7 Particle Transport and Loss

Now that we have seen how complicated a mixture a plasma can be, it is time to take a step back and consider the simplest possible plasma made only of electrons, one type of ion and one type of neutral. The *transport* of charged particles is dealt with in this section. For low pressure plasmas this deals with the basic question of how particles move to the walls. We also look at what happens right next to any solid walls in contact with the plasma.

7.1 Diffusion

Particles in a gas undergo a *random walk* process as they move about. Collisions change the direction of motion and the particles move in straight lines between collisions. Because we are dealing with enormous numbers of particles, we can assume that the changes in direction are completely random, and this means that there is no preferred direction for the particles to move in. If we start by labelling a bunch of particles in a region A and then follow them, we will find them spreading out in all directions as they move about and collide with other particles and each other.

Now consider two separated (equal volume) regions A and B. If we label *all* the particles in these regions and follow them, we'll find that after a time some of the particles from A have ended up in B and vice versa. However, if A and B initially contained unequal numbers of particles ($n_A > n_B$, say) then we expect that more from A have ended up in B than the other way round.

The random walk process thus tends to equalise the density in the gas, smoothing out any peaks and filling in any hollows, and this applies equally well whether we are considering neutral molecules or charged particles. The *diffusion* of particles can be expressed by Fick's law which relates the resulting flux $\mathbf{\Gamma}$ to the density gradient which causes it,

$$\mathbf{\Gamma} = -D\nabla n \tag{7.1}$$

where D is the *diffusion coefficient* and ∇n gives the concentration gradient in the x, y and z directions and is given by

$$\nabla n = \mathbf{i}\frac{\partial n}{\partial x} + \mathbf{j}\frac{\partial n}{\partial y} + \mathbf{k}\frac{\partial n}{\partial z} \tag{7.2}$$

In some important cases the concentration is almost uniform in two of the directions (y and z say) so there is no variation in n in these directions. Thus $\partial n/\partial y$ and $\partial n/\partial z$ are both zero and we have

$$\nabla n = \mathbf{i}\frac{dn}{dx} \tag{7.3}$$

and Fick's law in one dimension becomes

$$\Gamma = -D\nabla n = -D\frac{dn}{dx}. \tag{7.4}$$

The spatial variations in density will be smoothed more quickly if the particles move faster and collide less often. The diffusion coefficient is given by

$$D = \frac{k_B T}{m\nu_m} \tag{7.5}$$

where ν_m is the *momentum transfer* collision frequency. This is not the same as ν_c since we are now concerned not just with how often collisions occur, but also how much the direction changes in each collision. The two frequencies are related by

$$\nu_m = \nu_c(1 - \langle\cos(\theta)\rangle) \tag{7.6}$$

with the cosine of θ, the scattering angle, averaged over all collisions in the expression. For isotropic scattering, $\langle\cos(\theta)\rangle = 0$ and $\nu_m = \nu_c$. The diffusion process we have just described is called *free diffusion* and it clearly does not apply to charged particles when an electric field is present. In that case there certainly is a preferred direction since the particles accelerate in the direction of (or opposite to) the field.

7.2 Mobility

When an electric field is present, charged particles are accelerated but their motion is disrupted by collisions with the neutrals. Let us consider the equation of motion for electrons

$$m_e \frac{dv_e}{dt} = -eE + \sum_n m_e \Delta v_n \delta(t - t_n). \tag{7.7}$$

In the second term Δv_n represents the change in velocity during the nth collision at time t_n (δ is the Dirac delta function). Collisions occur randomly and there is a huge number of electrons, so we'd like to average over time to find what happens to all the electrons as a group. This averaging gives

$$m_e \frac{dv_e}{dt} = -eE + m_e v_e \nu_m \tag{7.8}$$

where v_e now represents an average electron velocity. This equation of motion is interesting, because the second term on the right hand side has the form of a frictional drag (proportional to the velocity and opposing the motion). As a group, we can expect the electrons to eventually reach a terminal velocity in the medium of the (stationary) background gas. Equation 7.8 can be integrated to give this average velocity, which is called the *drift velocity*

$$v_d = -\frac{eE}{m_e \nu_m}. \tag{7.9}$$

We see that the electron drift is simply proportional to the strength of the field and the proportionality constant is called the *mobility*

$$\mu_e = \frac{e}{m_e \nu_m}. \tag{7.10}$$

Notes:

- The mobility measures how easily electrons or ions move through a neutral gas. It depends on the particle mass as well as the density of the gas and the cross section for elastic collisions.
- We have tacitly assumed that the *magnitude* of the electron velocity does not change during a collision with a neutral. This is a good approximation for electrons but not for ions. However, we can still define a mobility for ions by $\mu_i = e/m_i\nu_m$ where ν_m is the momentum transfer collision frequency for the ions.
- Since the collision cross section in general depends on the particle energy, the linear dependence in equation 7.9 is not strict. The mobility is still very useful for modelling plasmas and making estimates.

- Comparing equation 7.10 with equation 7.5) gives the Einstein relation

$$\mu = \frac{e}{k_B T} D. \tag{7.11}$$

This is a thermodynamic relation between the two *transport coefficients* which can often simplify calculations.

7.3 Ambipolar Diffusion

The total flux of charged particles in a gas consists of both the diffusion due to the density gradients and the drift in the electric field

$$\Gamma_{e,i} = \mp \mu_{e,i} n_{e,i} E - D_{e,i} \nabla n_{e,i}. \tag{7.12}$$

The subscripts refer to the electrons or ions and the sign depends on the sign of the particle charge. Since the plasma is quasineutral, $n_e = n_i = n$, and the gradients are the same. Consider now a small volume in the plasma. The fluxes of electrons and ions out of (or into) this volume must be equal. Any inequality results in a charge imbalance which generates an electric field that opposes the charge buildup by equalising the fluxes. Since electrons and ions have different mobilities and diffusion coefficients, a small electric field always exists in *nonuniform* plasmas where $\nabla n \neq 0$. This field can be found by equating the fluxes

$$-\mu_e n E - D_e \nabla n = \mu_i n E - D_i \nabla n \tag{7.13}$$

and solving for E. The flux of electrons (or ions) can then be written in the form of Fick's law

$$\Gamma_{e,i} = -D_a \nabla n \tag{7.14}$$

with the *same* diffusion coefficient

$$D_a = \frac{D_i \mu_e + D_e \mu_i}{\mu_e + \mu_i} \tag{7.15}$$

governing the motion of both species. This is *ambipolar* diffusion, which just means that both types of charges move together.

Notes:

- $D_e \gg D_i$ and $\mu_e \gg \mu_i$ so $D_i < D_a < D_e$. Ambipolar diffusion results in faster diffusion for the ions and slower diffusion for the electrons. The faster electrons drag the ions 'behind' them. We can approximate D_a by

$$D_a \approx D_i + D_e \frac{\mu_i}{\mu_e}. \tag{7.16}$$

- There seems to be a contradiction in using $n_e = n_i$ and finding that there is an electric field present (i.e. $n_e \neq n_i$). Is this a serious flaw in the argument?

- If $T_e = T_i$ the Einstein relations give $D_a = 2D_i$. In industrial plasmas we often have $T_e \gg T_i$ which leads to

$$D_a \approx D_i \frac{T_e}{T_i}. \tag{7.17}$$

- The character of the motion is quite different for the electrons and the ions. The fast, mobile electrons move about a great deal with the net flux quite small in comparison with the separate fluxes due to the field and the density gradient

$$\mu_e n E \approx D_e \nabla n. \tag{7.18}$$

In contrast, the ions are dragged along by the field

$$\Gamma_i \approx \mu_i n E \tag{7.19}$$

and their thermal motion is less important. We can thus normally assume that the electrons are in Boltzmann equilibrium but this is not true for the ions.

- At very low pressures collisions between ions and neutrals are rare and it is reasonable to assume that the ions fall ballistically through the ambipolar field. This is sometimes called the 'free fall' regime. At slightly higher pressures collisions become important but the drift velocity is still higher than the thermal velocity. A better estimate of the collision frequency is $\nu_m \approx |u_i|/\lambda_i$ where u_i is the ion drift velocity.