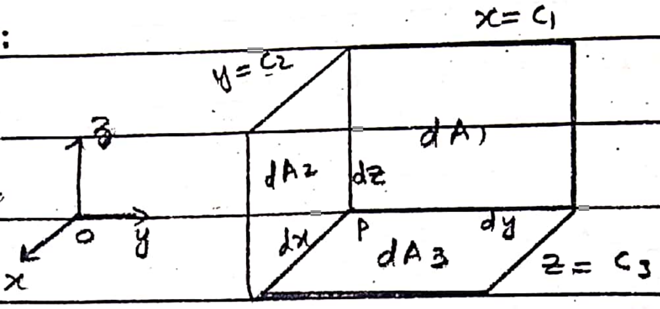
EXPRESSIONS FOR ARC LENGTH, AREA AND VOLUME ELEMENTS IN RECTANGULAR CARTESIAN COORDINATES:

In rectangular coordinates

$u_1 = x \qquad u_2 = y \qquad u_3 = z$

$h_1 = 1 \qquad h_2 = 1 \qquad h_3 = 1$

i) ARC LENGTH ELEMENT:



In orthogonal curvilinear coordinates

$\vec{r} = \vec{r}\,(u_1, u_2, u_3)$

$d\vec{r} = \frac{\partial \vec{r}}{\partial u_1} du_1 + \frac{\partial \vec{r}}{\partial u_2} du_2 + \frac{\partial \vec{r}}{\partial u_3} du_3$

$d\vec{r} = h_1 du_1 \hat{e}_1 + h_2 du_2 \hat{e}_2 + h_3 du_3 \hat{e}_3$

$(ds)^2 = d\vec{r} \cdot d\vec{r}$

$(ds)^2 = h_1^2 (du_1)^2 + h_2^2 (du_2)^2 + h_3^2 (du_3)^2$

In rectangular coordinates, it becomes

$(ds)^2 = (1)^2 (dx)^2 + (1)^2 (dy)^2 + (1)^2 (dz)^2$

$(ds)^2 = (dx)^2 + (dy)^2 + (dz)^2$

$ds = \sqrt{(dx)^2 + (dy)^2 + (dz)^2}$

ii) AREA ELEMENT:

In orthogonal curvilinear coordinates

$dA_1 = h_2 h_3 du_2 du_3$

$dA_2 = h_1 h_3 du_1 du_3$

$dA_3 = h_1 h_2 du_1 du_2$

In rectangular Cartesian Coordinates

$dA_1 = (1)(1)\, dy\, dz = dy\, dz$

$dA_2 = (1)(1)\, dx\, dz = dx\, dz$

$dA_3 = (1)(1)\, dx\, dy = dx\, dy$

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VOLUME ELEMENT:

In orthogonal curvilinear coordinates

$dV = h_1 h_2 h_3 \, du_1 \, du_2 \, du_3$

In rectangular cartesian coordinates

$dV = (1)(1)(1) \, dx \, dy \, dz$

$dV = dx \, dy \, dz$

EXPRESSION FOR JACOBIAN IN RECTANGULAR CARTESIAN COORDINATES:

In orthogonal curvilinear coordinates

$J = h_1 h_2 h_3$

In rectangular cartesian coordinates

$J = (1)(1)(1)$

$J = 1$

EXPRESSIONS FOR GRADIENT, DIVERGENCE, CURL AND LAPLACIAN IN RECTANGULAR CARTESIAN COORDINATES:

In rectangular coordinate system

$u_1 = x \qquad u_2 = y \qquad u_3 = z \qquad \hat{e}_1 = \hat{i} \quad \hat{e}_2 = \hat{j} \quad \hat{e}_3 = \hat{k}$

$h_1 = 1 \qquad h_2 = 1 \qquad h_3 = 1 \qquad \vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$

EXPRESSION FOR GRADIENT:

In orthogonal curvilinear coordinates

$$\nabla\psi = \frac{1}{h_1}\frac{\partial\psi}{\partial u_1}\hat{e}_1 + \frac{1}{h_2}\frac{\partial\psi}{\partial u_2}\hat{e}_2 + \frac{1}{h_3}\frac{\partial\psi}{\partial u_3}\hat{e}_3$$

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In rectangular cartesian coordinates

$$\nabla\psi = \frac{1}{1}\frac{\partial\psi}{\partial x}\hat{i} + \frac{1}{1}\frac{\partial\psi}{\partial y}\hat{j} + \frac{1}{1}\cdot\frac{\partial\psi}{\partial z}\hat{k}$$

$$\nabla\psi = \frac{\partial\psi}{\partial x}\hat{i} + \frac{\partial\psi}{\partial y}\hat{j} + \frac{\partial\psi}{\partial z}\hat{k}$$

EXPRESSION FOR DIVERGENCE:

In orthogonal curvilinear coordinates

$$\nabla\cdot\vec{A} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}(A_1h_2h_3) + \frac{\partial}{\partial u_2}(A_2h_1h_3) + \frac{\partial}{\partial u_3}(A_3h_1h_2)\right]$$

In rectangular cartesian coordinates

$$\nabla\cdot\vec{A} = \frac{1}{(1)(1)(1)}\left[\frac{\partial}{\partial x}(A_1(1)(1)) + \frac{\partial}{\partial y}(A_2(1)(1)) + \frac{\partial}{\partial z}(A_3(1)(1))\right]$$

$$\nabla\cdot\vec{A} = \partial A_1/\partial x + \partial A_2/\partial y + \partial A_3/\partial z$$

EXPRESSION FOR CURL:

In orthogonal curvilinear coordinates

$$\nabla\times\vec{A} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\hat{e}_1 & h_2\hat{e}_2 & h_3\hat{e}_3 \\ \partial/\partial u_1 & \partial/\partial u_2 & \partial/\partial u_3 \\ h_1A_1 & h_2A_2 & h_3A_3 \end{vmatrix}$$

In rectangular cartesian coordinates.

$$\nabla\times\vec{A} = \frac{1}{(1)(1)(1)}\begin{vmatrix} (1)\hat{e} & (1)\hat{j} & (1)\hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (1)A_1 & (1)A_2 & (1)A_3 \end{vmatrix}$$

$$\nabla\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_1 & A_2 & A_3 \end{vmatrix}$$

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EXPRESSION FOR LAPLACIAN:

In orthogonal curvilinear coordinates

$$\nabla^2\psi = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}\left(\frac{h_2h_3}{h_1}\frac{\partial\psi}{\partial u_1}\right)+\frac{\partial}{\partial u_2}\left(\frac{h_1h_3}{h_2}\frac{\partial\psi}{\partial u_2}\right)+\frac{\partial}{\partial u_3}\left(\frac{h_1h_2}{h_3}\frac{\partial\psi}{\partial u_3}\right)\right]$$

In rectangular cartesian coordinates

$$\nabla^2\psi = \frac{1}{(1)(1)(1)}\left[\frac{\partial}{\partial x}\left((1)\frac{\partial\psi}{\partial x}\right)+\frac{\partial}{\partial y}\left((1)\frac{\partial\psi}{\partial y}\right)+\frac{\partial}{\partial z}\left((1)\frac{\partial\psi}{\partial z}\right)\right]$$

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}$$

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