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The Need for a New Theory of Space and Time

5.1 SPACE AND TIME REVISITED

Perhaps the most astonishing idea underpinning Einstein's Special Theory of Relativity is the rejection of the assumption that both space and time are absolute. Since the whole of Part I of this book was built on such an assumption, it means we will have to start all over again. Of course that is not to say that Newton's theory is useless, for whatever Einstein's theory says, it had better be experimentally indistinguishable from Newton's theory for a very wide range of phenomena. Before we attempt to figure out what Einstein's theory actually says, we should first be very clear on what exactly it means to say that space and time are absolute.

Intuitively, absolute space means that we can imagine a gigantic fixed frame of reference against which the positions of events can unambiguously be determined. Of course the actual co-ordinates of an event will depend upon where the origin of the reference frame is[1] but its position vector will nevertheless specify a unique position in absolute space. Absolute time is also very intuitive. We can imagine the Universe being filled with tiny clocks all synchronised with each other and ticking at exactly the same rate. The time of an event can unambiguously be measured by the time registered on a clock located close to the event (these are imaginary clocks so we don't worry too much about the fact it isn't really practicable to put clocks everywhere). Again, although the actual time of an event will depend upon when we set the clocks to zero it still specifies a unique moment in time. The consequences of absolute space and time are clear: there is no argument about how long a body is (it is the distance between two points in absolute space), or whether

[1] They'll also depend upon the orientation of our axes and on our choice of co-ordinate system (e.g. cartesian or spherical polar).

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or not two events occured simultaneously (which means they occured at the same absolute time).

If absolute space really existed, as Newton imagined it did, then it follows that there exists a set of very special frames of reference. Namely, all those frames which are at rest in the absolute space. Inertial frames are then those frames which are moving with some constant velocity relative to absolute space. It is interesting that long before Einstein, absolute space was under attack. Since no experiments have ever been performed that are able to identify a special inertial frame it follows that we cannot figure out which inertial frames are at rest in absolute space. Therefore, as far as physics is concerned we can dispense with the idea of absolute space in favour of the democracy of inertial frames. Physicists now take the equality of inertial frames so seriously that they have elevated it to the status of a fundamental principle: the Principle of (Special) Relativity. By postulating this relativity principle, absolute space is dismissed from physics and consigned to the realm of philosophy. Newton's theory itself obeys the relativity principle, and as such does not require the notion of absolute space. However it does assume that time is absolute.

Let's prepare the ground for later developments and gain some experience of thinking about events in space and time. Consider two inertial frames of reference, S and S' and suppose an event occurs at a time t and has Cartesian co-ordinates (x, y, z) in S. The question is, what are the corresponding co-ordinates measured in S'? To answer this we need to be more explicit and say how the S and S' move relative to each other. We'll take their relative motion to be as illustrated in Figure 5.1, i.e. S' moves at a speed v relative to S and in a direction parallel to the x-axis. Let's also suppose that their origins O and O' coincide at time $t = t' = 0$. Clearly the y and z co-ordinates of the event are the same in both frames:

$$y = y', \tag{5.1a}$$

$$z = z'. \tag{5.1b}$$

More interesting is the relationship between the co-ordinates x and x'. Common sense tells us that

$$x = x' + vt. \tag{5.2}$$

Of course this is the correct answer but only provided we assume absolute time, i.e. that $t = t'$. The proof goes like this. Firstly, we need to recognise that the

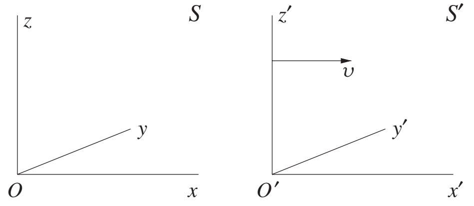

Figure 5.1 The two frames of reference S and S' moving with relative speed v in the sense shown.

relationship must be of the linear form

$$x = ax' + bt \tag{5.3}$$

where a and b are to be determined. We'll not dwell on this, but if the relationship were not linear then it would violate the relativity principle. We also know that the point $x' = 0$ travels along the x-axis with speed v, i.e. along the line $x = vt$. From this it follows that $b = v$. The relativity principle can be used again to figure out a since one can equally well think of S' as being at rest and S as moving along the negative x-axis with speed v. This implies that

$$x' = ax - vt. \tag{5.4}$$

Substituting for x' into Eq. (5.3) implies that $a = 1$ and we have proved the result. Notice that we did not need to invoke the idea of absolute space to derive this result: all that was needed was the relativity principle and the assumption that time is absolute. The equations (5.1) and (5.2) tell us how to relate the co-ordinates of an event in two different inertial frames and they are often referred to as the Galilean transformations.

In what follows we shall often speak of 'observers'. These are the real or fictitious people who we suppose are interested in recording the co-ordinates of events using a specified system of co-ordinates. For example, we might say that 'if an observer at rest in S measures an event to occur at the point (x, y, z) then an observer at rest in S' will measure the same event to occur at (x', y', z') where the co-ordinates in the two frames are related to each other by the Galilean transformations.'

Example 5.1.1 *A rigid rod of length 1m is at rest and lies along the x-axis in an inertial frame S. Show that if space and time are universal, the rod is also 1m long as determined by an observer at rest in an inertial frame S' which moves at a speed v relative to S in the positive x direction.*

Solution 5.1.1 *It is tempting to think that this result is so self-evident that it needs no proof but as we shall see, it is not true in Einstein's theory so it is a good idea for us to work through the proof here assuming that Eq. (5.2) holds. We shall also go very slowly and spell out explicity exactly how the length is measured in each inertial frame. For this question this level of analysis may be a little over the top but it will prepare us well for later, trickier, problems.*

We can refer to Figure 5.1 and imagine two observers, one at rest in S and the other at rest in S'. Suppose that the observer at rest in S measures the positions of each end of the rigid rod. In doing so, she records the space and time co-ordinates of two events. The first event is the measurement of one end of the rod and the second event is the measurement of the other end of the rod. To specify an event we need to specify four numbers: the three spatial co-ordinates and the time at which the event took place. The first event has co-ordinates $(x_1, 0, 0)$ and occurs at time t whilst the second event has co-ordinates $(x_2, 0, 0)$ and also occurs at time t. Obviously these two events take place at the same time since that is what we mean by making a measurement of length: we measure the positions of the ends of *the rod at an instant in time. We are told that* $x_2 - x_1 = 1\,m$ *and asked to find the corresponding length as measured by an observer at rest in* S'.

Our second observer makes their measurement of the length of the rod. Let's suppose they do it at a time t' *(the two observers don't have to measure the length at the same time so* t' *does not have to equal* t*). Again there are two events, the measurement of one end of the rod at* $(x_1', 0, 0)$ *and the measurement of the other end of the rod at* $(x_2', 0, 0)$. *Now using the Galilean transformations it follows that* $x_1' = x_1 - vt'$ *and* $x_2' = x_2 - vt'$ *from which it follows that* $x_2' - x_1' = x_2 - x_1 = 1m$*, i.e. both observers agree on the length of the rod.*

The relationships between measurements of events in different inertial frames under the assumption of absolute time is called 'Galilean relativity'. According to Galiliean relativity, all observers will agree on things like the length of a rod or whether or not two events are simultaneous. It is now time to question the validity of this simple and intuitive relativity theory.

5.2 EXPERIMENTAL EVIDENCE

We are going to need some pretty compelling reason to give up the Galilean view of space and time. In this section we'll motivate the need for something different and we start with the 1887 experiment of Michelson and Morley.

5.2.1 The Michelson-Morley experiment

Is it possible to chase after a beam of light? In classical physics the answer seems to be a resounding 'yes'. We can even imagine running at close to the speed of light whilst shining a torch ahead of us. If we run fast enough then we might expect to see the light travelling slowly out of the front of the torch and when we reach light speed the torch is finally rendered useless. Thinking like this we are imagining that the light travels in a medium, just as every other wave we know of in Nature, and that its speed of propagation is fixed relative to the medium. The uselessness of our torch as we reach the speed of light is in this way entirely analogous to the phenomenon whereby a jet aircraft travelling at the speed of sound cannot be heard until it has passed by. This is a natural way to think, i.e. light needs a medium to support its vibrations, but it is wrong and the experiment that proves[2] it is the Michelson-Morley experiment.

If light is a wave travelling through some medium, which is historically referred to as the 'ether', then it should travel at a fixed speed c relative to the ether. This means that different observers in different inertial frames will all measure different speeds for a beam of light. Similarly an observer can tell if they are moving relative to the ether by sending out two (or more) beams of light in non-parallel directions. Only if the two beams travel at the same speed is the observer entitled to say they are at rest relative to the ether and from any difference in speeds the observer will be able to determine their speed relative to the ether. As an aside,

[2] Actually it does not strictly prove the absence of a medium. Rather it provides some very compelling evidence.