Box 7.5 (See Figure 7.5)

For a current source and sink, the potential V_P at any point P in the ground is equal to the sum of the voltages from the two electrodes, such that: $V_P = V_A + V_B$ where V_A and V_B are the potential contributions from the two electrodes, A(+I) and B(−I).

The potentials at electrode M and N are:

$$V_M = \frac{\rho I}{2\pi}\left[\frac{1}{AM} - \frac{1}{MB}\right], \quad V_N = \frac{\rho I}{2\pi}\left[\frac{1}{AN} - \frac{1}{NB}\right].$$

However, it is far easier to measure the potential difference, δV_{MN}, which can be rewritten as:

$$\delta V_{MN} = V_M - V_N = \frac{\rho I}{2\pi}\left\{\left[\frac{1}{AM} - \frac{1}{MB}\right] - \left[\frac{1}{AN} - \frac{1}{NB}\right]\right\}$$

Rearranging this so that resistivity ρ is the subject:

$$\rho = \frac{2\pi\delta V_{MN}}{I}\left\{\left[\frac{1}{AM} - \frac{1}{MB}\right] - \left[\frac{1}{AN} - \frac{1}{NB}\right]\right\}^{-1}$$

7.3 ELECTRODE CONFIGURATIONS AND GEOMETRIC FACTORS

7.3.1 General case

The final expression in Box 7.5 has two parts, namely a resistance term (R; units Ω) and a term that describes the geometry of the electrode configuration being used (Box 7.6) and which is known as the *geometric factor* (K; units m). In reality, the sub-surface ground does not conform to a homogeneous medium and thus the resistivity obtained is no longer the 'true' resistivity but the *apparent resistivity* (ρ_a) which can even be negative. It is very important to remember that the apparent resistivity is not a physical property of the sub-surface

Box 7.6 The geometric factor (see Figure 7.5)

The geometric factor (K) is defined by the expression:

$$K = 2\pi\left[\frac{1}{AM} - \frac{1}{MB} - \frac{1}{AN} + \frac{1}{NB}\right]^{-1}.$$

Where the ground is not uniform, the resistivity so calculated is called the *apparent resistivity* (ρ_a):

$$\rho_a = RK, \text{ where } R = \delta V/I.$$

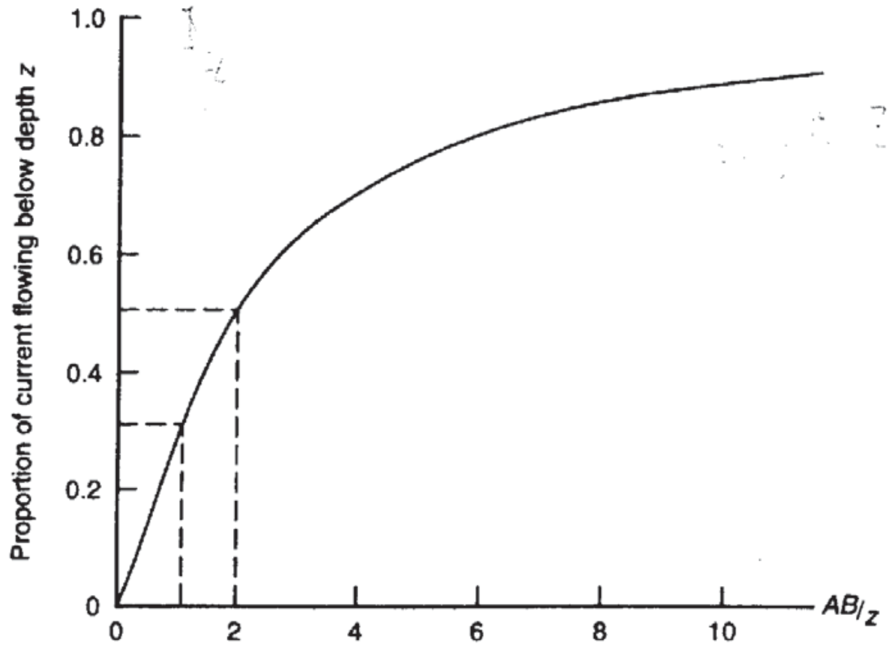


Figure 7.6 Proportion of current flowing below a depth z (m); AB is the current electrode half-separation

media, unlike the true resistivity. Consequently, all field resistivity data are apparent resistivity while those obtained by interpretation techniques are 'true' resistivities.

Figure 7.6 shows that, in order for at least 50% of the current to flow through an interface at a depth of z metres into a second medium, the current electrode separation needs to be at least twice – and preferably more than three times – the depth. This has obvious practical implications, particularly when dealing with situations where the depths are of the order of several hundreds of metres, so requiring very long cable lengths that can produce undesirable inductive coupling effects. For very deep soundings where the electrode separation is more than several kilometres, telemetering the data becomes the only practical solution (e.g. Shabtaie *et al.* 1980, 1982). However, it should be emphasised that it is misleading to equate the depth of penetration with the current electrode separation as a general rule of thumb in the region of a resistivity survey. This aspect is discussed in Section 7.3.3.

7.3.2 Electrode configurations

The value of the apparent resistivity depends on the geometry of the electrode array used, as defined by the geometric factor K. There are three main types of electrode configuration, two of which are named after their originators – Frank Wenner (1912a,b) and Conrad Schlumberger – and a range of sub-types (Table 7.2 and Figure 7.7). The geometric factors for these arrays are given in Box 7.7 and a worked example for the Wenner array is given in Box 7.8. Arrays highlighted in bold in Table 7.2 are those most commonly used.

Table 7.2 Electrode configurations (see also Figure 7.7)

Wenner arrays	**Standard Wenner**
	Offset Wenner
	Lee-partitioning array
	Tripotential (α, β and γ arrays)
Schlumberger array	**Standard Schlumberger**
	Brant array
	Gradient array
Dipole–dipole arrays	**Normal** (axial or polar)
	Azimuthal
	Radial
	Parallel
	Perpendicular
	Pole–Dipole
	Equatorial
	Square (special form of equatorial)

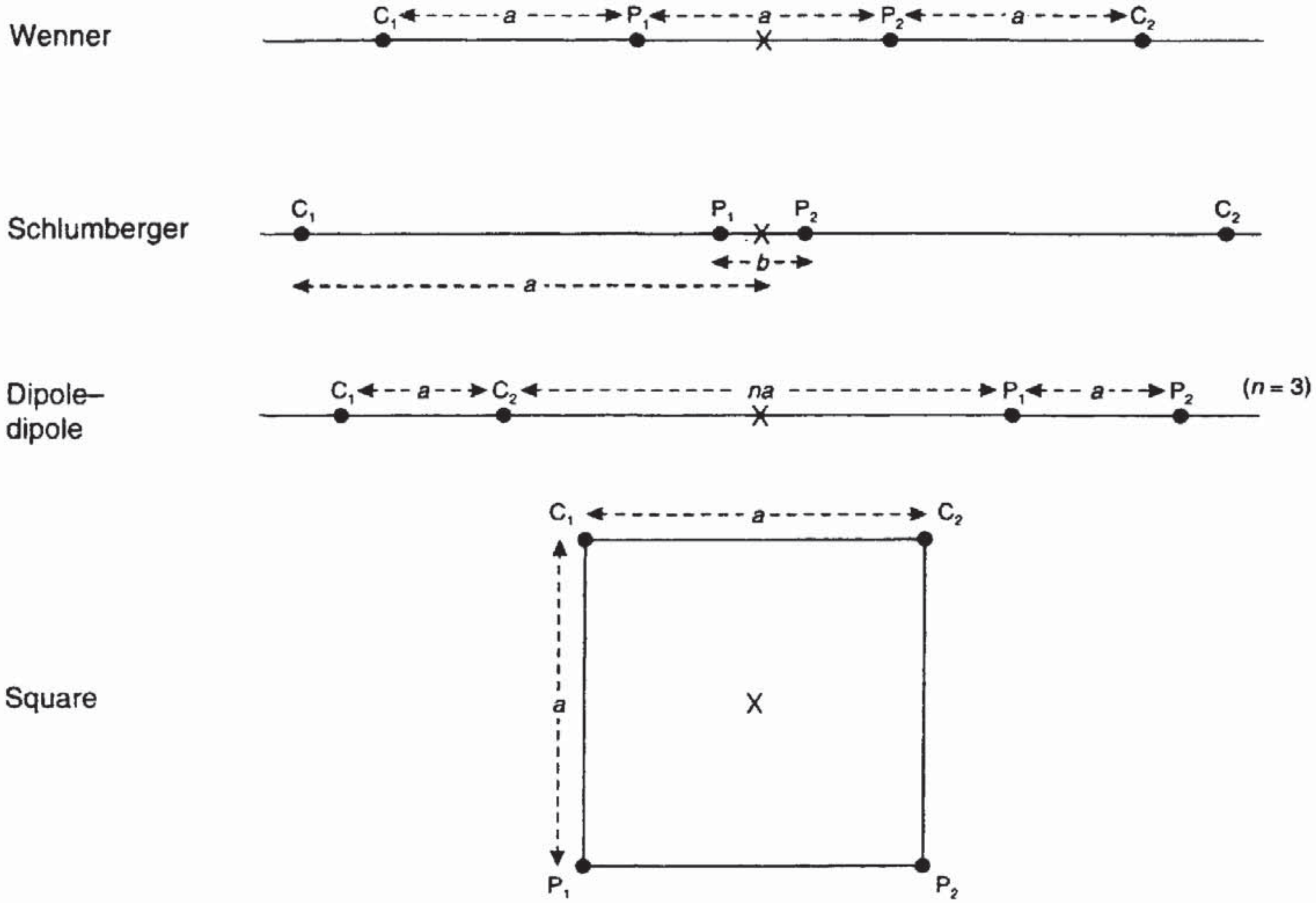


Figure 7.7 Electrode configurations used in electrical surveys

Dipole–dipole arrays have been used extensively by Russian geophysicists since 1950, and especially in Canada, particularly for 'induced polarisation' surveys (see Chapter 9) in mineral exploration, and in the USA in groundwater surveys (Zohdy 1974). The term 'dipole' is misapplied in a strict sense because the inter-electrode separation for each of the current or potential electrode pairs should be insignificant with respect to the length of the array, which it is not. However, the term is well established in its usage.

Box 7.7 Apparent resistivities for given geometric factors for electrode configurations in Figure 7.7

Wenner array: $\rho_a = 2\pi a R$ (alpha/beta arrays)
$\rho_a = 3\pi a R$ (gamma rays)

Two-electrode: $\rho_a = 2\pi s R$

Lee array: $\rho_a = 4\pi a R$

Schlumberger array: $\rho_a = \dfrac{\pi a^2}{b}\left[1 - \dfrac{b^2}{4a^2}\right]R;\quad a \geqslant 5b$

Gradient array: $\rho_a = 2\pi \dfrac{L^2}{a}\dfrac{1}{G}R$

$$\text{where } G = \frac{1-X}{(Y^2+(1-X)^2)^{3/2}} + \frac{1+X}{(Y^2+(1+X)^2)^{3/2}}$$

$$\text{and } X = x/L,\ Y = y/L$$

Dipole–dipole array: $\rho_a = \pi n(n+1)(n+2)aR$

Pole–dipole array: $\rho_a = 2\pi n(n+1)aR$

Square array: $\rho_a = \pi a(2+\sqrt{2})R$

These different types and styles of electrode configuration have particular advantages, disadvantages and sensitivities. Factors affecting the choice of array type include the amount of space available to lay out an array and the labour-intensity of each method. Other important considerations are the sensitivity to lateral inhomogeneities (Habberjam and Watkins 1967a; Barker 1981) and to dipping interfaces (Broadbent and Habberjam 1971).

A graphic example of the different responses by the three main electrode configurations is given by so-called 'signal contribution sections' (Barker 1979) shown in Figure 7.8. These sections are contoured plots of the contribution made by each unit volume of the sub-surface to the voltage measured at the surface.

[illegible]

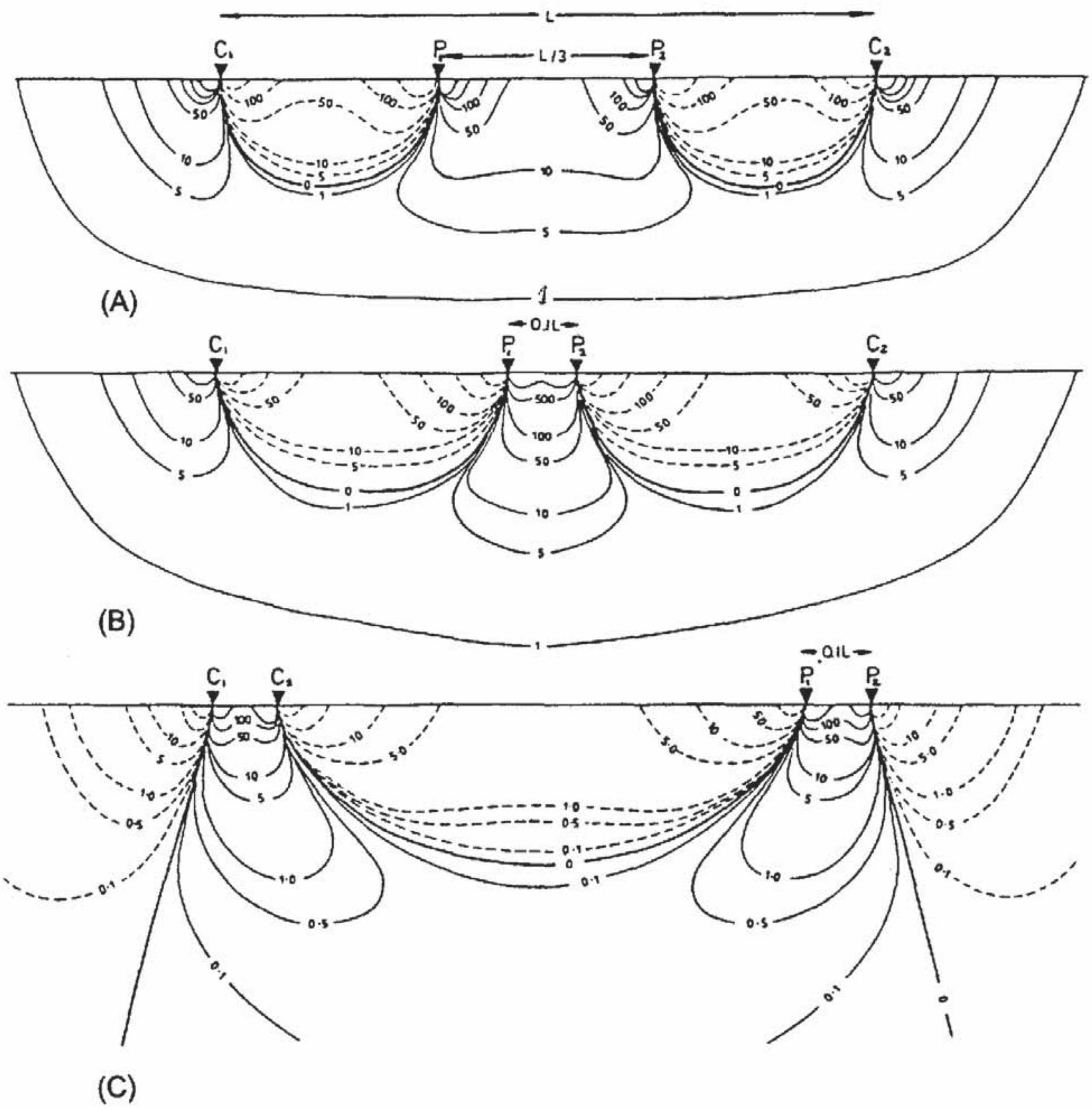


Figure 7.8 Signal contribution sections for: (A) Wenner, (B) Schlumberger and (C) dipole–dipole configurations. Contours indicate the relative contributions made by discrete volume elements of the sub-surface to the total potential difference measured between the two potential electrodes P_1 and P_2. From Barker (1979), by permission

Box 7.8 Worked example of how to calculate a geometric factor

Using the expression previously defined in Box 7.6 (see also Figure 7.5), and substituting in the correct values for the *Wenner array*:

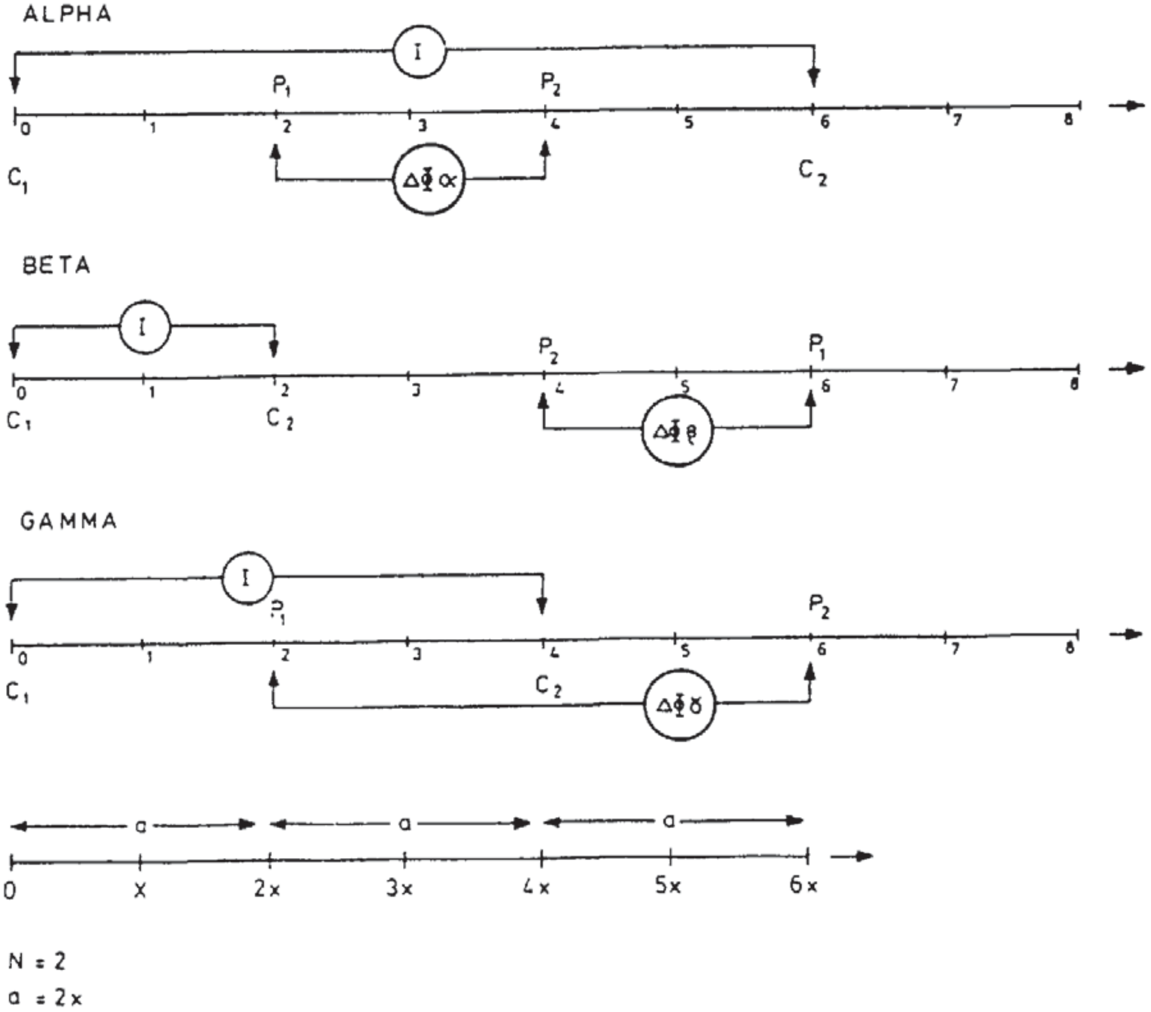
$$K = 2\pi\left[\frac{1}{a}-\frac{1}{2a}-\frac{1}{2a}+\frac{1}{a}\right]^{-1} = 2\pi\left[\frac{2}{a}-\frac{2}{2a}\right]^{-1}$$
$$= 2\pi a.$$

Hence, as $\rho_a = KR$, $\rho_a = 2\pi aR$.

Figure 7.8A shows the signal contribution for a Wenner array. In the near-surface region, the positive and negative areas cancel each other out and the main response, which originates from depth, is largely flat (see the 1 unit contour). This indicates that for horizontally, layered media, the Wenner array has a high vertical resolution. The Schlumberger array has almost as high a vertical resolution, but note that the form of the signal contribution at depth is now concave upwards (Figure 7.8B). For the dipole–dipole array (Figure 7.8C), the lobate form of the signal contribution indicates that there is a poor vertical resolution and that the array is particularly sensitive to deep lateral resistivity variations, making it an unsuitable array for depth sounding (Bhattacharya and Patra 1968). Nevertheless, this sensitivity can be utilised in resistivity profiling (see Section 7.4.3).

A modified electrode array (Lee partitioning array) was devised by Lee (Lee and Schwartz 1930) in an attempt to reduce the undesirable effects of near-surface lateral inhomogeneities. An alternative tripotential method was proposed by Carpenter (1955) and by Carpenter and Habberjam (1956) which combined the apparent resistivities obtained for the alpha, beta and gamma rays (Figure 7.9). The

Figure 7.9 Wenner tripotential electrode configurations for $N = 2$. x is the fixed interelectrode separation, and the active electrode separation is $2x$. From Ackworth and Griffiths (1985), by permission



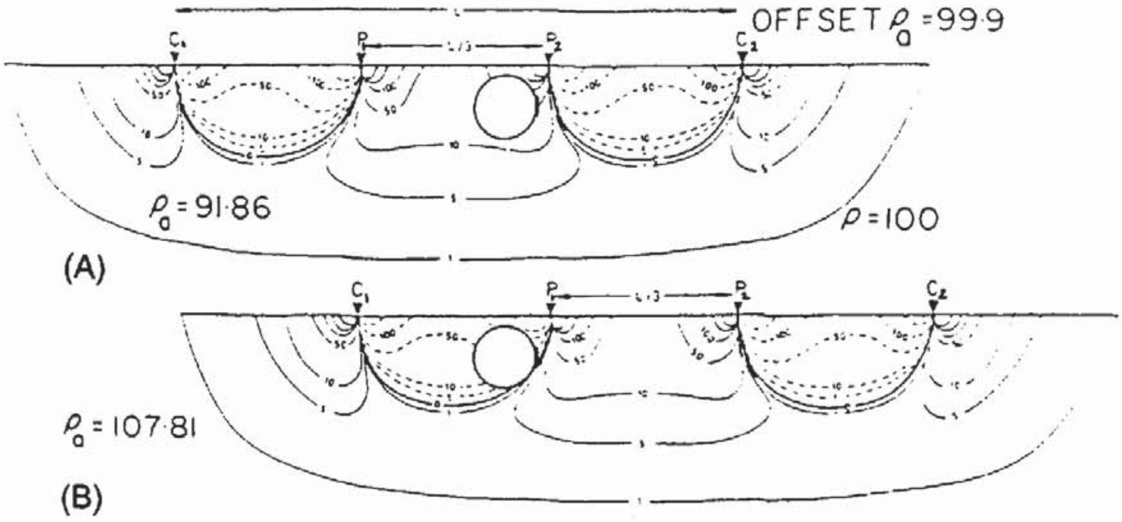


Figure 7.10 (A) Signal contribution section for a Wenner array with a conducting sphere (negative K) in a positive region in a medium with resistivity 100 Ω m. (**B**) Offset Wenner electrodes in which the sphere is now in a negative region. Distortion of contours due to the presence of the sphere is not shown. From Barker (1981), by permission

method has been discussed further by Ackworth and Giffiths (1985). A smoothing technique using the tripotential method was produced by Habberjam and Watkins (1967a).

An alternative technique, called the *Offset Wenner* method (Barker 1981), has been readily adopted for its ease of use. The method is extremely simple in concept. Figure 7.10 shows a single contribution section for a standard Wenner array. A conducting sphere buried in a semi-infinite homogeneous medium with true resistivity of 100 Ω m is located in a positive region of the signal contribution section (Figure 7.10A). The corresponding apparent resistivity, calculated using an exact analytical method (Singh 1976), is 91.86 Ω m. Offsetting the Wenner array one spacing to the right (Figure 7.10B), the previously positive areas are now negative and vice versa, and the buried sphere is located in a negative region resulting in an apparent resistivity of 107.81 Ω m. The average of these two apparent resistivities is 99.88 Ω m, thereby reducing the error due to a lateral inhomogeneity from around ±8% to only 0.1%.

One array that is seldom used, but which has two major advantages, is the square array. This is a special form of the equatorial dipole–dipole array for $n = 1$. The square array is particularly good for determining lateral azimuthal variations in resistivity. By swapping P_1 and C_2, the square is effectively rotated through 90° and thus the apparent resistivity can be determined for two orthogonal directions. For ground that is largely uniform, the two resistivities should be the same, but where there is a difference in resistivity due to a form of anisotropy (*transverse anisotropy* as it is measured only in the $x - y$ plane), the two resistivities will differ. The ratio of the two resistivities is an indication of the transverse anisotropy. Profiles and maps of transverse anisotropy can be interpreted qualitatively to indicate anomalous ground. The second advantage of the square array is that it lends itself to rapid grid mapping. By moving two electrodes at

Table 7.3 Comparison of dipole–dipole, Schlumberger, square and Wenner electrode arrays

Criteria	Wenner	Schlumberger	Dipole–dipole	Square
Vertical resolution	√√√	√√	√	√√
Depth penetration	√	√√	√√√	√√
Suitability to VES	√√	√√√	√	×
Suitability to CST	√√√	×	√√√	√√√
Sensitivity to orientation	Yes	Yes	Moderate	No
Sensitivity to lateral inhomogeneities	High	Moderate	Moderate	Low
Labour intensive	Yes (no*)	Moderate (no*)	Moderate (no*)	Yes
Availability of interpretational aids	√√√	√√√	√√	√

√ = poor; √√ = moderate; √√√ = good; × = unsuitable
* When using a multicore cable and automated electrode array

a time, the square can be moved along the transect. By increasing the dimensions of the square, and thus generally increasing the depth penetration and repeating the same survey area, three-dimensional models of the resistivity distribution can be obtained. Of all the electrode configurations, the square array is the least sensitive to steeply dipping interfaces (Broadbent and Habberjam 1971) and thus it can cope in situations where the sub-surface media are not horizontally-layered. Being a particularly labour-intensive field method, it is best restricted to small-scale surveys where the electrode separation is only of the order of a few metres. This technique has particular value in 3-D mapping of buried massive ice and in shallow archaeological investigations, for example.

A general guide to the suitability of the dipole–dipole, Schlumberger, square and Wenner electrode configurations is given in Table 7.3. An important consideration for the suitability of a given array is the scale at which it is to be deployed. For example, a square array is not really appropriate for depth sounding ('vertical electrical sounding'; VES) or for 'constant separation traversing' (CST) with a large square side; whereas it is perhaps better than either the Wenner or Schlumberger arrays for applications concerned with very shallow depths (< 2 m), such as in archaeological investigations. While the main electrode configurations are now well established in their major applications, small-scale mini-resistivity surveys have yet to realise their full potential.

7.3.3 Media with contrasting resistivities

A geological section may show a series of lithologically defined interfaces which do not necessarily coincide with boundaries identified electrically. For example, in an unconfined sandstone aquifer,