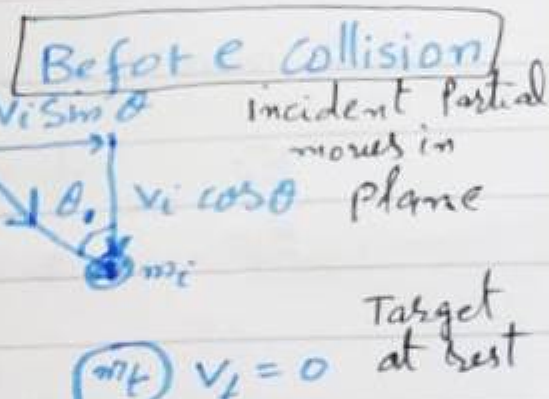
Elastic Collisions

Conservation of momentum along vertical direction

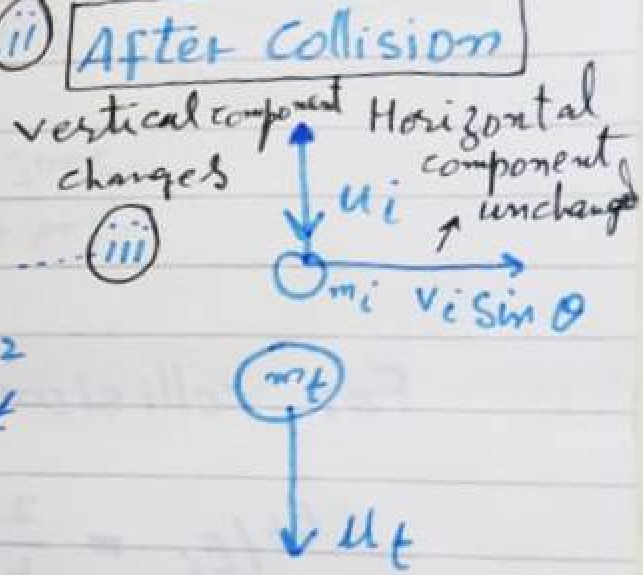


$$m_i v_i \cos\theta = m_i u_i + m_t u_t \quad \text{...(i)}$$

Solving (i) for u_i

$$u_i = m_i v_i \cos\theta - m_t u_t / m_i \quad \text{...(ii)}$$

Conservation of energy



Target moves along vertical direction

$$\frac{1}{2} m_i v_i^2 = \frac{1}{2} m_i (u_i^2 + v_i^2 \sin^2\theta) + \frac{1}{2} m_t u_t^2 \quad \text{...(iii)}$$

$$\frac{1}{2} m_i v_i^2 = \frac{1}{2} m_i u_i^2 + \frac{1}{2} m_i v_i^2 \sin^2\theta + \frac{1}{2} m_t u_t^2$$

$$m_i v_i^2 (1 - \sin^2\theta) = m_i u_i^2 + m_t u_t^2$$

$$m_i v_i^2 \cos^2\theta = m_i u_i^2 + m_t u_t^2 \quad \text{...(iv)}$$

Putting value of u_i from (ii) in (iv)

$$m_i v_i^2 \cos^2\theta = m_i \left[\frac{m_i v_i \cos\theta - m_t u_t}{m_i}\right]^2 + m_t u_t^2$$

$$m_i v_i^2 \cos^2\theta = \frac{m_i}{m_i^2}\{m_i^2 v_i^2 \cos^2\theta + m_t^2 u_t^2 - 2 m_i m_t v_i u_t \cos\theta\} + m_t u_t^2$$

$$m_i^2 v_i^2 \cos^2\theta = m_i^2 v_i^2 \cos^2\theta + m_t^2 u_t^2 - 2 m_i m_t v_i u_t \cos\theta + m_i m_t u_t^2$$

$$m_t u_t (m_t u_t - 2 m_i v_i \cos\theta + m_i u_t) = 0$$

$$m_t u_t - 2 m_i v_i \cos\theta + m_i u_t = 0$$

$$u_t (m_i + m_t) = 2 m_i v_i \cos\theta$$

$$\boxed{u_t = \frac{2 m_i v_i \cos\theta}{m_i + m_t}} \quad \text{...(V)}$$

The fractional energy transferred from m_i to mass m_t is

$$\frac{E_t}{E_i} = \frac{\frac{1}{2} m_t u_t^2}{\frac{1}{2} m_i v_i^2} = \frac{\cancel{1} m_t}{\cancel{m_i v_i^2}} \times \frac{4 \cancel{m_i^2} \cancel{v_i^2} \cos^2\theta}{(m_i + m_t)^2}$$

$$= \frac{4 m_i m_t \cos^2\theta}{(m_i + m_t)^2} \text{ — (vi)} ; \langle \cos^2\theta \rangle = \frac{1}{2}$$

$$= \frac{2 m_i m_t}{(m_i + m_t)^2} \text{ — (vii)}$$

(1) For collision between identical particle, $m_i = m_t$

$$E_t/E_i = \frac{2 m_i^2}{4 m_i^2} = \frac{1}{2}$$

For head on collision

$\theta = 0$, $\cos 0 = 1$

$$E_t/E_i = \frac{4 m_i^2}{4 m_i^2} = 1$$

(2) Light particle collides a heavy particle $m_i << m_t$

$$E_t/E_i = \frac{4 m_i m_t}{(m_i + m_t)^2} \cos^2\theta = \frac{2 m_i m_t}{(m_i + m_t)^2}$$

$$= \frac{2 m_i}{m_t}$$

$\theta = 0$ (head on)

$$E_t/E_i = \frac{4 m_i m_t}{m_t^2} = 4 \frac{m_i}{m_t}$$

electron collides with carbon monoxide molecule

$$2 m_i / 28 \times 1840 m_i = 10^{-4}$$

$$4 m_i / 28 \times 1840 m_i \simeq 10^{-4}$$

Inelastic Collision (PTO for diagram)

$$m_i v_i \cos\theta = m_i u_i + m_t u_t \quad \text{--- (i)}$$

conservation of momentum along vertical direction

$$u_i = m_i v_i \cos\theta - m_t u_t / m_i \quad \text{.... (ii)}$$

Conservation of energy

$$\frac{1}{2} m_i v_i^2 = \frac{1}{2} m_i (u_i^2 + v_i^2 \sin^2\theta) + \frac{1}{2} m_t u_t^2 + 2\Delta U$$

$$m_i v_i^2 (1 - \sin^2\theta) = m_i u_i^2 + m_t u_t^2 + 2\Delta U \quad \text{.... (iii)}$$

Putting value of u_i from (ii) in (iii)

$$m_i v_i^2 \cos^2\theta = m_i \left(\frac{m_i v_i \cos\theta - m_t u_t}{m_i}\right)^2 + m_t u_t^2 + 2\Delta U$$

$$= \frac{m_i}{m_i^2} (m_i^2 v_i^2 \cos^2\theta + m_t^2 u_t^2 - 2 m_i m_t v_i u_t \cos\theta) + m_t u_t^2 + 2\Delta U$$

$$m_i v_i^2 \cos^2\theta = m_i v_i^2 \cos^2\theta + \frac{1}{m_i}(m_t^2 u_t^2 - 2 m_i m_t v_i u_t \cos\theta) + m_t u_t^2 + 2\Delta U$$

$$2\Delta U = -\frac{m_t^2}{m_i} u_t^2 - m_t u_t^2 + 2 m_t v_i u_t \cos\theta$$

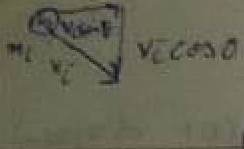
$$2\Delta U = -\frac{m_t}{m_i}(m_t + m_i) u_t^2 + 2 m_t v_i u_t \cos\theta \quad \text{--- (iv)}$$

$$2 \frac{d(\Delta U)}{du_t} = -\frac{2 m_t}{m_i}(m_t + m_i) u_t + 2 m_t v_i \cos\theta = 0$$

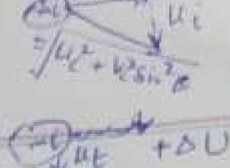
$$u_t = \frac{2 m_t v_i \cos\theta}{\frac{2 m_t}{m_i}(m_t + m_i)} = \frac{m_i v_i \cos\theta}{(m_i + m_t)} \quad \text{--- (v)}$$

Putting value of u_t from eq. (v) in eq (iv)

Before



After



$$2\Delta U = -\frac{m_t}{m_i}(m_i + m_t)\frac{m_i^2 v_i^2 \cos^2\theta}{(m_i+m_t)^2} + 2 v_i m_i \cos\theta \cdot \frac{m_i v_i \cos\theta}{(m_i + m_t)}$$

$$\cancel{\Delta U = \frac{2 m_t^2 v_i^2 \cos^2\theta}{2(m_i+m_t)} - \frac{m_i m_t v_i^2 \cos^2\theta}{2(m_i+m_t)}}$$

$$2\Delta U = -\frac{m_i m_t v_i^2 \cos^2\theta}{(m_i+m_t)} + \frac{2 m_i m_t v_i^2 \cos^2\theta}{(m_i+m_t)} = \frac{m_i m_t v_i^2 \cos^2\theta}{(m_i+m_t)}$$

$$\Delta U = \frac{m_i m_t v_i^2 \cos^2\theta}{2(m_i+m_t)} \quad \text{---- (vi)}$$

~~Fractional~~ KE of particle 1 (energy transferred) into internal energy of particle 2 ~~from mass m_i to m_t is~~

$$\frac{\Delta U}{\frac{1}{2} m_i v_i^2} = \frac{m_i m_t v_i^2 \cos^2\theta}{2(m_i+m_t)} \cdot \frac{2}{m_i v_i^2} = \frac{m_t \cos^2\theta}{(m_i+m_t)}$$

For head on collision; $\theta = 0$

$$= \frac{m_t}{(m_i+m_t)} \simeq 1$$

KE of mass 1 transferred into KE of second particles

$$\frac{\frac{1}{2} m_t u_t^2}{\frac{1}{2} m_i v_i^2} = \frac{\frac{1}{2} m_t u_t^2}{\frac{1}{2} m_i} \times \left(\frac{2 m_t u_t \cos\theta}{\frac{m_t}{m_i}(m_i+m_t)u_t^2 + 2\Delta U} \right)^2$$

from eq (IV) $\quad 2\Delta U + \frac{m_t}{m_i}(m_i+m_t)u_t^2 = 2 m_t v_i u_t \cos\theta$

$$v_i = \frac{\frac{m_t}{m_i}(m_i+m_t)u_t^2 + 2\Delta U}{2 m_t u_t \cos\theta}$$

If $\Delta U = 0$, then fractional energy transferred is

$$= \frac{m_t u_t^2}{m_i} \times \frac{4 m_t^2 u_t^2 \cos^2\theta}{\frac{m_t^2}{m_i^2}(m_i+m_t)^2 u_t^4} = \frac{4 m_i m_t \cos^2\theta}{(m_i+m_t)^2}$$