CURVILINEAR COORDINATES:

Given a point P with rectangular coordinates (x, y, z)

$$x = x(u_1, u_2, u_3)$$

$$y = y(u_1, u_2, u_3)$$

$$z = z(u_1, u_2, u_3)$$

we can associate a unique set of coordinates (u_1, u_2, u_3)

$$u_1 = u_1(x, y, z)$$

$$u_2 = u_2(x, y, z)$$

$$u_3 = u_3(x, y, z)$$

called the curvilinear coordinates of the point P.

→ Any point P can be defined in space not only by rectangular coordinates (x, y, z) but also by curvilinear coordinates (u_1, u_2, u_3).

TRANSFORMATION OF COORDINATES:

The set of equations

$x = x(u_1, u_2, u_3)$ $\quad y = y(u_1, u_2, u_3)$ $\quad z = z(u_1, u_2, u_3)$

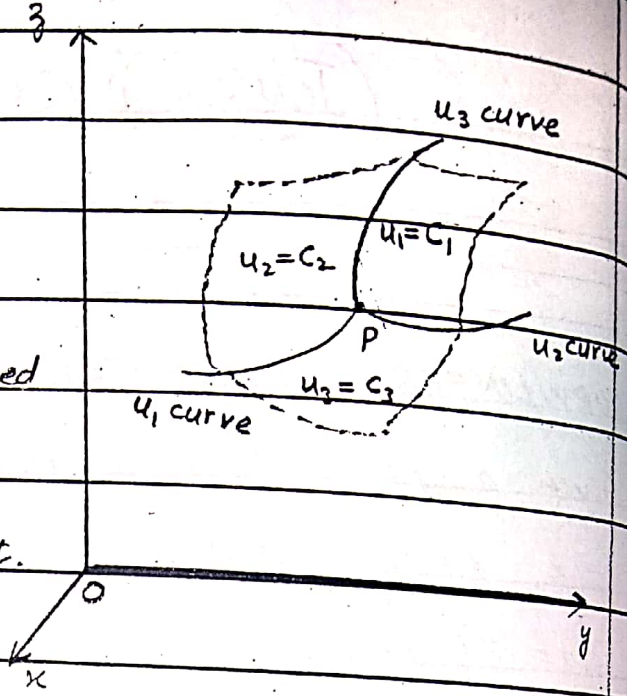
and

$u_1 = u_1(x, y, z)$ $\quad u_2 = u_2(x, y, z)$ $\quad u_3 = u_3(x, y, z)$

define the transformation of coordinates.

COORDINATE SURFACES:

The coordinate surfaces (or level surfaces) are the families of surfaces obtained by setting the coordinate equations equal to a constant.



If c_1, c_2, c_3 are constants then the surfaces

$u_1 = c_1$ $\qquad u_2 = c_2$ $\qquad u_3 = c_3$.

are called coordinate surfaces.

COORDINATE CURVES:

The coordinate surfaces are generally curved and each pair of coordinate surfaces intersect in curves called coordinate curves in space.

→ u_1- coordinate curve is that along which only u_1 varies while u_2 and u_3 are constants.

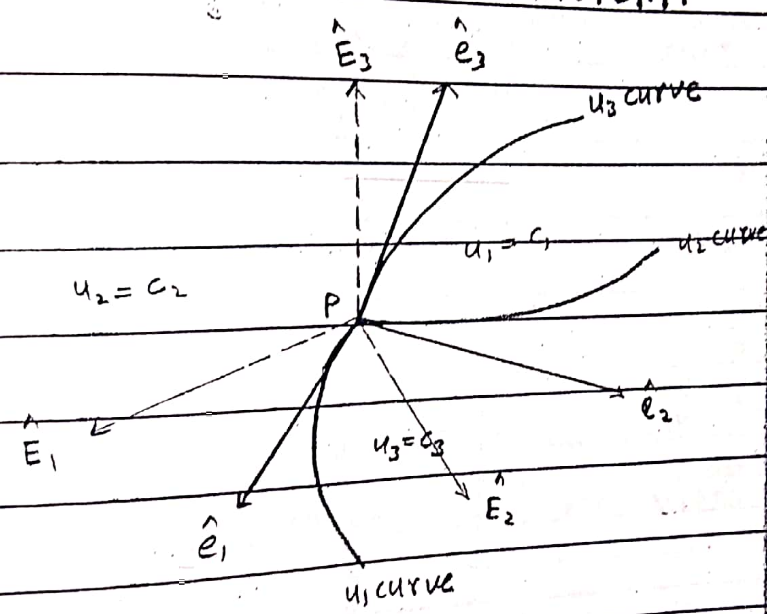
→ u_2- coordinate curve is that along which only u_2 varies while u_1 and u_3 are constants.

→ u_3- coordinate curve is that along which only u_3 varies while u_2 and u_1 are constants.

CURVILINEAR COORDINATE SYSTEM:

Since the three coordinate curves are generally not straight lines, as in rectangular coordinate system, Such a coordinate system is called the curvilinear coordinate system.

UNIT VECTORS IN CURVILINEAR COORDINATE SYSTEM:



UNIT TANGENT VECTORS:

Position vector of a point P $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$x = x(u_1, u_2, u_3)$

$y = y(u_1, u_2, u_3)$

$z = z(u_1, u_2, u_3)$ Thus $\vec{r} = \vec{r}(u_1, u_2, u_3)$

Tangent vector to u_1-coordinate curve at P $= \dfrac{\partial \vec{r}}{\partial u_1} = \hat{e}$

Tangent vector to u_2-coordinate curve at P $= \dfrac{\partial \vec{r}}{\partial u_2} =$

Tangent vector to u_3-coordinate curve at P $= \dfrac{\partial \vec{r}}{\partial u_3} =$

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Unit Tangent vector to u_1-coordinate curve at $P = \hat{e}_1$

$$\hat{e}_1 = \frac{\partial \vec{r}/\partial u_1}{|\partial \vec{r}/\partial u_1|}$$

$$\Rightarrow \frac{\partial \vec{r}}{\partial u_1} = \left|\frac{\partial \vec{r}}{\partial u_1}\right| \hat{e}_1$$

$$\Rightarrow \partial \vec{r}/\partial u_1 = h_1 \hat{e}_1 \quad , h_1 = \left|\partial \vec{r}/\partial u_1\right|$$

Unit Tangent vector to u_2-coordinate curve at $P = \hat{e}_2$

$$\hat{e}_2 = \frac{\partial \vec{r}/\partial u_2}{|\partial \vec{r}/\partial u_2|}$$

$$\Rightarrow \frac{\partial \vec{r}}{\partial u_2} = \left|\frac{\partial \vec{r}}{\partial u_2}\right| \hat{e}_2$$

$$\Rightarrow \frac{\partial \vec{r}}{\partial u_2} = h_2 \hat{e}_2 \quad , h_2 = \left|\partial \vec{r}/\partial u_2\right|$$

Unit Tangent vector to u_3-coordinate curve at $P = \hat{e}_3$

$$\hat{e}_3 = \frac{\partial \vec{r}/\partial u_3}{|\partial \vec{r}/\partial u_3|}$$

$$\Rightarrow \frac{\partial \vec{r}}{\partial u_3} = \left|\frac{\partial \vec{r}}{\partial u_3}\right| \hat{e}_3$$

$$\Rightarrow \frac{\partial \vec{r}}{\partial u_3} = h_3 \hat{e}_3 \quad , h_3 = \left|\partial \vec{r}/\partial u_3\right|$$

→ Here

$$h_1 = \left|\frac{\partial \vec{r}}{\partial u_1}\right| \qquad h_2 = \left|\frac{\partial \vec{r}}{\partial u_2}\right| \qquad h_3 = \left|\frac{\partial \vec{r}}{\partial u_3}\right|$$

are called the scalar factors.

→ In general, h_1, h_2, h_3 are functions of u_1, u_2, u_3 and $h_1 \neq 0$, $h_2 \neq 0$, $h_3 \neq 0$. Hence $\hat{e}_1, \hat{e}_2, \hat{e}_3$ are also functions of u_1, u_2, u_3.

UNIT NORMAL VECTORS :

Normal vector to the surface $u_1 = c_1$ at $P = \nabla u_1$

Normal vector to the surface $u_2 = c_2$ at $P = \nabla u_2$

Normal vector to the surface $u_3 = c_3$ at $P = \nabla u_3$