs to be emphasized more to overcome the problems of the misconceptions. Hence, the next section is briefly highlighting about the misconceptions in Mathematics and the importance of understanding in Mathematics.

With the appreciation and consideration of these thoughts, the researcher of the present research study is intended to try-out the experiment for the Mathematics teaching-learning within the structural levels of SOLO Taxonomy developed on 'structure for increasing complexity in Understanding' with the view as, "Teachers must actively cultivate the learner's intellectual skills rather than just to impart knowledge. Likewise, teachers need to assess their learners' intellectual skills, not just their capacity to memorise information". With the intent to emphasize more on Conceptual understanding in Mathematics, the next section is explaining about the Mathematical Understanding, Misconceptions – that is gaps in understanding and then about the said structure – SOLO Taxonomy.

1.5 IMPROVISATION OF MATHEMATICAL UNDERSTANDING

The concept of Understanding is very wider. Several theories and models had been developed to understand and practice the term 'Understanding' appropriately. But here, very brief description on Understanding has been given as the concern is only to study the attainment as well as the measurement of the Understanding through the levels of SOLO Taxonomy. As the present research study is concerned with the Understanding in terms to maximise the learning in Mathematics and the same thing is the central idea of the SOLO taxonomy as taken for the present research study. Following is about the Mathematical Understanding elaborated based on the few

theories only. Before elaborating more on it, first have brief explanation on the meaning of ‘Comprehension’ as usually considered same as the (full) Understanding.

As ‘Comprehension’ is one of the layers in cognitive domain of Bloom’s taxonomy referred to or related with the Understanding aspects of the learning. But it is quite different from the actual or complete meaning of the understanding. According to **Bloom et al. (1956)**, probably the largest general class of intellectual abilities and skills emphasized in schools and colleges are those which involve Comprehension. The Comprehension represents the lowest level of Understanding. It refers to a type of Understanding or apprehension such that the individual knows what is being communicated and can make use of the material or idea being communicated without necessarily relating it to other material or seeing its fullest implications. For instance, we commonly expect Comprehension of a Physics demonstration, a geologic formation viewed on a field trip, a building illustrating a particular architectural feature, a musical work played by an orchestra. And, of course, it mean to comprehension of the above phenomena when presented in verbal, pictorial or symbolic form on paper.

According to **Meel (2003)** as drafted in a paper on “Models and theories of Mathematical Understanding: Comparing Pirie and Kieren’s model of the growth of Mathematical Understanding and APOS (Action, Process, Object and Schema) Theory”, that even though the term “Understanding: has been freely used in Mathematics education literature, the search for a concise definition of “Understanding” has been going on for years.

In terms to develop students’ Mathematical Understanding, **Pirie & Kieren (1992a)** identified four critical tenets that teachers must hold when creating a constructivist environment for encouraging Mathematical learning and understanding: (a) Although a teacher may have the intension to move students toward particular Mathematics learning goals, she will be well aware that such progress may not be achieved by some of the students and may not be achieved as expected by others. (b) In creating an environment or providing opportunities for children to modify their Mathematical understanding, the teacher will act upon the belief that there are different pathways to similar Mathematical understanding. (c) The teacher will be aware that different

people will hold different Mathematical understanding and (d) The teacher will know that for any topic there are different levels of understanding but that these are never achieved 'once and for all'. Also, underlying the discussion has been belief that "teachers cannot give understanding to the students. Only the student can build her own understanding. The role of the teacher is to provoke and enable this growth." The provocation and enabling of growth reaches beyond simply asking students to work on high level Mathematics and includes the generation of opportunities to promote understanding. The teacher can address the individualized understandings of a student by provoking movement to an outer layer of understanding, by invoking folding back to a previous level of understanding and by encouraging students to validate their own reasoning. With these understandings, the researcher prompted to experiment aforesaid theory through the instructional strategy along with the levels of SOLO Taxonomy. Next sub-section is briefly highlighting on the meaning and means of misconceptions in Mathematics.

1.5.1 Misconceptions: The Gaps In Understanding The Mathematics

As it is learnt from the above section that we need to more emphasize on the understanding of Mathematical concepts and if gaps remain in the process of understanding tend to create misconceptions. Misconception, what does it mean is drawn out from various definitions as: **McCloskey (1983)** mentioned about misconception as a belief or an idea that is not based on correct understanding or correct information. **Carey (1985)** stated as misconception as a wrong belief or wrong opinion as a result of improper understanding of facts. **Fowler & Joude (1987)** defined misconceptions as an inaccurate understanding of a concept, the misuse of a concept name, the incorrect classification of concept examples, confusion between differing concepts, improper hierarchical relationships, or over or under generalizing of concepts. According to **Read (2004)**, a number of term such as mistake, misunderstanding, miscomprehension, error, misinterpretation, misbelieve are synonymously used for misconception. **Peckmez (2010)**, the term misconception also refers to the inappropriate understanding of any idea or concept. **Mayer (2011)** defines misconception is a conclusion that is wrong because it is based on faulty thinking or facts that are wrong.

As a consequence, students experience difficulty in understanding and internalising any new concept. Identifying the exact nature of student misconceptions is difficult through regular classrooms interactions. Generally, student misconceptions persist until students recognise that their understanding is flawed.

According to **Wipro & EI (2006)** conducted study on ‘Student Learning in Metros’ to find out the gaps in the way children learn even in popular schools and stated as students develop cognitive understanding of the world around them through interactions based on their daily experiences. Teachers and schools help build this understanding. As said, ‘Misconceptions’ are concepts that students acquire when they get incomplete answers to their questions or when exposed to incorrect facts. These result in cognitive gaps in their understanding. Students try to fill these gaps by formulating their own notions, attributing meanings and by drawing conclusions that might seem to them as logical. The resulting misunderstandings or alternative concepts formulated by the students, if not challenged, interfere with subsequent learning.

There are many concepts and topics in Mathematics where misunderstandings could occur during the teaching-learning process of Mathematics and many times it happens that it could not be detected at early or desired stages. Such misunderstanding could be caused for such misconceptions. Emphasising on giving proper understanding should be an appropriate way or remedy to minimize the misconceptions. In Mathematics, understanding does not mean to understand the procedural or mechanical way of (step by step) solving the examples or exercises, but it should be more refer to understand the concepts logically, in-depth and in a correct manner as well about its applications. The next section as given below, is elaborating about the meaning of conceptual understanding.

1.5.2 Conceptual Understanding: Necessity For Mathematics Education

While according to **Hiebert & Grouws (2007)**, one of the strongest results in recent research is that the most important feature in effective teaching is giving students "opportunity to learn". Teachers can set expectations, time, kinds of tasks, questions, acceptable answers and type of discussions that will influence students' opportunity to learn. This must involve both skill efficiency and conceptual understanding.

The conceptual understanding, as **Hiebert & Grouws (2007)** mentioned as two of the most important features of teaching in the promotion of conceptual understanding are attending explicitly to concepts and allowing students to struggle with important Mathematics. Both of these features have been confirmed through a wide variety of studies. Explicit attention to concepts involves making connections between facts, procedures and ideas. These connections can be made through explanation of the meaning of a procedure, questions comparing strategies and solutions of problems, noticing how one problem is a special case of another, reminding students of the main point, discussing how lessons connect, and so on. Deliberate and productive struggle with making connections of Mathematical ideas refers to the fact that when students exert effort with important Mathematical ideas, even if this struggle initially involves confusion and errors, the end result is greater learning. This has been shown to be true whether the struggle is due to challenging, well-implemented teaching, or due to faulty teaching the students must struggle to make sense of.

The **Institute of Education Sciences (2003)** stated as, this is often seen as one of the strong points in Mathematics teaching in East Asian countries, where teachers typically devote about half of their time to making connections. At the other extreme is the U.S.A., where essentially no connections are made in school classrooms.

As the **Perkins (1993)**, concisely captures the need to teach for understanding: “We must teach for understanding in order to realize the long term payoffs of education. Now the questions to consider are: What is understanding? and How do we teach for understanding?” We often talk about understanding in terms of mental constructs, such as schema, models and structures or in terms of learning performances, such as explaining, reasoning, analyzing, interpreting, relating, comparing, making analogies, abstracting, conjecturing and generalizing. Researchers have asked, ‘Is understanding a mental state or a performance?’ It is useful to consider it as both. More specifically, researchers have identified key components of the nature of understanding:

- ♣ **Connections** - Making connections has long been identified as a key component of understanding.
- ♣ **Structures** - Related to and extending the idea of connections is the notion of mental structures or schemas

- ♣ **Performances** - Researchers debate about whether understanding is a performance or a mental state. For practitioners, it seems productive to consider it as both. Furthermore, these two notions of understanding are related, in that robust mental schemas help enable understanding performances
- ♣ **Constructing Knowledge** - Understanding is often characterized in terms of students constructing knowledge
- ♣ **Depth and Type of Knowledge** - An analysis of understanding also includes analyzing the nature of knowledge. Two aspects of knowledge particularly relevant to the discussion here are depth of knowledge and type of knowledge. For example, in a review of cognitive science research **William (1980)** describes three levels of knowledge: rote, inflexible, and deep structure, and **Star (2005)** argues for the value of both procedural and conceptual knowledge, as long as both are deep.

With such understandings on the Conceptual Understanding, SOLO taxonomy as the new framework has been chosen to experiment in the current research study. Before the descriptions on SOLO Taxonomy, an overview on various taxonomies practiced in Education are briefly as below. Also, explained the majorly used Bloom's taxonomy in terms to understand the insights of the SOLO Taxonomy.

1.6 TAXONOMIES OF LEARNING: THE HIGHLIGHTS

As **Thomas (2004)** reported about various taxonomies in a draft on "Learning Taxonomies in Cognitive, Affective and Psychomotor domains". According to report, the defining characteristic of taxonomies is not only that they categorise, but that the categories are ordered. In the case of Bloom's taxonomy for instance, this meant that, in the sequence of cognitive categories as 'remember', 'understand', 'apply', 'analyse', 'evaluate', 'create' where each category includes the previous one. The use of taxonomies sometimes tends to focus on levels of demand and complexity associated with the different categories and on the ability and/or skill needed rather than on an analysis of the task and the type of cognitive operations and knowledge which can be used to complete the task.

Reasons for using Taxonomy are majorly to frame the objectives, design and implement the instructional strategies systematically and to measure the desired

learning outcomes. Perhaps, **Anderson and Krathwohl (2001)** gave six reasons for categorizing objectives in taxonomy as: (1) It permits educators to examine objectives from the student's point of view. (2) It helps educators consider the panorama of possibilities in education — teaching for higher-order objectives and learning how to learn. (3) it helps educators see the integral relationship between knowledge and cognitive processes inherent in objectives, (4) it makes life easier — examiners can easily identify the 'demand' of a question by knowing the framework, so guesswork is removed, (5) It makes more readily apparent the consistency, or lack of it, among the stated objectives for a unit, the way it was taught, and how learning was assessed, (6) It helps educators make better sense of the wide variety of terms that are used in education — the precision in the taxonomy improves communication and understanding of what is to be taught and assessed.

In general ways, the purpose or intention of any taxonomy is to provide a common understanding, on the part of the users, of what to teach and learners what to learn (often by using specific verbs, such as 'identify' or 'analyse'). This greater clarity about what students must be able to know and do is intended to: (i) ensure that learners learn – and not just to pass examinations and (ii) improve the efficiency and effectiveness of the assessment or examining process by making sure that the assessment is directly related to the purpose of learning. Taxonomies can be used to: (i) define the syllabus or course for teachers so that they know what needs to be taught and to what extent, (ii) give clear objectives to learners for their course of learning, (iii) ensure that learners are not set over-simplistic or over-complicated assessment tasks for their course of learning, (iv) facilitate assessment of learning, (v) facilitate the grading of learners.

There are various Taxonomies used in framing educational or learning objectives. Following is a figure-1.3 presenting an overview on some of the taxonomies while some more were searched by the researcher are mentioned here as: (a) Robert Gagne's Learning Taxonomy; (b) Bloom's Taxonomy; (c) Harrow's Taxonomy Of The Psychomotor Domain; (d) Simpson's Taxonomy Of The Psychomotor Domain; (e) Thomas' Taxonomy Of The Psychomotor Domain; (f) Krathwohl's Taxonomy Of The Affective Domain; (g) Anderson And Krathwohl (Bloom Revised); (h) Marzano's New Taxonomy; (i) Fink's Taxonomy Of Significant Learning; (j)

Structure Of The Observed Learning Outcome (SOLO); (k) Scottish Credit And Qualifications Framework and (l) Framework Of Achievement.

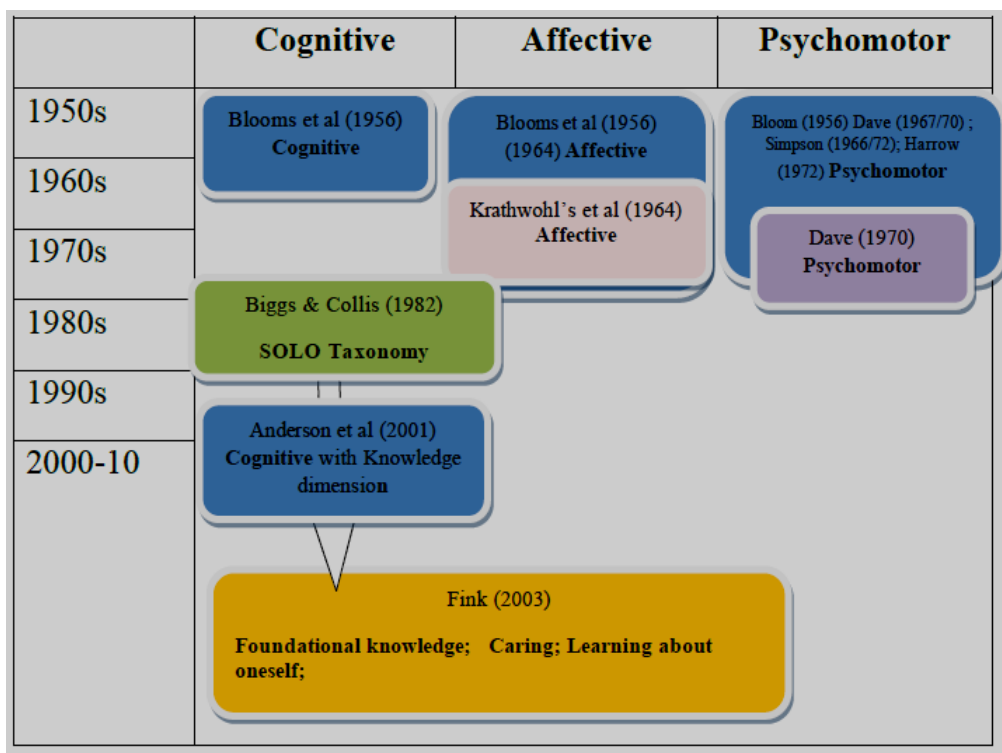


Figure – 1.3: Overview of development of Taxonomies and their domains
(Source: <http://www.ucd.ie/t4cms/taxonomies3.pdf>)

Following are the tables from table-1.1 to 1.8 representing some of the taxonomies and very briefly explained as to have idea about the kind of taxonomies practiced in the field of education.

Gagne’s learning taxonomy does not specify the three primary domains recognized today (cognitive, psychomotor, and affective). This taxonomy was developed in terms to identify the Internal and External Conditions of Learning. And is broken down and shows its relationship to KSAs (Knowledge, Skill and Attitudes) and the learning domains.

Table: 1.1
Gagne’s Taxonomy

KSA	Learning Domain	Gagne’s Taxonomy
Knowledge	Cognitive	<ul style="list-style-type: none"> ◆ Intellectual Skills (Discrimination, concrete concept, rule using, problem solving) ◆ Cognitive Strategy

		◆ Verbal Information (Labels, facts, bodies of knowledge)
Skill	Psychomotor	Motor Skills
Attitude	Affective	Attitude

(Source: <http://www.rockymountainalchemy.com/whitepapers/rma-wp-learning-taxonomies.pdf>)

Benjamin Bloom, M. Englehart, E. Furst, W. Hill, and David R. Krathwohl worked together to develop a taxonomy of educational objectives to measure the cognitive domain of human behaviour. In general it is known as Bloom's Taxonomy, is a set of three hierarchical models used to classify educational learning objectives into levels of complexity and specificity.

Translating affective goals into observable behaviours or performance objectives is challenging, at best. Objectives from the affective domain are typically measured through survey instruments or similar tools. It's also worth noting that Krathwohl worked on the team that developed the taxonomy commonly referred to as "Bloom's Taxonomy." Following is the presentation on levels of Cognitive Model (domain), Affective and Psychomotor models of the taxonomy.

Table: 1.2
Bloom et al.'s (Bloom(1956)) Taxonomy

Level	Description
Cognitive Domain	
Knowledge	Recall and remember information.
Comprehension	Understand the meaning, translation, interpolation and interpretation of instructions and problems. State a problem in one's own words. Establish relationships between dates, principles, generalizations or values
Application	Use a concept in a new situation or unprompted use of an abstraction. Applies what was learned in the classroom into novel situations in the workplace. Facilitate transfer of knowledge to new or unique situations.
Analysis	Separates material or concepts into component parts so that its organizational structure may be understood. Distinguishes between facts and inferences

Synthesis	Builds a structure or pattern from diverse elements. Put parts together to form a whole, with emphasis on creating a new meaning or structure. Originality and creativity.
Evaluation	Make judgments about the value of ideas or materials.
Affective domain (Krathwohl's Affective)	
Receiving phenomena	Awareness, willingness to hear, selected attention.
Responding to phenomena	Active participation on the part of the learners. Attends and reacts to a particular phenomenon. Learning outcomes may emphasize compliance in responding, willingness to respond, or satisfaction in responding (motivation).
Valuing	The worth or value a person attaches to a particular object, Phenomenon or behaviour. This ranges from simple acceptance to the more complex state of commitment.
Organization	Organizes values into priorities by contrasting different values, resolving conflicts between them, and creating a unique value system. The emphasis is on comparing, relating, and synthesizing values.
Internalizing values	Has a value system that controls their behaviour. The behaviour is pervasive, consistent, predictable, and most importantly, characteristic of the learner.
Psychomotor Domain (Dave's Psychomotor)	
Imitation	Includes repeating an act that has been demonstrated or explained, and it includes trial and error until an appropriate response is achieved.
Manipulation	Includes repeating an act that has been demonstrated or explained, and it includes trial and error until an appropriate response is achieved.
Precision	Response is complex and performed without hesitation.
Articulation	Skills are so well developed that the individual can modify movement patterns to fit special requirements or to meet a problem situation.
Naturalization	Response is automatic. One acts "without thinking."

(Source: <http://www.csus.edu/indiv/blommstaxonomy>)

The revised Bloom's Cognitive domain has a hierarchy of categories that capture the process of learning, from simply remembering information to creating something new: Remember, Understand, Apply, Analyse, Evaluate and Create. To these levels has been added a knowledge dimension (factual, conceptual, procedural and

metacognitive). Table given below indicates the structure of Anderson's taxonomy (Bloom's revised taxonomy) and some verbs that might be useful in writing learning outcomes.

Table: 1.3
Anderson's et al. (2001) Cognitive Revised Domain

	Factual Knowledge	Conceptual Knowledge	Procedural Knowledge	Metacognitive Knowledge
Remember	List	Describe	Tabulate	Appropriate Use
Understand	Summarize	Interpret	Predict	Execute
Apply	Classify	Experiment	Calculate	Construct
Analyse	Order	Explain	Differentiate	Achieve
Evaluate	Rank	Assess	Conclude	Action
Create	Combine	Plan	Compose	Actualise

(Source: <http://www.ucd.ie/teaching>)

Anita Harrow developed a taxonomy for children with special physical needs. This taxonomy is better suited to assessing ability to perform a task or activity or to sports and recreation activities than to the typical physical activities performed in the workplace. Anita Harrow's taxonomy for the psychomotor domain is organised according to the degree of coordination including involuntary responses as well as learned capabilities.

Table: 1.4
Harrow's (1972) Taxonomy of the Psychomotor Domain

Level	Description
Reflex Movements (Level 1)	Actions elicited without learning in response to some stimuli.
Basic Fundamental Movements (Level 2)	Inherent movement patterns which are formed by combining reflex movements (serve as the basis for complex skilled movements).
Perceptual (Level 3)	Interpretation of various stimuli that enable one to make adjustments to the environment. Visual, auditory, kinaesthetic, or tactile discrimination. Suggests cognitive as well as psychomotor behaviour.
Physical Activities (Level 4)	Endurance, strength, vigor, and agility which produce a sound, efficiently functioning body.
Skilled Movements (Level 5)	The result of acquisition of a degree of efficiency when performing a complex task.
Non-Discursive Communication (Level 6)	Communication through bodily movements ranging from facial expressions through sophisticated choreography.

(Source: <http://cehdclass.gmu.edu/ndabbagh/Resources/IDKB/harrowstax.htm/>)

Elizabeth Simpson's taxonomy is focused on the progression of a skill from guided response (i.e., doing what you are told to do) to reflex or habitual response (i.e., not having to think about what you're doing), then includes origination as the highest level (i.e., invention of a new way to perform a task). Simpson built this taxonomy on the work of Bloom and others.

Table: 1.5
Simpson's (1066) Taxonomy of the Psychomotor Domain

Level	Description
Perception (Level 1)	The process of becoming aware of objects, qualities, etc. by way of senses. Basic in situation-interpretation-action chain leading to motor activity. May include sensory stimulation, cue selection, translation.
Set (Level 2)	Readiness for a particular kind of action or experience. This readiness or preparatory adjustment may be mental, physical or emotional.
Guided Response (Level 3)	Overt behavioural act of an individual under guidance of an instructor, or following model or set criteria. May include imitation of another person, or trial and error until appropriate response obtained.
Mechanism (Level 4)	Occurs when a learned response has become habitual. At this level the learner has achieved certain confidence and proficiency or performance. The act becomes part of his/her repertoire of possible responses to stimulus and demands of situations.
Complex (Level 5)	Overt Response Performance of a motor act that is considered complex because of movement pattern required. May include resolution of uncertainty, i.e., done without hesitation; and automatic performance, finely coordinated with great ease and muscle control.
Adaptation (Level 6)	Altering motor activities to meet demands of problematic situations.
Origination (Level 7)	Creating new motor acts or ways of manipulating materials out of skills, abilities and understandings developed in the psychomotor area.

(Source: <http://users.rowan.edu/~cone/curriculum/psychomotor.htm>)

Thomas' can be used to define minimum psychomotor requirements for a task or set of tasks which define a job. Thomas' is organized in a basic hierarchical structure (although not all levels are necessarily "below" other levels, nor are lower levels necessarily "prerequisites" for higher levels), building in complexity and origination. However, Thomas does not focus on the habitualization or reflex of the task, merely on the ability to perform the foundational verb behaviour. Although the fundamental approach is different, Thomas' taxonomy borrows pieces from both Harrow & Simpson.

Table: 1.6
Thomas' (2004) Taxonomy of the Psychomotor Domain

Level	Description
Perception (Level 1)	Gathering stimuli through the senses.
Communication (Level 2)	Physical aspects of communication (e.g., organizing a persuasive speech would rely on elements of the cognitive domain, but actually delivering that speech would be psychomotor).
Movement (Level 3)	Physical movement from simple body positioning to complex locomotion.
Strength (Level 4)	Actions or tasks requiring a degree of physical strength and/or endurance.
Dexterity (Level 5)	Tasks requiring hand control and skill.
Coordination (Level 6)	Synchronization of multiple physical activities.
Operation of Tools & Equipment (Level 7)	Actions and skills associated with operating tools and/or pieces of equipment.
Construction (Level 8)	Activities or tasks involved in building or constructing an object or structure.
Art (Level 9)	Refined and/or skilled actions associated with creating art.

(Source: <http://www.rockymountainalchemistry.com/whitepapers/rma-wp-learning-taxonomies.pdf>)

Robert Marzano developed a taxonomy to respond to the shortcomings of the widely used Bloom's Taxonomy. Marzano's model of thinking skills incorporates a wider range of factors that affect how students think and provides a more research-based theory to help teachers improve their students' thinking.

Table: 1.7
Marzano and Kendall's (2000) The New Taxonomy

Level Of Difficulty	Process / Description	
6. Self System Thinking	Examining Importance	Examining Efficacy Examining Emotions
5. Metacognition	Examining Motivation Specifying Goals	Process Monitoring Monitoring Clarity Monitoring Accuracy
4. Knowledge Utilization	Investigating Experimenting	Problem-Solving Decision-Making
3. Analyzing	Specifying Generalizing Analysing	Classifying Matching

2. Comprehension	Symbolizing	Integrating
1. Retrieval	Executing	Recalling Recognising

(Source: http://www.kidsrsu.org/images/uploads/Marzano_New_Taxonomy_Chart_with_verbs_3.16.121.pdf)

Fink presents a taxonomy that is not hierarchical. In addition it covers a broader cross section of domains with the exception of a psychomotor domain. It is similar to Anderson’s taxonomy (2001) in its emphasis is on metacognition (learning to learn) and also includes more affective aspects such as the ‘human dimension’ and ‘caring: identifying/changing one’s feelings’.

Table: 1.8
Fink’s (2009) Taxonomy

Dimension	Description
Foundational Knowledge	Understand and remember
Application	Critical, creative and practical thinking; problem solving
Integration	Make connections among ideas, subjects, people
Human Dimensions	Learning about and changing one’s self; understanding and interacting with others
Caring	Identifying/changing one’s feelings, interests, values.
Learning to learn	Learning how to ask and answer questions, becoming a self-directed learner

(Source: <http://www.ucd.ie/t4cms/taxonomies3.pdf>)

An advantage of the use of taxonomies is that it provides a means to describe and compare outcomes and assessments. The general advantages of Taxonomies are means to get (a) Clarity; (b) High-level learning; (c) Systematic approach; (d) Confidence in assessment.

Apart from the aforesaid taxonomies, one more taxonomy known as ‘SOLO Taxonomy’ that is opted by the researcher for the present research is elaborated next in detail. The focal point of this SOLO taxonomy is different from the other taxonomies as explained above. It merely focusing on the aspects of “Progressive Understanding” in the learning and arranged in a hierarchal order through five levels of taxonomy.

1.7 THE S.O.L.O. (SOLO) TAXONOMY: AN ABSTRACT VIEW

As per the **Biggs & Tang (2007)**, the SOLO is the short form of ‘Structure of Observed Learning Outcomes’. It was developed by an Australian educational psychologist and novelist **John B. Biggs** for assessing the quality of learning outcomes and the model of ‘Constructive Alignment’ for designing teaching and assessment. It describes level of increasing complexity in a student's understanding of a subject through five stages as (i) Pre-structural, (ii) Uni-structural, (iii) Multi-structural, (iv) Relational and (v) Extended Abstract. Also, it is claimed to be applicable to any subject area. Not all students get through all five stages, of course, and indeed not all teaching is designed to take them all the way. Below are the explanation and diagrammatic presentation (figure-1.4 and 1.5) about the five stages (levels) of SOLO taxonomy.

Level – 1: The Pre-Structural Level

Here the student does not have any kind of understanding but uses irrelevant information and/or misses the point altogether. Scattered pieces of information may have been acquired, but they are unorganized, unstructured, and essentially void of actual content or relation to a topic or problem.

Level - 2: The Uni-Structural Level

The student can deal with one single aspect and make obvious connections. The student can use terminology, recite (remember things), perform simple instructions/algorithms, paraphrase, identify, name, count, etc.

Level - 3: The Multi-Structural Level

At this level the student can deal with several aspects but these are considered independently and not in connection. Metaphorically speaking; the student sees the many trees, but not the forest. He is able to enumerate, describe, classify, combine, apply methods, structure, execute procedures, etc.

Level – 4: The Relational Level

At level four, the student may understand relations between several aspects and how they might fit together to form a whole. The understanding forms a structure and now he does see how the many trees form a forest. A student may thus have

the competence to compare, relate, analyze, apply theory, explain in terms of cause and effect, etc.

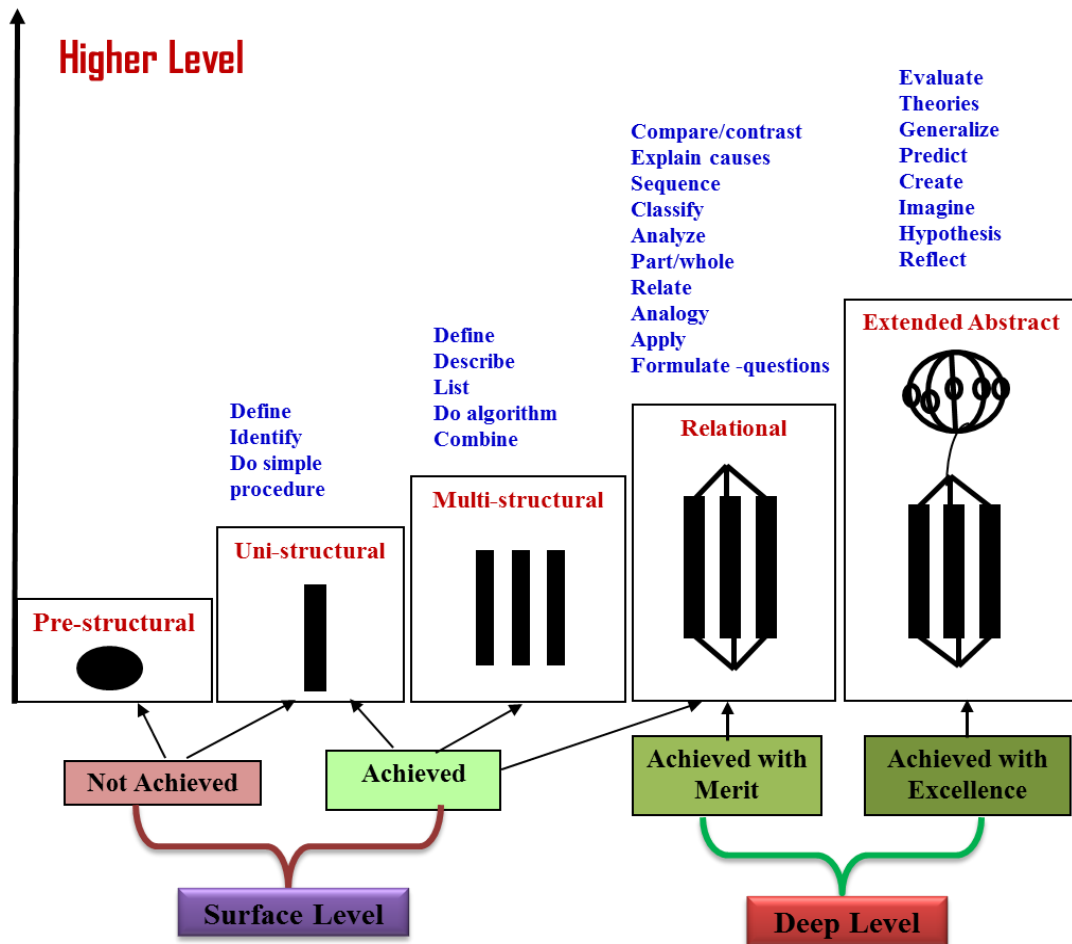


Figure-1.4: Levels of SOLO Taxonomy
 (Source: http://pamhook.com/wiki/The_Learning_Process)

These levels of understanding (logic) can be represented diagrammatically, as shown in following figure.

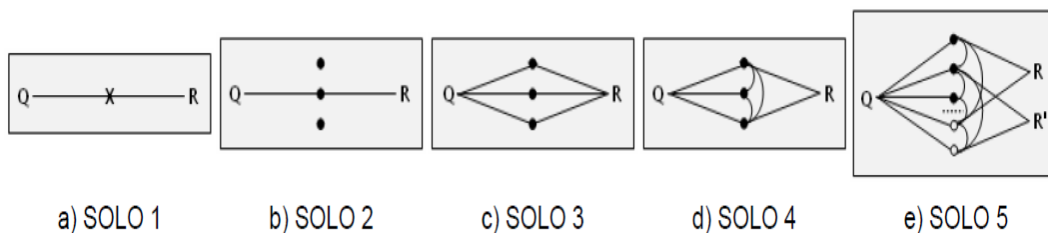


Figure-1.5: Visualization of the SOLO-levels 1-5. (Based on Biggs and Collis (1982))
 (Source: <http://crpit.com/confpapers/CRPITV88Brabrand.pdf>)

Level – 5: The Extended Abstract Level

At this level, which is the highest, a student may generalize structure beyond what was given, may perceive structure from many different perspectives, and transfer ideas to new areas. He may have the competence to generalize, hypothesize, criticize, theorize, etc.

Through these five levels, how constructivist approach could be addressed and how gradually it align the progress with the successive levels of the taxonomy as explained by the Biggs is elaborated below.

1.7.1 Means Of Constructive Alignment With SOLO Levels

According to the **Biggs & Tang (2007)**, Constructive Alignment is a principle used for devising teaching and learning activities, and assessment tasks that directly address the learning outcomes intended in a way not typically achieved in traditional lectures, tutorial classes and examinations. Constructive alignment was devised by Professor John B. Biggs, and represents a bonding between a constructivist understanding of the nature of learning, and an aligned design for outcomes-based teaching education.

Constructive Alignment is the underpinning concept behind the current requirements for programme specification, declarations of Learning Outcomes (LOs) and assessment criteria, and the use of criterion based assessment. There are two basic concepts behind Constructive Alignment:

- ◆ Learners construct meaning from what they do to learn. This concept derives from cognitive psychology and constructivist theory, and recognizes the importance of linking new material to concepts and experiences in the learner's memory, and extrapolation to possible future scenarios via the abstraction of basic principles through reflection.
- ◆ The teacher makes a deliberate alignment between the planned learning activities and the learning outcomes. This is a conscious effort to provide the learner with a clearly specified goal, a well designed learning activity or activities that are appropriate for the task, and well designed assessment criteria for giving feedback to the learner.

A branch of educational evaluation theory has emerged that focuses on Constructive Alignment as a key element in effective educational design.

1.7.2 Psychological Aspects Of The SOLO Taxonomy

As **Hattie & Brown (2004)** explained how and which Psychological aspects concerned with the SOLO Taxonomy are given here. According to their study, in SOLO model, it is closely related to the existing notion of Piaget's stage of cognitive development which proposes a number of developmental stages demonstrating increasing abstraction from sensory-motor (infancy: Birth to 2 yrs), iconic (early childhood of preschool: 2 to 7 yrs), concrete-symbolic (childhood to adolescence: 7 to 11 yrs), formal (early adulthood: 11 yrs & up) through to post formal (adulthood).

Biggs & Collis (1982) based model on the notion that in any “learning episode, both qualitative and quantitative learning outcomes are determined by a complex interaction between teaching procedures and student characteristics”. They emphasised the roles played by (i) the prior knowledge that the student has of the content relating to the episode, (ii) the student's motives and intentions about the learning, and (iii) the student's learning strategies. As a consequence, the levels are ordered in terms of various characteristics - from the concrete to the abstract, an increasing number of organising dimensions, increasing consistency, and the increasing use of organising or relating principles. It was developed to assess the qualitative outcomes of learning in a range of school and college situations and in most subject areas.

There are four major ways that the four levels (unistructural, multistructural, relational, extended abstract) can increase in complexity are briefed below:

Capacity: Each level of the SOLO taxonomy increases the demand on the amount of working memory or attention span. At the surface levels (unistructural and multistructural), a student need only encode the given information and may use a recall strategy to provide an answer. At the deep levels (relational or extended abstract), a student needs to think not only about more things at once, but also how those objects inter-relate.

Relationship: Each level of SOLO refers to a way in which the question and the response interrelate. A unistructural response involves thinking only in terms of one aspect and thus there is no relationship possible. The multistructural level involves a many aspects but there is no attention of relationship between these aspects. At the relational level, the student needs to analyse and identify an appropriate relationship between the many ideas. And at the extended abstract level, the student needs to generalise to situations not experienced or beyond the given environment.

Consistency and Closure: These refer to two opposing needs felt by the learner. On the one hand, the student wants to come to a conclusion and thus answer or close the question. But on the other hand, the student wants to experience consistency so that there is no contradiction between the question posed, the material given, and the answer provided. Often, when there is a greater need for closure, less information is utilised resulting in an answer/response that is less consistent. In contrast, when a high level of need for consistency is required, a student may utilise more information when conceiving an answer but may not be able to reach closure if external factors do not permit. At the unistructural level, the student often seizes on immediate recall information but at the extended abstract level, the student must integrate potentially inconsistent ideas and must tolerate the possibility of inconsistency across contexts.

Structure: The unistructural response takes one relevant piece of information to link the question to the answer. The multistructural response takes several pieces and links them to the question. The relational response identifies and makes use of an underlying conceptual structure and the extended abstract requires a generalised structure such that the student demonstrates an extension beyond the original given context.

These psychological aspects are necessary to consider as while dealing with the domain of Understanding and it takes place during the learning of Mathematics too.

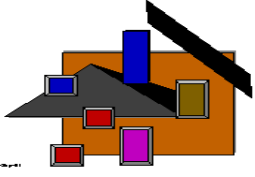
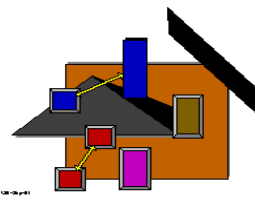
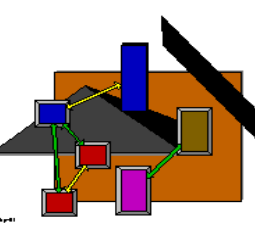

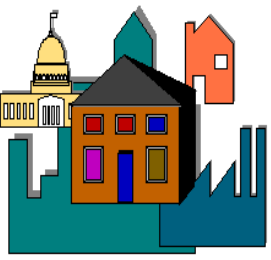
1.7.3 The Application Of S.O.L.O. Taxonomy: S.O.L.O. Based Examples

Three examples have been taken to describe how conceptual understanding progressing through the levels of SOLO Taxonomy i.e. from the Pre-structural level to the level of Extended abstract. Here example-1 is taken from Mathematics of lower

grade to justify that these levels could be practiced at any grade-level or possibly for any concepts of the Mathematics.

Example-1: Geometrical Shapes

Table – 1.9:
Example-1: Understanding of Geometrical Shapes through SOLO Levels

Levels Of S.O.L.O.	Pictorial Presentation Of Understanding	Description
Pre-structural		Here students are simply acquiring bits of unconnected information, which have no organisation and make no sense.
Uni-structural		Simple and obvious connections are made, but their significance is not grasped. (connections made based on shapes or colours like blue with blue as well red with red)
Multi-structural		A number of connections may be made, but the meta-connections between them are missed, as is their significance for the whole. (connections made using the ideas behind the colours and shapes)
Relational		The student is now able to appreciate the significance of the parts in relation to the whole. (ideas developed to arrange objects with the context of the whole object)
Extended Abstract		The student is making connections not only within the given subject area, but also beyond it, able to generalise and transfer the principles and ideas underlying the specific instance. (develop ideas for the surroundings or the relational thoughts of the object)

(Source : <http://www.learningandteaching.info/learning/solo.htm>)

From the above example-1, it seems from the proceedings for the learning from Pre-structural to Extended abstract that, such framework could be used in lower grades also in which all the levels of the SOLO taxonomy could be addressed or practiced.

Example - 2: “Why Cheetahs have spots?”

Table 1.10:

Example-2: SOLO level-wise proceedings of example of ‘Why Cheetahs have spots?’

Levels of S.O.L.O.	Answer/s	Characteristics	Remarks
Prestructural	“I don’t know”	No attempt to answer the question	❖ Give no answer ❖ Repeat what was said in the class
	“So that they are different from Lions”	The response is irrelevant	❖ Guesses the answer
Unistructural	“It makes them hard to see”	True, but with no explanation	❖ Gives simple correct answer ❖ Based on a quick decision ❖ No explanation
Multistructural	“The spots are formed by Melanin in the skin. The spots camouflaged them while hunting”	True, but with no explanation	❖ Uses two or more explanation ❖ No integration of additional knowledge
Relational	“The spots are formed by Melanin in the skin which is a chemical reaction during embryotic development. The spots evolved after mutation to camouflaged them while hunting”	Biological explanation and link it to evolution.	❖ Integrates prior knowledge ❖ Links ideas to explain information
Extended Abstract	“The spots are formed by Melanin in the skin which is a chemical reaction during embryotic development. The spots evolved after mutation to camouflage	Extended Abstract thought	❖ Goes beyond what has been taught ❖ Uses logical deductions to frame the answer

	<p>them while hunting. Stripes will not be an advantage that is why the 'king' cheetah is so rare in the wild. The spots are Mathematical pattern that can be described with partial differential equations.”</p>		
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(Source: Killen & Hattingh (2004), <http://www.cut.ac.za/healthdiscussion/solotax.pdf>)

The explanation given by **Killen & Hattingh (2004)** as, this example illustrates the importance of deciding in advance what level of understanding is required. A 6th-grade teacher, for example, might not expect learners to have anything more than a multi-structural level of understanding. She might have an outcome such as: 'Learners will be able to give simple biological and environmental reasons for animals having different skin colours'. A 12th-grade teacher, however, might expect students to have a relational level of understanding. She might have an outcome such as: 'Learners will be able to explain how various genetic factors determine the skins colorations of animals'. At undergraduate level, an outcome designed to encourage extended abstract thinking might be: 'Learners will be able to compare various theories about how biological and environmental factors interact to produce animal characteristics that are inherited'.

Example – 3: Matchstick Houses - Patterns In Number

When teaching understanding the patterns in number/algebra, is a common task to provide students with a diagram of a pattern (e.g., house outlines made with matchsticks). It is then possible to devise a series of questions that explore both the surface and deep thinking around the objects and principles involved in pattern making.

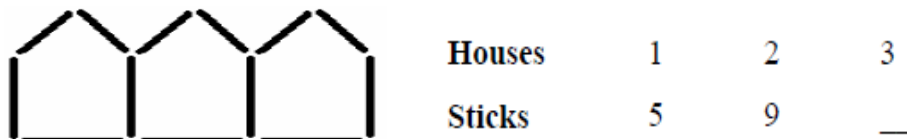


Figure – 1.6: Example of Matchstick Houses

A simple **Uni-structural** question (one idea) requires elicitation of a response based on handling one aspect of the given data; *“How many sticks are needed for 3*

houses?” This task can be answered most simply by counting the number of sticks shown in the diagram to come up with the answer of 13.

The next level, **Multi-structural**, is to require two or more ideas that are handled independently or serially. For example, *“How many sticks are needed for each of these three houses?”* requires the learner to take the given pattern and count the sticks for each house (5 each). To require deep thinking, the teacher needs to frame a question about finding a **Relationship** within the given material, rather than persist with surface approaches of count or draw-and-count: For example, *“If 52 houses require 209 sticks, how many sticks do you need to be able to make 53 houses?”* (Answer: 213). In order to respond, a child must detect that for every additional house four more sticks are required, regardless of how many houses there are.

Extended abstraction within the domain of algebra is commonly achieved through explicit attention to more general rules that apply in all cases, whether such rules are expressed in words or algebraic terminology. Such an extended abstract task would be *“Make up a rule to count how many sticks are needed for any number of houses”*. This demands a response that identifies not only the four sticks per house but also the need for one more to close off the last house in the series (**e.g. $S = 4H + 1$**). If a student provided this response, it would demonstrate understanding not only the relationship of sticks to houses but also the abstract extension that applies to all cases regardless of actual numbers.

This example-3 had taken in the context of chapter from Mathematics of class-IX – introducing the concept of ‘Linear equation in two variables’. In the above example, formula is formulated by using two variables S(sticks) & H(houses).

From the explanations, it can be seen that how knowledge is constructed according to all the levels of SOLO taxonomy and how interaction/ responses proceeding the learning from known to unknown knowledge. At entry level it could be assumed that learners are familiar with the figures made by matchsticks, Mathematical operations, concept of pattern/s and means of single variable/s. Such known knowledge could be utilised in further proceedings for the construction / discovery of unknown knowledge

as 'Linear equation in two variables' within the framework of SOLO taxonomy. Ahead, some of the advantages of the SOLO taxonomy are mentioned.

1.7.4 Advantages Of SOLO Model For The Evaluation Of Students Learning

Following are some advantages of the SOLO Taxonomy as precisely stated by the **Hook & Mills (2012)** as: There are several advantages of the SOLO model over the Bloom taxonomy in the evaluation of student learning.

- ❖ These advantages concern not only item construction and scoring, but incorporate features of the process of evaluation that pay attention to how students learn, and how teachers devise instructional procedures to help students use progressively more complex & cognitive processes.
- ❖ Unlike the Bloom taxonomy, which tends to be used more by teachers than by students, the SOLO can be taught to students such that they can learn to write progressively more difficult answers or prompts.
- ❖ There is a closer parallel to how teachers teach and how students learn.
- ❖ Both teachers and students often progress from more surface to deeper constructs and this is mirrored in the four levels of the SOLO taxonomy.
- ❖ There is no necessary progression in the manner of teaching or learning in the Bloom taxonomy.
- ❖ The SOLO taxonomy not only suggests an item writing methodology, but the same taxonomy can be used to score the items. The marker assesses each response to establish either the number of ideas (one = unistructural; two = multistructural), or the degree of interrelatedness (directly related or abstracted to more general principles). This can lead to more dependability of scoring.
- ❖ Similarly, teachers could be encouraged to use the 'plus one' principle when choosing appropriate learning material for students. That is, the teacher can aim to move the student one level higher in the taxonomy by appropriate choice of learning material and instructional sequencing.

1.7.5 Observing From Bloom's Taxonomy To S.O.L.O. Taxonomy

Indian education system follows the Bloom's Taxonomy to design instructional strategies in terms to achieve instructional objectives means the concrete statements framed to fulfil the desired goals. Also this taxonomy is used to evaluate the

performances or learning of the students. But for the present research study, SOLO Taxonomy has been used to design instructional strategy. Present research study is not meant to compare both (or other) taxonomies, but few literature reviews are presented here below to have comparative insights of both the taxonomies. SOLO Taxonomy is described in the next section, here have a brief about Bloom's Taxonomy.

Bloom's Taxonomy is developed by Benjamin Bloom, basically consist of three domains as (i) Cognitive, (ii) Affective and (iii) Psychomotor. Each domain consist further levels and here concern is with Cognitive domain. This domain is having the levels as (a) Knowledge, (b) Comprehension, (c) Application, (d) Analysis, (e) Synthesis, and (f) Evaluation.

According to **Emeny (2014)**, the SOLO Taxonomy is a notion that describes the stages of learning that students go through to reach a real depth of understanding on a topic. It outlines the journey from surface to deep learning. SOLO is John Hattie's taxonomy of choice and is currently being studied in depth at his Visible Learning Labs (Osiris Educational Outstanding Teaching Conference, 2014). It is seen by Hattie and other academics as having many advantages over other taxonomies, in particular that of Benjamin Bloom.

Quoted advantages over Bloom's Taxonomy include: **(1) The SOLO Taxonomy emerged from in-classroom research whereas Bloom's Taxonomy was theorized from a proposal by a committee of educators. (2) SOLO is a taxonomy about teaching and learning vs. Bloom's which is about knowledge. (3) SOLO is based on progressively more challenging levels of cognitive complexity. It is argued this is less clear within Bloom's Taxonomy. (4) It is claimed that educators and students agree more consistently which level a piece of student work has reached on the SOLO Taxonomy than on Bloom's Taxonomy. (5) SOLO is more simple to understand and apply than Bloom's making it more accessible for students to grasp, even primary phase.**

Moreover, **Emeny (2014)** shared the experience as, whilst interesting from a mostly academic perspective, these advantages are unlikely to grab the coal-face busy professional teacher and convince them to go SOLO in their planning. I had the same thought originally until I understood how incredibly simple SOLO is and that it

seemed to ‘work’ for a math classroom a lot better than Bloom’s Taxonomy does. It makes sense to me as a good summarization of what I have learned from experience as the way students learn in math.

However, as per the **Hook & Mills (2012)** reported as the SOLO Taxonomy provides a measure of cognitive learning outcomes or understanding of thinking that teachers have felt comfortable adopting. This hierarchical model is comprehensive, supported by objective criteria, and used across different subjects and on differing types of assignments. Teachers enjoy the way that SOLO represents student learning of quite diverse material in stages of ascending structural complexity, and that these stages display a similar sequence across tasks. Furthermore, surface or deep levels of understanding can be planned for and assessed by coding a student’s thinking performance against unistructural, multistructural, relational, or extended abstract categories. Using visual symbols to represent levels of understanding in SOLO means that coding for complexity of thinking can be undertaken by both student and teacher, allowing “where should we go next?” decisions and thinking interventions to more accurately target student learning needs.

According to **Hook & Mills (2012)**, following points are highlighting on comparative aspects of both Bloom’s and SOLO Taxonomy.

- ♣ Most of the evaluations are philosophical treatises noting, among other criticisms, that there is no evidence for the invariance of these stages, or claiming that the taxonomy is not based on any known theory of learning or teaching.
- ♣ The Bloom taxonomy presupposes that there is a necessary relationship between the questions asked and the responses to be elicited, whereas in the SOLO taxonomy both the questions and the answers can be at differing levels.
- ♣ Whereas Bloom separates 'knowledge' from the intellectual abilities or process that operates on this 'knowledge', the SOLO taxonomy is primarily based on the processes of understanding used by the students when answering the prompts. Knowledge, therefore, permeates across all levels of the SOLO taxonomy.
- ♣ Hierarchy. Bloom has argued that his taxonomy is related not only to complexity but also to an order of difficulty such that problems requiring behaviour at one level should be answered more correctly before tackling problems requiring

behaviour at a higher level. Although there may be measurement advantages to this increasing difficulty, this is not a necessary requirement of the SOLO method. It is possible for an item at the relational level, for example, to be constructed so that it is less difficult than an item at the unistructural level. For example, an item aiming to elicit relational responses might be 'How does the movement of the Earth relative to the sun define day and night'. This may be easier (depending on instruction, etc.) than a unistructural item that asks 'What does celestial rotation mean?'

- ♣ Bloom's taxonomy is not accompanied by criteria for judging the outcome of the activity (Ennis, 1985), whereas SOLO is explicitly useful for judging the outcomes. Take for example, a series of art questions suggested by Hamblen (1984).

As researcher of the present research study also learnt from this comparative knowledge and desired to experiment the SOLO Taxonomy in terms to study in fair manner and experience the feasibility of the said taxonomy while shifting focus from usual practices of Bloom's taxonomy to SOLO taxonomy with the Mathematics teaching-learning.

1.8 RATIONALE OF THE STUDY

Understanding about "Learning to learn" requires the learner to think about the strengths and weaknesses of their own understanding as well thinking when they are learning and to make thoughtful decisions on what to do next. Students of all ages can use SOLO levels, rubrics and frameworks to answer the questions: What am I learning?; How is it going?; and What do I do next ?

The idea for the present research study propagated from the questions like (i) how to bring some variations in Mathematics teaching-learning?, (ii) how to enhance understanding in Mathematics among the learners?, (iii) how to follow the concept of constructivism? and (iv) how to bring change in assessment criteria to evaluate actual learning/understanding in Mathematics? To address such queries and thoughts, researcher had conducted investigation through various literatures and research studies where researcher found about SOLO taxonomy. Hence, researcher had studied

more about SOLO taxonomy and appreciated to implement/ experiment it for Mathematics teaching-learning as well as for assessment.

From the literature reviews, it is learnt that Mathematics is the logic based subject where mastery or the grip on this subject can be acquired by better understanding and the thinking. Perhaps, **Gandhi & Varma (2007)** stated as it is important to note that present status of the Mathematics is the ‘rote memorization (of the examples also)’ and the ‘traditional way of teaching’ which are responsible for the poor performance of the children as well as for lowering the standard of Mathematics education. The perfunctory process that undergoes in the teaching of school Mathematics according to **Schoenfeld (1987)** is: (i) A task is selected by the teacher to introduce a technique or skill, (ii) The technique is illustrated, (iii) More exercises are provided for practice in the illustrated skill.

According to **Gandhi & Varma (2007)**, a large number of students have rarely understood Mathematics in its right perspective and meaning. Learning to think Mathematically involves a great deal more than having large amounts of knowledge. But this notion should get reflected in the pedagogy being practiced by teachers in Mathematics classes. Apologetically, the roots of the tragedy lie in the structure of how Mathematics is taught. One understands how to think Mathematically when one is resourceful, flexible and efficient in one’s ability to deal with new problems in Mathematics. The available research suggests that there may be better ways for students to learn Mathematics than mere listening to their teachers followed by drill.

The lifelong learning is necessary and meant to stress on the actual understanding (i.e. relational, not the instrumental) of the concepts and progressively leads to the higher thinking. To deal with such issues, new strategies or approaches are always required to research or to apply in order to avoid mechanical or boredom practices and to inculcate the active and constructive practices. As the explanation given by the **Killen & Hattingh (2004)**, about the understanding is that the demonstrations of learning depend upon the learner's understanding. If that understanding is shallow, the demonstration of learning cannot be complex or sophisticated. Understanding may be regarded as the ability to provide explanations, or the ability to think logically, or to solve unfamiliar problems, or to reinterpret objective knowledge, or to view things

from multiple perspectives - to mention just a few possibilities. A particular educator's idea of what it means to 'understand' will influence the way that person tries to help learners to understand and how they attempt to assess their learners' understanding.

Why SOLO? SOLO is the new theory basically leads to consistent thinking and understanding. Also **UNICEF (2005b)** stated as SOLO is a true hierarchic taxonomy – increasing in quantity and quality of thought. It is a potent instrument in differentiating curriculum and provides cognitive challenge for learners. Use of SOLO allows us to balance the cognitive demand of the questions that we ask and to scaffold students into deeper thinking and meta-cognition. It also allows teachers and learners to ask deeper questions without creating new ones. Also, the feature of 'Constructive Alignment' intends towards the constructive approach and outcome based instructional designs in terms to encourage the student-centred teaching-learning processes.

After extensive reading on SOLO taxonomy, researcher was motivated to study this new theory in a practical manner and to observe its implications for the gaps identified in Mathematics. Also there is a positive favor about the SOLO taxonomy used for assessment aspects as learnt from several reviews. Also useful for measuring the progress or the level of understanding (surface or deep) at any level of any aspects related with Mathematics. The Researcher of the present research study also appreciates that, SOLO taxonomy may be having any issues or limitations but for the present study, researcher is looking forward and will try to find out the applications of this taxonomy whether (in practical sense) it resulted in positive or negative manner. In this study, researcher is not intended to compare this theory or taxonomy with other theories or taxonomy (not even Bloom's taxonomy).

The researcher is intended to implement this study on the students of the 9th grade as looking to the maturity or developmental stage at this age-grade group, where the teaching-learning in Mathematics can be carried to all the five levels of the SOLO Taxonomy. All the Instruments and instructional strategies prepared will be based on the SOLO taxonomy and will be used to measure for the - (i) teaching-learning process, (ii) teaching of the teacher (self) and (iii) learning of the learners. All these three criteria will be measured at all five levels of the SOLO taxonomy. The

Researcher also having the clarification that the purpose of the present research study established with the SOLO taxonomy is to provide learners with a detailed description of their current understanding of some particular fact, concept, principle or process and the purpose is not to label learners as higher or lower achievers or performers.

According to **Spady (1994)**, there is always an expectation that a learners' response/s can be improved through instruction and/or experience. This notion of 'capacity for continual improvement' is consistent with one of the basic principles of OBE (Outcome Based Education) is that - all learners can succeed if they have appropriate opportunities and time to learn. The SOLO taxonomy enables teachers to make inferences about the depth of learners' understanding by examining the way they structure their oral or written responses to open-ended questions. (A response may be anything from a short oral answer to a lengthy essay). For the present study also, the researcher has given more concerns about how to improve the responses of the students through the systematic interactions or questioning.

Thus, considering to these thoughts expressed in this section, researcher was proposed to opt the present experimental research study. The next section is detailing on the particulars of the present research study that is brief methodology of this current research study.

1.9 STATEMENT OF THE PROBLEM

Developing And Implementing Instructional Strategy On The Structure Of Observed Learning Outcomes (SOLO) Taxonomy For Mathematics Of Class-IX

1.10 OBJECTIVES OF THE STUDY

- (1) To develop the SOLO Taxonomy based instructional strategy for Mathematics of Class-IX.
- (2) To implement the developed SOLO Taxonomy based instructional strategy in Mathematics of Class-IX.
- (3) To study the effectiveness of the developed SOLO Taxonomy based instructional strategy with respect to the chapter-wise achievement of the group studied through developed instructional strategy.

- (4) To study the effectiveness of the developed SOLO Taxonomy based instructional strategy with respect to the overall achievement of the group studied through developed instructional strategy.
- (5) To study the effectiveness of the developed SOLO Taxonomy based instructional strategy with respect to the SOLO Level-wise achievement of the group studied through developed instructional strategy.
- (6) To study the effectiveness of the developed SOLO Taxonomy based instructional strategy with respect to chapter-wise reactions of the group studied through developed instructional strategy.
- (7) To study the effectiveness of the developed SOLO Taxonomy based instructional strategy with respect to the overall reactions of the group studied through developed instructional strategy.

1.11 HYPOTHESES

- H₁:** There will be no significant difference between the mean scores of Achievement test observed for a chapter-Heron's Formula at Post-test among the group studied through developed instructional strategy and the group studied through conventional mode.
- H₂:** There will be no significant difference between the mean scores of Achievement test observed for a chapter-Linear Equation In Two Variables at Post-test among the group studied through developed instructional strategy and the group studied through conventional mode.
- H₃:** There will be no significant difference between the mean scores of Achievement test observed for a chapter-Quadrilaterals at Post-test among the group studied through developed instructional strategy and the group studied through conventional mode.
- H₄:** There will be no significant difference between the mean scores of Achievement test observed for a chapter-Statistics at Post-test among the group studied through developed instructional strategy and the group studied through conventional mode.
- H₅:** There will be no significant difference between the mean scores of Achievement test observed for a chapter-Probability at Post-test among the group studied through developed instructional strategy and the group studied through conventional mode.

- H₆:** There will be no significant difference between the mean scores of Overall Achievement Test observed among the group studied through developed instructional strategy and the group studied through conventional mode.
- H₇:** There will be no significant difference between the mean scores of Overall Achievement Test observed among the group studied through developed instructional strategy and the group studied through conventional mode at Pre-structural level of the SOLO Taxonomy.
- H₈:** There will be no significant difference between the mean scores of Overall Achievement Test observed among the group studied through developed instructional strategy and the group studied through conventional mode at Uni-structural level of the SOLO Taxonomy.
- H₉:** There will be no significant difference between the mean scores of Overall Achievement Test observed among the group studied through developed instructional strategy and the group studied through conventional mode at Multi-structural level of the SOLO Taxonomy.
- H₁₀:** There will be no significant difference between the mean scores of Overall Achievement Test observed among the group studied through developed instructional strategy and the group studied through conventional mode at Relational level of the SOLO Taxonomy.
- H₁₁:** There will be no significant difference between the mean scores of Overall Achievement Test observed among the group studied through developed instructional strategy and the group studied through conventional mode at Extended Abstract level of the SOLO Taxonomy.
- H₁₂:** There will be no significant difference in the reactions for the learning experiences gained for a chapter-Heron's Formula by the group studied through the developed instructional strategy.
- H₁₃:** There will be no significant difference in the reactions for the learning experiences gained for a chapter-Linear Equation In Two Variables by the group studied through the developed instructional strategy.
- H₁₄:** There will be no significant difference in the reactions for the learning experiences gained for a chapter-Quadrilaterals by the group studied through the developed instructional strategy..

H₁₅: There will be no significant difference in the reactions for the learning experiences gained for a chapter-Statistics by the group studied through the developed instructional strategy.

H₁₆: There will be no significant difference in the reactions for the learning experiences gained for a chapter-Probability by the group studied through the developed instructional strategy.

H₁₇: There will be no significant difference in overall reactions received for the developed instructional strategy by the group studied through the developed instructional strategy.

1.12 EXPLANATION OF THE TERMS

1.12.1 Instructional Strategy: Instructional strategy is inclusive of all the learning experiences provided by the researcher in order to achieve the desired educational objectives. The learning experiences for the present study will be designed based on the SOLO taxonomy and all its level in Mathematics and will also include activities for group and individual, experiments, demonstrations, brainstorming/ discussions. The instructional strategy will include lesson-planning, testing, various methodologies like Lecture, Heuristic, CAM, Games, puzzles, role plays and the use of technology, or Information and Communication Technology(ICT).

1.12.2 SOLO Taxonomy: SOLO stands for the Structure of Observed Learning Outcomes. SOLO Taxonomy is the five layered (pre-structural, uni-structural, multi-structural, relational, extended abstract) hierarchical framework proposes for the progressive (surface to deep) teaching-learning and measures the learning outcomes. This framework will be used to design lesson plans, rubrics, observation schedules, test-items.

1.13 OPERATIONALIZATION OF THE TERMS

1.13.1 Achievement: Achievement will be measured by the scores of both, the group studied through developed instructional strategy and the group studied through conventional mode which will be gained from the

achievement test conducted in Mathematics at Post-test level by the researcher.

1.13.2 Effectiveness: Effectiveness will be measured in terms of significance of differences between post-test mean scores achieved through achievement test by the group studied through developed instructional strategy and the group studied through the conventional mode. Effectiveness will also be observed based on the responses of the students during the implementation of the instructional strategy in Mathematics.

1.13.3 Reactions: Reactions will be reflective responses expressed by the group studied through developed instructional strategy on their overall learning experiences provided by the researcher during the implementation of the developed instructional strategy in Mathematics.

1.14 DELIMITATION OF THE STUDY

The present study was delimited in the following manner.

- a) Only English medium secondary schools of the Vadodara city those following the syllabus of the Central Board of Secondary Education (CBSE).
- b) The experiment of the research study was conducted during an academic year of 2014-2015.
- c) The study was also being delimited to the selected five chapters of Class-IX Mathematics. Selected (selection criteria is shown in the following figure) Chapters were (i) Heron's Formula, (ii) Linear Equations in Two Variables, (iii) Quadrilaterals, (iv) Statistics, and (v) Probability. (as in figure-1.7)

1.15 SCHEME OF CHAPTERIZATION

Conceptual framework is framed in the first chapter as the introduction to the present research study. Conceptual framework is comprised with the points starting with the Introduction, Mathematics education in India: An overview, School Mathematics education: a polity perspective, Focusing on secondary school Mathematics education, Mathematics teaching and learning: The concerns, Improvisation Of Mathematical understanding, Taxonomies of learning: The Highlights, The SOLO

Taxonomy: An abstract view followed by the rationale of the study, statement of the problem, objectives of the study, hypotheses, explanation of the terms, operationalization of the terms and delimitation of the study. Further, the thesis has been presented chapter-wise in the following way.

The second chapter devoted to review of related literature and its implications. The third chapter presents the plot and processes of the study by giving details of the methodology adopted, at length describes about the development and implementation of an instructional strategy, procedure for the data collection and the techniques used to analyze data. The fourth chapter describe the data analysis and interpretation as well the conclusion of objectives and testing the hypotheses. The fifth chapter is the summary of the entire research work carried out with major findings, implication of the study, suggestions for further research and discussion.

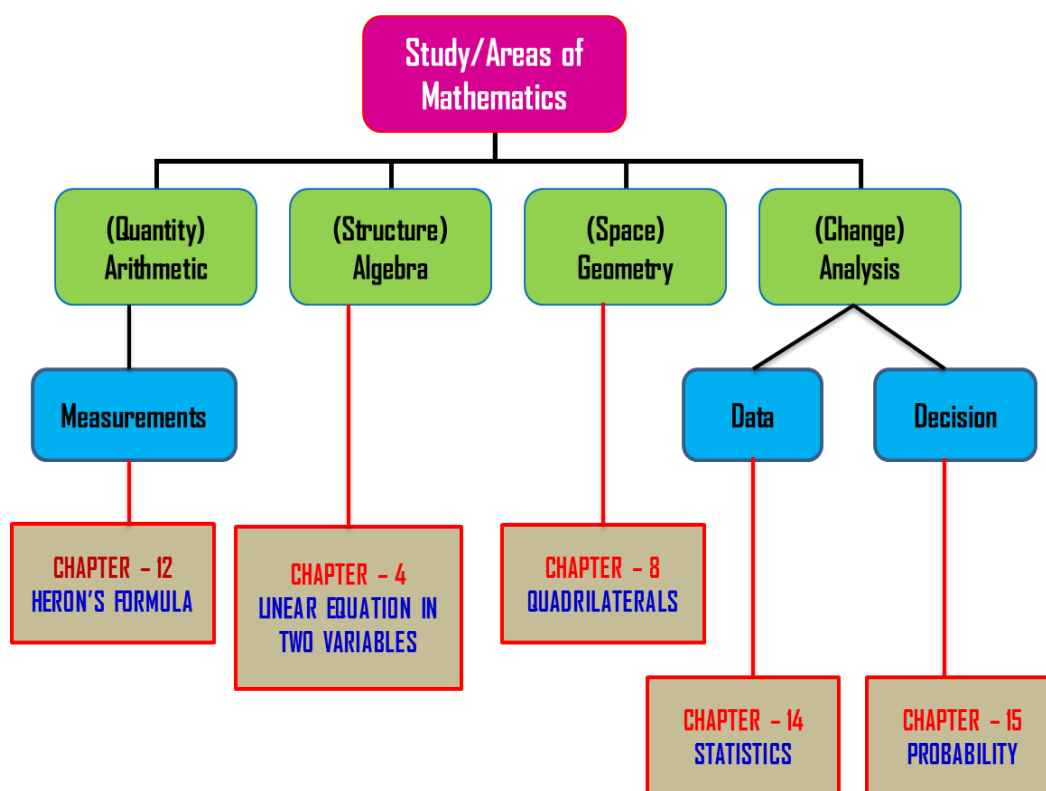


Figure-1.7: Selection for the chapters of class-IX Mathematics for the experimental study

