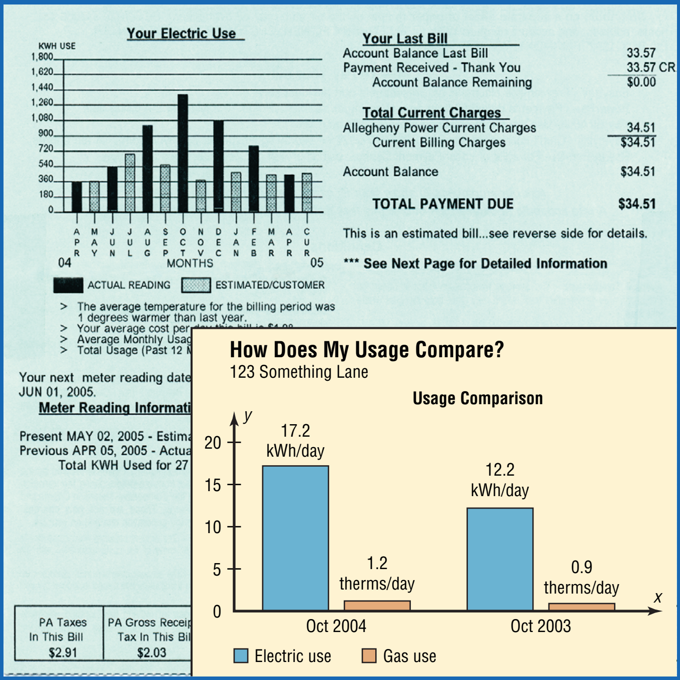


CHAPTER 2

Frequency Distributions and Graphs

Objectives

After completing this chapter, you should be able to

1. Organize data using frequency distributions.
2. Represent data in frequency distributions graphically using histograms, frequency polygons, and ogives.
3. Represent data using Pareto charts, time series graphs, and pie graphs.
4. Draw and interpret a stem and leaf plot.

Outline

Statistics Today

How Serious Are Hospital Infections?

According to an article in the *Pittsburgh Tribune Review,* hospital infections occur in nearly 2 million patients every year. Just how serious a problem is this? It is very serious since the article further reports that one out of every six patients who develop an infection while in the hospital dies. In the first 3 months of 2004, hospitals in Pennsylvania reported that there were 2253 hospital-acquired infections, and 388 deaths resulted from these infections. That is about 17%. The type and number of infections are shown in the following table.

Type of infection	Infections reported	Number of deaths	Death rate
Urinary tract	931	99	10.6%
Surgical site	229	6	2.6
Pneumonia	291	100	34.4
Bloodstream	410	107	26.1
Other	392	76	Varies
	2253	388	

Looking at the numbers presented in a table does not have the same impact as presenting numbers in a well-drawn chart or graph. The article did not include any graphs. This chapter will show you how to construct appropriate graphs to represent data and help you to get your point across to your audience.

See Statistics Today—Revisited at the end of the chapter for some suggestions on how to represent the data graphically.

2–1 Introduction

When conducting a statistical study, the researcher must gather data for the particular variable under study. For example, if a researcher wishes to study the number of people who were bitten by poisonous snakes in a specific geographic area over the past several years, he or she has to gather the data from various doctors, hospitals, or health departments.

To describe situations, draw conclusions, or make inferences about events, the researcher must organize the data in some meaningful way. The most convenient method of organizing data is to construct a *frequency distribution.*

After organizing the data, the researcher must present them so they can be understood by those who will benefit from reading the study. The most useful method of presenting the data is by constructing *statistical charts* and *graphs.* There are many different types of charts and graphs, and each one has a specific purpose.

This chapter explains how to organize data by constructing frequency distributions and how to present the data by constructing charts and graphs. The charts and graphs illustrated here are histograms, frequency polygons, ogives, pie graphs, Pareto charts, and time series graphs. A graph that combines the characteristics of a frequency distribution and a histogram, called a stem and leaf plot, is also explained.

2–2 Organizing Data

Objective 1

Organize data using frequency distributions.

Suppose a researcher wished to do a study on the number of miles that the employees of a large department store traveled to work each day. The researcher first would have to collect the data by asking each employee the approximate distance the store is from his or her home. When data are collected in original form, they are called **raw data.** In this case, the data are

1	2	6	7	12	13	2	6	9	5
18	7	3	15	15	4	17	1	14	5
4	16	4	5	8	6	5	18	5	2
9	11	12	1	9	2	10	11	4	10
9	18	8	8	4	14	7	3	2	6

Since little information can be obtained from looking at raw data, the researcher organizes the data into what is called a *frequency distribution.* A frequency distribution consists of *classes* and their corresponding *frequencies.* Each raw data value is placed into a quantitative or qualitative category called a **class.** The **frequency** of a class then is the number of data values contained in a specific class. A frequency distribution is shown for the data set above.

Class limits (in miles)	Tally	Frequency
1–3	~~////~~ ~~////~~	10
4–6	~~////~~ ~~////~~ ////	14
7–9	~~////~~ ~~////~~	10
10–12	~~////~~ /	6
13–15	~~////~~	5
16–18	~~////~~	5
		Total 50

Now some general observations can be made from looking at the data in the form of a frequency distribution. For example, the majority of employees live within 9 miles of the store.

A **frequency distribution** is the organization of raw data in table form, using classes and frequencies.

Unusual Stat

Of Americans 50 years old and over, 23% think their greatest achievements are still ahead of them.

The classes in this distribution are 1–3, 4–6, etc. These values are called *class limits.* The data values 1, 2, 3 can be tallied in the first class; 4, 5, 6 in the second class; and so on.

Two types of frequency distributions that are most often used are the *categorical frequency distribution* and the *grouped frequency distribution.* The procedures for constructing these distributions are shown now.

Categorical Frequency Distributions

The **categorical frequency distribution** is used for data that can be placed in specific categories, such as nominal- or ordinal-level data. For example, data such as political affiliation, religious affiliation, or major field of study would use categorical frequency distributions.

Example 2–1

Twenty-five army inductees were given a blood test to determine their blood type. The data set is

A	B	B	AB	O
O	O	B	AB	B
B	B	O	A	O
A	O	O	O	AB
AB	A	O	B	A

Construct a frequency distribution for the data.

Solution

Since the data are categorical, discrete classes can be used. There are four blood types: A, B, O, and AB. These types will be used as the classes for the distribution.

The procedure for constructing a frequency distribution for categorical data is given next.

Step 1 Make a table as shown.

A Class	B Tally	C Frequency	D Percent
A			
B			
O			
AB			

Step 2 Tally the data and place the results in column B.

Step 3 Count the tallies and place the results in column C.

Step 4 Find the percentage of values in each class by using the formula

$$\% = \frac{f}{n} \cdot 100\%$$

where f = frequency of the class and n = total number of values. For example, in the class of type A blood, the percentage is

$$\% = \frac{5}{25} \cdot 100\% = 20\%$$

Percentages are not normally part of a frequency distribution, but they can be added since they are used in certain types of graphs such as pie graphs. Also, the decimal equivalent of a percent is called a *relative frequency.*

Step 5 Find the totals for columns C (frequency) and D (percent). The completed table is shown.

A Class	B Tally	C Frequency	D Percent
A	~~////~~	5	20
B	~~////~~ //	7	28
O	~~////~~ ////	9	36
AB	////	4	16
		Total 25	100

For the sample, more people have type O blood than any other type.

Grouped Frequency Distributions

When the range of the data is large, the data must be grouped into classes that are more than one unit in width, in what is called a **grouped frequency distribution.** For example, a distribution of the number of hours that boat batteries lasted is the following.

Unusual Stat

Six percent of Americans say they find life dull.

Class limits	Class boundaries	Tally	Frequency	Cumulative frequency
24–30	23.5–30.5	///	3	3
31–37	30.5–37.5	/	1	4
38–44	37.5–44.5	~~////~~	5	9
45–51	44.5–51.5	~~////~~ ////	9	18
52–58	51.5–58.5	~~////~~ /	6	24
59–65	58.5–65.5	/	1	25
			25	

The procedure for constructing the preceding frequency distribution is given in Example 2–2; however, several things should be noted. In this distribution, the values 24 and 30 of the first class are called *class limits.* The **lower class limit** is 24; it represents the smallest data value that can be included in the class. The **upper class limit** is 30; it represents the largest data value that can be included in the class. The numbers in the second column are called **class boundaries.** These numbers are used to separate the classes so that there are no gaps in the frequency distribution. The gaps are due to the limits; for example, there is a gap between 30 and 31.

Students sometimes have difficulty finding class boundaries when given the class limits. The basic rule of thumb is that *the class limits should have the same decimal place value as the data, but the class boundaries should have one additional place value and end in a 5.* For example, if the values in the data set are whole numbers, such as 24, 32, 18, the limits for a class might be 31–37, and the boundaries are 30.5–37.5. Find the boundaries by subtracting 0.5 from 31 (the lower class limit) and adding 0.5 to 37 (the upper class limit).

$$\text{Lower limit} - 0.5 = 31 - 0.5 = 30.5 = \text{lower boundary}$$

$$\text{Upper limit} + 0.5 = 37 + 0.5 = 37.5 = \text{upper boundary}$$

If the data are in tenths, such as 6.2, 7.8, and 12.6, the limits for a class hypothetically might be 7.8–8.8, and the boundaries for that class would be 7.75–8.85. Find these values by subtracting 0.05 from 7.8 and adding 0.05 to 8.8.

Unusual Stat

One out of every 100 people in the United States is color-blind.

Finally, the **class width** for a class in a frequency distribution is found by subtracting the lower (or upper) class limit of one class from the lower (or upper) class limit of the next class. For example, the class width in the preceding distribution on the duration of boat batteries is 7, found from $31 - 24 = 7$.

The class width can also be found by subtracting the lower boundary from the upper boundary for any given class. In this case, $30.5 - 23.5 = 7$.

Note: Do not subtract the limits of a single class. It will result in an incorrect answer.

The researcher must decide how many classes to use and the width of each class. To construct a frequency distribution, follow these rules:

1. *There should be between 5 and 20 classes.* Although there is no hard-and-fast rule for the number of classes contained in a frequency distribution, it is of the utmost importance to have enough classes to present a clear description of the collected data.
2. *It is preferable but not absolutely necessary that the class width be an odd number.* This ensures that the midpoint of each class has the same place value as the data. The **class midpoint** X_m is obtained by adding the lower and upper boundaries and dividing by 2, or adding the lower and upper limits and dividing by 2:

$$X_m = \frac{\text{lower boundary} + \text{upper boundary}}{2}$$

or

$$X_m = \frac{\text{lower limit} + \text{upper limit}}{2}$$

For example, the midpoint of the first class in the example with boat batteries is

$$\frac{24 + 30}{2} = 27 \qquad \text{or} \qquad \frac{23.5 + 30.5}{2} = 27$$

The midpoint is the numeric location of the center of the class. Midpoints are necessary for graphing (see Section 2–3). If the class width is an even number, the midpoint is in tenths. For example, if the class width is 6 and the boundaries are 5.5 and 11.5, the midpoint is

$$\frac{5.5 + 11.5}{2} = \frac{17}{2} = 8.5$$

Rule 2 is only a suggestion, and it is not rigorously followed, especially when a computer is used to group data.

3. *The classes must be mutually exclusive.* Mutually exclusive classes have nonoverlapping class limits so that data cannot be placed into two classes. Many times, frequency distributions such as

Age
10–20
20–30
30–40
40–50

are found in the literature or in surveys. If a person is 40 years old, into which class should she or he be placed? A better way to construct a frequency distribution is to use classes such as

Age
10–20
21–31
32–42
43–53

4. *The classes must be continuous.* Even if there are no values in a class, the class must be included in the frequency distribution. There should be no gaps in a

frequency distribution. The only exception occurs when the class with a zero frequency is the first or last class. A class with a zero frequency at either end can be omitted without affecting the distribution.

5. *The classes must be exhaustive.* There should be enough classes to accommodate all the data.
6. *The classes must be equal in width.* This avoids a distorted view of the data.

One exception occurs when a distribution has a class that is open-ended. That is, the class has no specific beginning value or no specific ending value. A frequency distribution with an open-ended class is called an **open-ended distribution.** Here are two examples of distributions with open-ended classes.

Age	Frequency
10–20	3
21–31	6
32–42	4
43–53	10
54 and above	8

Minutes	Frequency
Below 110	16
110–114	24
115–119	38
120–124	14
125–129	5

The frequency distribution for age is open-ended for the last class, which means that anybody who is 54 years or older will be tallied in the last class. The distribution for minutes is open-ended for the first class, meaning that any minute values below 110 will be tallied in that class.

Example 2–2 shows the procedure for constructing a grouped frequency distribution, i.e., when the classes contain more than one data value.

Example 2–2

These data represent the record high temperatures in °F for each of the 50 states. Construct a grouped frequency distribution for the data using 7 classes.

112	100	127	120	134	118	105	110	109	112
110	118	117	116	118	122	114	114	105	109
107	112	114	115	118	117	118	122	106	110
116	108	110	121	113	120	119	111	104	111
120	113	120	117	105	110	118	112	114	114

Source: *The World Almanac and Book of Facts.*

Solution

The procedure for constructing a grouped frequency distribution for numerical data follows.

Step 1 Determine the classes.

Find the highest value and lowest value: $H = 134$ and $L = 100$.

Find the range: R = highest value − lowest value = $H - L$, so

$$R = 134 - 100 = 34$$

Select the number of classes desired (usually between 5 and 20). In this case, 7 is arbitrarily chosen.

Find the class width by dividing the range by the number of classes.

$$\text{Width} = \frac{R}{\text{number of classes}} = \frac{34}{7} = 4.9$$

Unusual Stats

America's most popular beverages are soft drinks. It is estimated that, on average, each person drinks about 52 gallons of soft drinks per year, compared to 22 gallons of beer.

Round the answer up to the nearest whole number if there is a remainder: $4.9 \approx 5$. (Rounding *up* is different from rounding *off.* A number is rounded up if there is any decimal remainder when dividing. For example, $85 \div 6 = 14.167$ and is rounded up to 15. Also, $53 \div 4 = 13.25$ and is rounded up to 14. Also, after dividing, if there is no remainder, you will need to add an extra class to accommodate all the data.)

Select a starting point for the lowest class limit. This can be the smallest data value or any convenient number less than the smallest data value. In this case, 100 is used. Add the width to the lowest score taken as the starting point to get the lower limit of the next class. Keep adding until there are 7 classes, as shown, 100, 105, 110, etc.

Subtract one unit from the lower limit of the second class to get the upper limit of the first class. Then add the width to each upper limit to get all the upper limits.

$105 - 1 = 104$

The first class is 100–104, the second class is 105–109, etc.

Find the class boundaries by subtracting 0.5 from each lower class limit and adding 0.5 to each upper class limit:

99.5–104.5, 104.5–109.5, etc.

Step 2 Tally the data.

Step 3 Find the numerical frequencies from the tallies.

Step 4 Find the cumulative frequencies.

A cumulative frequency (cf) column can be added to the distribution by adding the frequency in each class to the total of the frequencies of the classes preceding that class, such as $0 + 2 = 2$, $2 + 8 = 10$, $10 + 18 = 28$, and $28 + 13 = 41$.

The completed frequency distribution is

Class limits	Class boundaries	Tally	Frequency	Cumulative frequency
100–104	99.5–104.5	//	2	2
105–109	104.5–109.5	~~////~~ ///	8	10
110–114	109.5–114.5	~~////~~ ~~////~~ ~~////~~ ///	18	28
115–119	114.5–119.5	~~////~~ ~~////~~ ///	13	41
120–124	119.5–124.5	~~////~~ //	7	48
125–129	124.5–129.5	/	1	49
130–134	129.5–134.5	/	1	50
			$n = \Sigma f = 50$	

The frequency distribution shows that the class 109.5–114.5 contains the largest number of temperatures (18) followed by the class 114.5–119.5 with 13 temperatures. Hence, most of the temperatures (31) fall between 109.5°F and 119.5°F.

Cumulative frequencies are used to show how many data values are accumulated up to and including a specific class. In Example 2–2, 28 of the total record high temperatures are less than or equal to 114°F. Forty-eight of the total record high temperatures are less than or equal to 124°F.

After the raw data have been organized into a frequency distribution, it will be analyzed by looking for peaks and extreme values. The peaks show which class or classes have the most data values compared to the other classes. Extreme values, called outliers, show large or small data values that are relative to other data values.

When the range of the data values is relatively small, a frequency distribution can be constructed using single data values for each class. This type of distribution is called an **ungrouped frequency distribution** and is shown next.

Example 2–3

The data shown here represent the number of miles per gallon that 30 selected four-wheel-drive sports utility vehicles obtained in city driving. Construct a frequency distribution, and analyze the distribution.

12	17	12	14	16	18
16	18	12	16	17	15
15	16	12	15	16	16
12	14	15	12	15	15
19	13	16	18	16	14

Source: *Model Year 1999 Fuel Economy Guide.* United States Environmental Protection Agency, October 1998.

Solution

Step 1 Determine the classes. Since the range of the data set is small ($19 - 12 = 7$), classes consisting of a single data value can be used. They are 12, 13, 14, 15, 16, 17, 18, 19.

Note: If the data are continuous, class boundaries can be used. Subtract 0.5 from each class value to get the lower class boundary, and add 0.5 to each class value to get the upper class boundary.

Step 2 Tally the data.

Step 3 Find the numerical frequencies from the tallies.

Step 4 Find the cumulative frequencies.

The completed ungrouped frequency distribution is

Class limits	Class boundaries	Tally	Frequency	Cumulative frequency
12	11.5–12.5	𝍸 /	6	6
13	12.5–13.5	/	1	7
14	13.5–14.5	///	3	10
15	14.5–15.5	𝍸 /	6	16
16	15.5–16.5	𝍸 ///	8	24
17	16.5–17.5	//	2	26
18	17.5–18.5	///	3	29
19	18.5–19.5	/	1	30

In this case, almost one-half (14) of the vehicles get 15 or 16 miles per gallon.

The steps for constructing a grouped frequency distribution are summarized in the following Procedure Table.

Procedure Table

Constructing a Grouped Frequency Distribution

Step 1 Determine the classes.

Find the highest and lowest value.

Find the range.

Select the number of classes desired.

Find the width by dividing the range by the number of classes and rounding up.

Select a starting point (usually the lowest value or any convenient number less than the lowest value); add the width to get the lower limits.

Find the upper class limits.

Find the boundaries.

Step 2 Tally the data.

Step 3 Find the numerical frequencies from the tallies.

Step 4 Find the cumulative frequencies.

When one is constructing a frequency distribution, the guidelines presented in this section should be followed. However, one can construct several different but correct frequency distributions for the same data by using a different class width, a different number of classes, or a different starting point.

Furthermore, the method shown here for constructing a frequency distribution is not unique, and there are other ways of constructing one. Slight variations exist, especially in computer packages. But regardless of what methods are used, classes should be mutually exclusive, continuous, exhaustive, and of equal width.

In summary, the different types of frequency distributions were shown in this section. The first type, shown in Example 2–1, is used when the data are categorical (nominal), such as blood type or political affiliation. This type is called a categorical frequency distribution. The second type of distribution is used when the range is large and classes several units in width are needed. This type is called a grouped frequency distribution and is shown in Example 2–2. Another type of distribution is used for numerical data and when the range of data is small, as shown in Example 2–3. Since each class is only one unit, this distribution is called an ungrouped frequency distribution.

All the different types of distributions are used in statistics and are helpful when one is organizing and presenting data.

The reasons for constructing a frequency distribution are as follows:

1. To organize the data in a meaningful, intelligible way.
2. To enable the reader to determine the nature or shape of the distribution.
3. To facilitate computational procedures for measures of average and spread (shown in Sections 3–2 and 3–3).
4. To enable the researcher to draw charts and graphs for the presentation of data (shown in Section 2–3).
5. To enable the reader to make comparisons among different data sets.

The factors used to analyze a frequency distribution are essentially the same as those used to analyze histograms and frequency polygons, which are shown in Section 2–3.

Interesting Fact

Male dogs bite children more often than female dogs do; however, female cats bite children more often than male cats do.

Applying the Concepts 2–2

Ages of Presidents at Inauguration

The data represent the ages of our presidents at the time they were first inaugurated.

57	61	57	57	58	57	61	54	68
51	49	64	50	48	65	52	56	46
54	49	50	47	55	55	54	42	51
56	55	54	51	60	62	43	55	56
61	52	69	64	46	54			

1. Were the data obtained from a population or a sample? Explain your answer.
2. What was the age of the oldest president?
3. What was the age of the youngest president?
4. Construct a frequency distribution for the data. (Use your own judgment as to the number of classes and class size.)
5. Are there any peaks in the distribution?
6. Identify any possible outliers.
7. Write a brief summary of the nature of the data as shown in the frequency distribution.

See page 93 for the answers.

Exercises 2–2

1. List five reasons for organizing data into a frequency distribution.

2. Name the three types of frequency distributions, and explain when each should be used.

3. Find the class boundaries, midpoints, and widths for each class.

a. 12–18

b. 56–74

c. 695–705

d. 13.6–14.7

e. 2.15–3.93

4. How many classes should frequency distributions have? Why should the class width be an odd number?

5. Shown here are four frequency distributions. Each is incorrectly constructed. State the reason why.

a.

Class	Frequency
27–32	1
33–38	0
39–44	6
45–49	4
50–55	2

b.

Class	Frequency
5–9	1
9–13	2
13–17	5
17–20	6
20–24	3

c.

Class	Frequency
123–127	3
128–132	7
138–142	2
143–147	19

d.

Class	Frequency
9–13	1
14–19	6
20–25	2
26–28	5
29–32	9

6. What are open-ended frequency distributions? Why are they necessary?

7. A survey was taken on how much trust people place in the information they read on the Internet. Construct a categorical frequency distribution for the data. A = trust in everything they read, M = trust in most of what they read, H = trust in about one-half of what they read, S = trust in a small portion of what they read. (Based on information from the *UCLA Internet Report.*)

M	M	M	A	H	M	S	M	H	M
S	M	M	M	M	A	M	M	A	M
M	M	H	M	M	M	H	M	H	M
A	M	M	M	H	M	M	M	M	M

8. The heights in inches of commonly grown herbs are shown. Organize the data into a frequency distribution with six classes, and think of a way in which these results would be useful.

18	20	18	18	24	10	15
12	20	36	14	20	18	24
18	16	16	20	7		

Source: *The Old Farmer's Almanac.*

9. The following data are the measured speeds in miles per hour of 30 charging elephants. Construct a grouped frequency distribution for the data. From the distribution, estimate an approximate average speed of a charging elephant. Use 5 classes. (Based on data in the *World Almanac and Book of Facts.*)

25	24	25	24	25
23	25	19	32	23
22	24	26	25	23
28	25	25	26	27
22	28	24	23	24
21	25	22	29	23

10. The total energy consumption in trillions of BTU for each of the 50 states in the United States is shown. Construct a frequency distribution using 10 classes, and analyze the nature of the data.

1,215	2,706	1,400	4,417	1,868
11,588	1,799	1,199	627	1,099
1,688	1,083	2,501	561	4,001
1,035	863	594	2,303	583
329	620	1,722	744	1,143
264	417	365	302	250
8,518	4,779	4,620	3,943	3,121
1,659	511	246	1,520	1,977
1,079	2,777	2,769	1,477	632
3,965	2,173	2,025	718	164

Source: *Energy Information Administration.*

11. The average quantitative GRE scores for the top 30 graduate schools of engineering are listed. Construct a frequency distribution with 6 classes.

767	770	761	760	771	768	776	771	756	770
763	760	747	766	754	771	771	778	766	762
780	750	746	764	769	759	757	753	758	746

Source: *U.S. News & World Report Best Graduate Schools.*

12. The number of unhealthy days in selected U.S. metropolitan areas is shown. Construct a frequency distribution with 7 classes. (The data in this exercise will be used in Exercise 22 in Section 3–2.)

61	88	40	5	12	12	18	23	1	15
6	81	50	21	0	27	5	13	0	24
5	1	32	12	23	93	38	29	16	0
1	22	36							

Source: *N.Y. Times Almanac.*

13. The ages of the signers of the Declaration of Independence are shown. (Age is approximate since only the birth year appeared in the source, and one has been omitted since his birth year is unknown.) Construct a frequency distribution for the data using 7 classes. (The data for this exercise will be used for Exercise 5 in Section 2–3 and Exercise 23 in Section 3–2.)

41	54	47	40	39	35	50	37	49	42	70	32
44	52	39	50	40	30	34	69	39	45	33	42
44	63	60	27	42	34	50	42	52	38	36	45
35	43	48	46	31	27	55	63	46	33	60	62
35	46	45	34	53	50	50					

Source: *The Universal Almanac.*

14. The number of automobile fatalities in 27 states where the speed limits were raised in 1996 is shown here. Construct a frequency distribution using 8 classes. (The data for this exercise will be used for Exercise 6 in Section 2–3 and Exercise 24 in Section 3–2.)

1100	460	85
970	480	1430
4040	405	70
620	690	180
125	1160	3630
2805	205	325
1555	300	875
260	350	705
1430	485	145

Source: *USA TODAY.*

15. The following data represent the ages of 47 of the wealthiest people in the United States. Construct a grouped frequency distribution for the data using 7 classes. Analyze the results in terms of peaks, extreme values, etc. (The information in this exercise will be used for Exercise 9 in Section 2–3 and Exercise 25 in Section 3–2.)

48	48	74	74	84	51	71
56	55	76	85	68	42	79
73	58	73	81	51	81	55
65	66	87	60	74	62	64
39	60	60	37	90	68	67
61	40	72	61	71	74	31
62	63	67	31	40		

Source: *Forbes.*

16. The acreage of the 39 U.S. National Parks under 900,000 acres (in thousands of acres) is shown here. Construct a frequency distribution for the data using 8 classes. (The data in this exercise will be used in Exercise 11 in Section 2–3.)

41	66	233	775	169
36	338	233	236	64
183	61	13	308	77
520	77	27	217	5
650	462	106	52	52
505	94	75	265	402
196	70	132	28	220
760	143	46	539	

Source: *The Universal Almanac.*

17. The heights (in feet above sea level) of the major active volcanoes in Alaska are given here. Construct a frequency distribution for the data using 10 classes. (The data in this exercise will be used in Exercise 9 in Section 3–2 and Exercise 17 in Section 3–3.)

4,265	3,545	4,025	7,050	11,413
3,490	5,370	4,885	5,030	6,830
4,450	5,775	3,945	7,545	8,450
3,995	10,140	6,050	10,265	6,965
150	8,185	7,295	2,015	5,055
5,315	2,945	6,720	3,465	1,980
2,560	4,450	2,759	9,430	
7,985	7,540	3,540	11,070	
5,710	885	8,960	7,015	

Source: *The Universal Almanac.*

18. During the 1998 baseball season, Mark McGwire and Sammy Sosa both broke Roger Maris's home run record of 61. The distances (in feet) for each home run follow. Construct a frequency distribution for each player, using 8 classes. (The information in this exercise will be used for Exercise 12 in Section 2–3, Exercise 10 in Section 3–2, and Exercise 14 in Section 3–3.)

McGwire				Sosa			
306	370	370	430	371	350	430	420
420	340	460	410	430	434	370	420
440	410	380	360	440	410	420	460
350	527	380	550	400	430	410	370
478	420	390	420	370	410	380	340
425	370	480	390	350	420	410	415
430	388	423	410	430	380	380	366
360	410	450	350	500	380	390	400
450	430	461	430	364	430	450	440
470	440	400	390	365	420	350	420
510	430	450	452	400	380	380	400
420	380	470	398	370	420	360	368
409	385	369	460	430	433	388	440
390	510	500	450	414	482	364	370
470	430	458	380	400	405	433	390
430	341	385	410	480	480	434	344
420	380	400	440	410	420		
377	370						

Source: *USA TODAY.*

Extending the Concepts

19. A researcher conducted a survey asking people if they believed more than one person was involved in the assassination of John F. Kennedy. The results were as follows: 73% said yes, 19% said no, and 9% had no opinion. Is there anything suspicious about the results?

Technology *Step by Step*

MINITAB
Step by Step

Make a Categorical Frequency Table (Qualitative or Discrete Data)

1. Type in all the blood types from Example 2–1 down C1 of the worksheet.

   A B B AB O O O B AB B B B O A O A O O O AB AB A O B A

2. Click above row 1 and name the column **BloodType.**
3. Select **Stat>Tables>Tally Individual Values.**

   The cursor should be blinking in the Variables dialog box. If not, click inside the dialog box.

4. Double-click C1 in the Variables list.
5. Check the boxes for the statistics: Counts, Percents, and Cumulative percents.
6. Click [OK]. The results will be displayed in the Session Window as shown.

Tally for Discrete Variables: BloodType

BloodType	Count	Percent	CumPct
A	5	20.00	20.00
AB	4	16.00	36.00
B	7	28.00	64.00
O	9	36.00	100.00
N=	25		

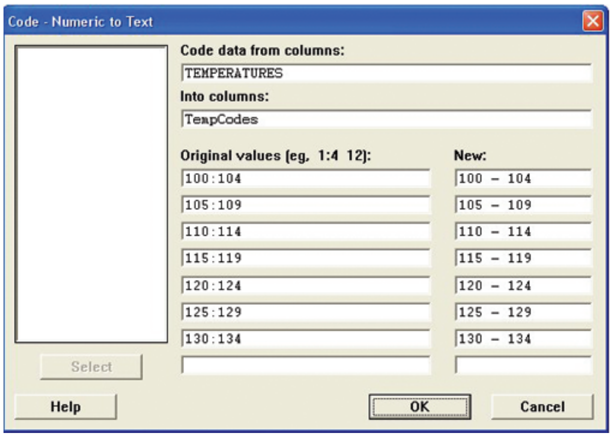
Make a Grouped Frequency Distribution (Quantitative Variable)

1. Select **File>New>New Worksheet.** A new worksheet will be added to the project.
2. Type the data used in Example 2–2 into C1. Name the column **TEMPERATURES.**
3. Use the instructions in the textbook to determine the class limits.

   In the next step you will create a new column of data, converting the numeric variable to text categories that can be tallied.
4. Select **Data>Code>Numeric to Text.**
   a) The cursor should be blinking in Code data from columns. If not, click inside the box, then double-click C1 Temperatures in the list. Only quantitative variables will be shown in this list.
   b) Click in the Into columns: then type the name of the new column, **TempCodes.**
   c) Press [Tab] to move to the next dialog box.
   d) Type in the first interval **100:104.**

      Use a colon to indicate the interval from 100 to 104 with no spaces before or after the colon.
   e) Press [Tab] to move to the New: column, and type the text category **100–104.**
   f) Continue to tab to each dialog box, typing the interval and then the category until the last category has been entered.

   The dialog box should look like the one shown.



5. Click [OK]. In the worksheet, a new column of data will be created in the first empty column, C2. This new variable will contain the category for each value in C1. The column C2-T contains alphanumeric data.

6. Click **Stat>Tables>Tally Individual Values,** then double-click TempCodes in the Variables list.
   a) Check the boxes for the desired statistics, such as Counts, Percents, and Cumulative percents.
   b) Click [OK].

   The table will be displayed in the Session Window. Eighteen states have high temperatures between 110°F and 114°F. Eighty-two percent of the states have record high temperatures less than or equal to 119°F.

Tally for Discrete Variables: TempCodes

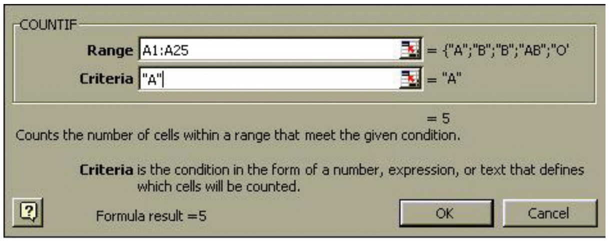
TempCodes	Count	Percent	CumPct
100–104	2	4.00	4.00
105–109	8	16.00	20.00
110–114	18	36.00	56.00
115–119	13	26.00	82.00
120–124	7	14.00	96.00
125–129	1	2.00	98.00
130–134	1	2.00	100.00
N=	50		

7. Click **File>Save Project As** . . . , and type the name of the project file, **Ch2-2.** This will save the two worksheets and the Session Window.

Excel
Step by Step

Categorical Frequency Table (Qualitative or Discrete Variable)

1. Select cell A1 and type in all the blood types from Example 2–1 down column A of the worksheet.
2. Type in the name **BloodType** in cell B1.
3. Select cell B2 and type in the four different blood types down the column.
4. Type in the name **Count** in cell C1.
5. Select cell C2. From the toolbar, select the paste function (f_x) option. Select Statistical from the Function category list. Select COUNTIF from the function name list.
6. In the dialog box, type in **A1:A25** in the Range. Type in the blood type corresponding to the corresponding value from column B.



7. After all the data have been counted, select cell C6 from the worksheet.
8. From the toolbar, select the sum (Σ) function. Then type in **C2:C5** and click [Enter].

Making a Grouped Frequency Distribution

1. Press **[Ctrl]-N** for a new worksheet.
2. Enter the data from Examples 2–2 and 2–4 in column A, one number per cell.
3. Select **Tools>Data Analysis.**

4. In Data Analysis, select Histogram and click the [OK] button.
5. In the Histogram dialog box, type **A1:A50** as the Input Range.
6. Select New Worksheet Ply, and check the Cumulative Percentage option. Click [OK].

By leaving the Chart output unchecked, the new worksheet will display the table only. It decides "bins" for the histogram itself (here it picked a bin size of 7 units), but you can also define your own bin range on the data worksheet.

2–3 Histograms, Frequency Polygons, and Ogives

Objective 2

Represent data in frequency distributions graphically using histograms, frequency polygons, and ogives.

After the data have been organized into a frequency distribution, they can be presented in graphical form. The purpose of graphs in statistics is to convey the data to the viewers in pictorial form. It is easier for most people to comprehend the meaning of data presented graphically than data presented numerically in tables or frequency distributions. This is especially true if the users have little or no statistical knowledge.

Statistical graphs can be used to describe the data set or to analyze it. Graphs are also useful in getting the audience's attention in a publication or a speaking presentation. They can be used to discuss an issue, reinforce a critical point, or summarize a data set. They can also be used to discover a trend or pattern in a situation over a period of time.

The three most commonly used graphs in research are as follows:

1. The histogram.
2. The frequency polygon.
3. The cumulative frequency graph, or ogive (pronounced o-jive).

An example of each type of graph is shown in Figure 2–1. The data for each graph are the distribution of the miles that 20 randomly selected runners ran during a given week.

The Histogram

The **histogram** is a graph that displays the data by using contiguous vertical bars (unless the frequency of a class is 0) of various heights to represent the frequencies of the classes.

Example 2–4

Construct a histogram to represent the data shown for the record high temperatures for each of the 50 states (see Example 2–2).

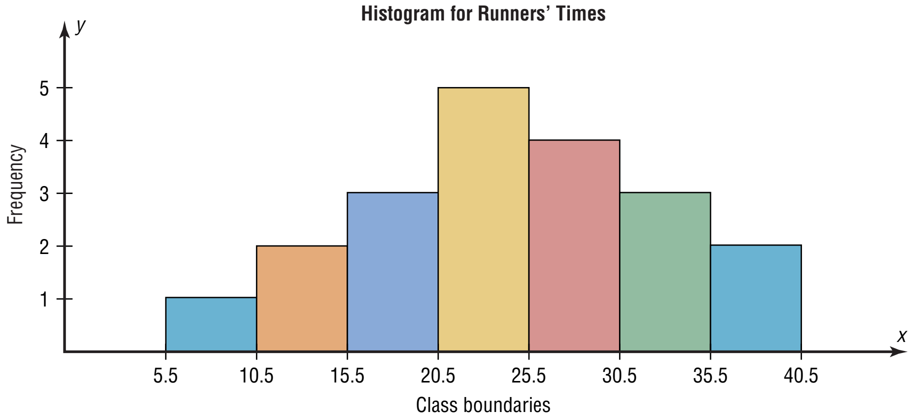
Class boundaries	Frequency
99.5–104.5	2
104.5–109.5	8
109.5–114.5	18
114.5–119.5	13
119.5–124.5	7
124.5–129.5	1
129.5–134.5	1

Solution

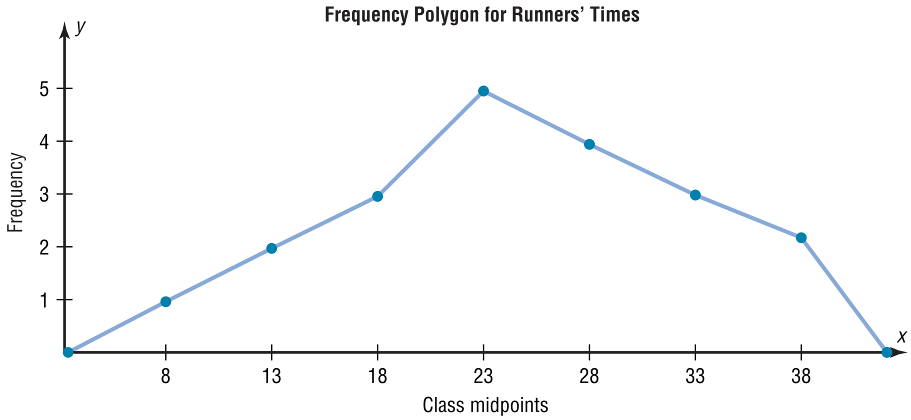
Step 1 Draw and label the x and y axes. The x axis is always the horizontal axis, and the y axis is always the vertical axis.

Figure 2–1

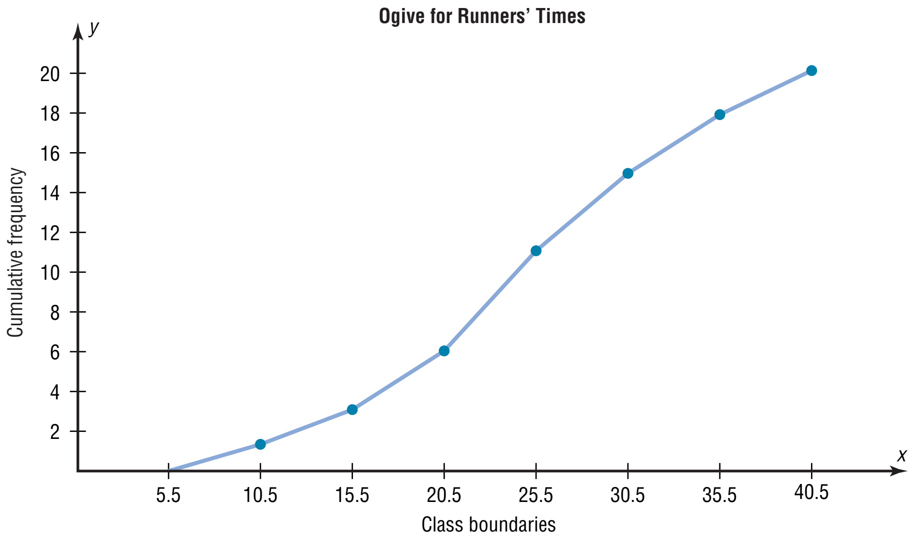
Examples of Commonly Used Graphs



(a) Histogram

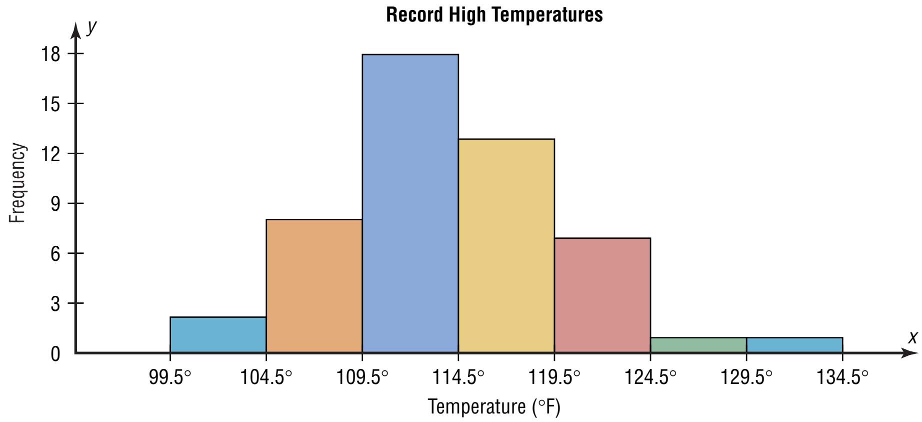


(b) Frequency polygon



(c) Cumulative frequency graph

Figure 2–2
Histogram for Example 2–4



Historical Note

Graphs originated when ancient astronomers drew the position of the stars in the heavens. Roman surveyors also used coordinates to locate landmarks on their maps.

The development of statistical graphs can be traced to William Playfair (1748–1819), an engineer and drafter who used graphs to present economic data pictorially.

Step 2 Represent the frequency on the *y* axis and the class boundaries on the *x* axis.

Step 3 Using the frequencies as the heights, draw vertical bars for each class. See Figure 2–2.

As the histogram shows, the class with the greatest number of data values (18) is 109.5–114.5, followed by 13 for 114.5–119.5. The graph also has one peak with the data clustering around it.

The Frequency Polygon

Another way to represent the same data set is by using a frequency polygon.

> The **frequency polygon** is a graph that displays the data by using lines that connect points plotted for the frequencies at the midpoints of the classes. The frequencies are represented by the heights of the points.

Example 2–5 shows the procedure for constructing a frequency polygon.

Example 2–5

Using the frequency distribution given in Example 2–4, construct a frequency polygon.

Solution

Step 1 Find the midpoints of each class. Recall that midpoints are found by adding the upper and lower boundaries and dividing by 2:

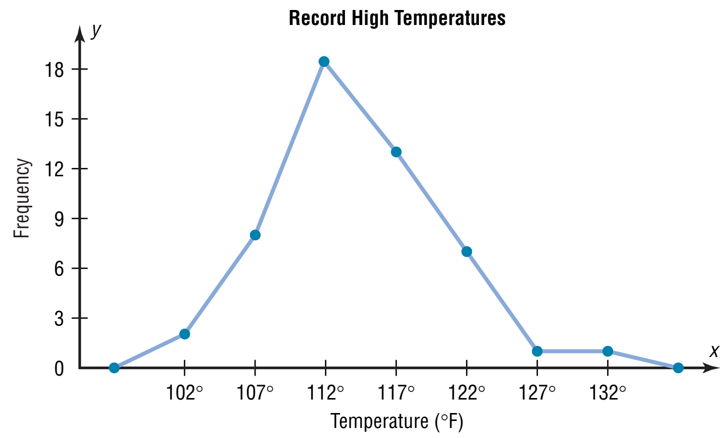
$$\frac{99.5 + 104.5}{2} = 102 \qquad \frac{104.5 + 109.5}{2} = 107$$

and so on. The midpoints are

Class boundaries	Midpoints	Frequency
99.5–104.5	102	2
104.5–109.5	107	8
109.5–114.5	112	18
114.5–119.5	117	13
119.5–124.5	122	7
124.5–129.5	127	1
129.5–134.5	132	1

Figure 2–3

Frequency Polygon for Example 2–5



Step 2 Draw the x and y axes. Label the x axis with the midpoint of each class, and then use a suitable scale on the y axis for the frequencies.

Step 3 Using the midpoints for the x values and the frequencies as the y values, plot the points.

Step 4 Connect adjacent points with line segments. Draw a line back to the x axis at the beginning and end of the graph, at the same distance that the previous and next midpoints would be located, as shown in Figure 2–3.

The frequency polygon and the histogram are two different ways to represent the same data set. The choice of which one to use is left to the discretion of the researcher.

The Ogive

The third type of graph that can be used represents the cumulative frequencies for the classes. This type of graph is called the *cumulative frequency graph* or *ogive.* The **cumulative frequency** is the sum of the frequencies accumulated up to the upper boundary of a class in the distribution.

The **ogive** is a graph that represents the cumulative frequencies for the classes in a frequency distribution.

Example 2–6 shows the procedure for constructing an ogive.

Example 2–6

Construct an ogive for the frequency distribution described in Example 2–4.

Solution

Step 1 Find the cumulative frequency for each class.

Class boundaries	Cumulative frequency
99.5–104.5	2
104.5–109.5	10
109.5–114.5	28
114.5–119.5	41
119.5–124.5	48
124.5–129.5	49
129.5–134.5	50

Figure 2–4

Plotting the Cumulative Frequency for Example 2–6

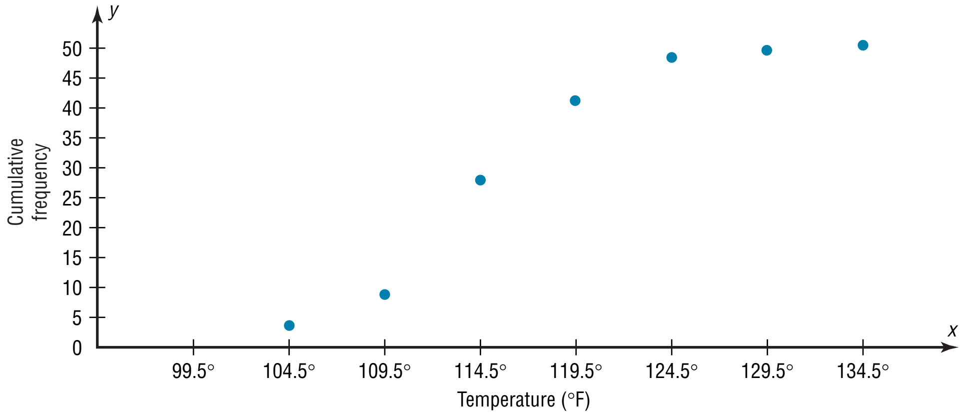
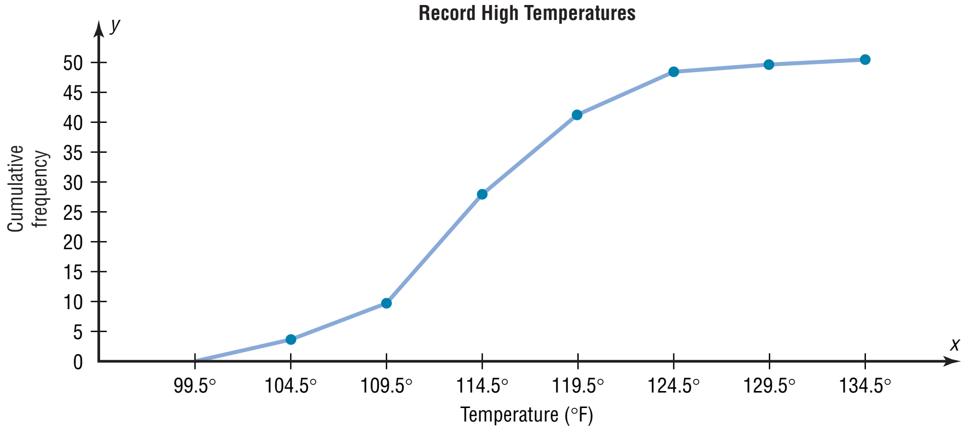


Figure 2–5

Ogive for Example 2–6



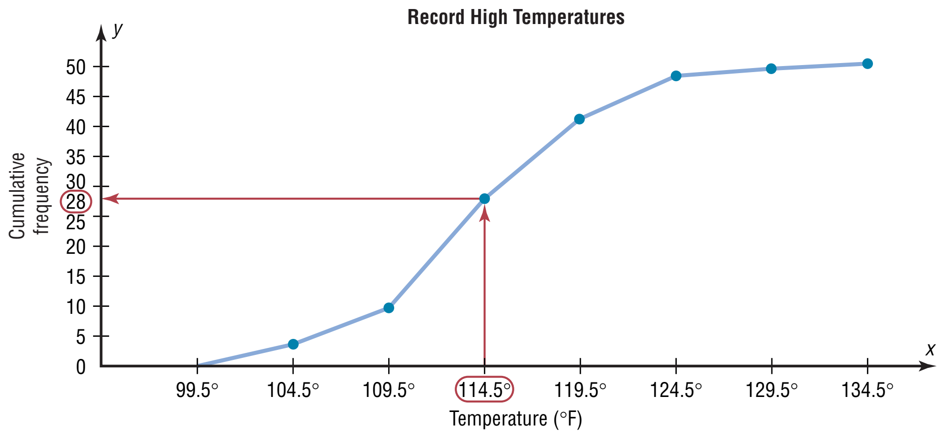
Step 2 Draw the *x* and *y* axes. Label the *x* axis with the class boundaries. Use an appropriate scale for the *y* axis to represent the cumulative frequencies. (Depending on the numbers in the cumulative frequency columns, scales such as 0, 1, 2, 3, . . . , or 5, 10, 15, 20, . . . , or 1000, 2000, 3000, . . . can be used. Do *not* label the *y* axis with the numbers in the cumulative frequency column.) In this example, a scale of 0, 5, 10, 15, . . . will be used.

Step 3 Plot the cumulative frequency at each upper class boundary, as shown in Figure 2–4. Upper boundaries are used since the cumulative frequencies represent the number of data values accumulated up to the upper boundary of each class.

Step 4 Starting with the first upper class boundary, 104.5, connect adjacent points with line segments, as shown in Figure 2–5. Then extend the graph to the first lower class boundary, 99.5, on the *x* axis.

Cumulative frequency graphs are used to visually represent how many values are below a certain upper class boundary. For example, to find out how many record high temperatures are less than 114.5°F, locate 114.5°F on the *x* axis, draw a vertical line up until it intersects the graph, and then draw a horizontal line at that point to the *y* axis. The *y* axis value is 28, as shown in Figure 2–6.

Figure 2–6
Finding a Specific Cumulative Frequency



The steps for drawing these three types of graphs are shown in the following Procedure Table.

Unusual Stat

Twenty-two percent of Americans sleep 6 hours a day or fewer.

Procedure Table

Constructing Statistical Graphs

Step 1 Draw and label the x and y axes.

Step 2 Choose a suitable scale for the frequencies or cumulative frequencies, and label it on the y axis.

Step 3 Represent the class boundaries for the histogram or ogive, or the midpoint for the frequency polygon, on the x axis.

Step 4 Plot the points and then draw the bars or lines.

Relative Frequency Graphs

The histogram, the frequency polygon, and the ogive shown previously were constructed by using frequencies in terms of the raw data. These distributions can be converted to distributions using *proportions* instead of raw data as frequencies. These types of graphs are called **relative frequency graphs.**

Graphs of relative frequencies instead of frequencies are used when the proportion of data values that fall into a given class is more important than the actual number of data values that fall into that class. For example, if one wanted to compare the age distribution of adults in Philadelphia, Pennsylvania, with the age distribution of adults of Erie, Pennsylvania, one would use relative frequency distributions. The reason is that since the population of Philadelphia is 1,478,002 and the population of Erie is 105,270, the bars using the actual data values for Philadelphia would be much taller than those for the same classes for Erie.

To convert a frequency into a proportion or relative frequency, divide the frequency for each class by the total of the frequencies. The sum of the relative frequencies will always be 1. These graphs are similar to the ones that use raw data as frequencies, but the values on the y axis are in terms of proportions. Example 2–7 shows the three types of relative frequency graphs.

Example 2–7

Construct a histogram, frequency polygon, and ogive using relative frequencies for the distribution (shown here) of the miles that 20 randomly selected runners ran during a given week.

Class boundaries	Frequency	Cumulative frequency
5.5–10.5	1	1
10.5–15.5	2	3
15.5–20.5	3	6
20.5–25.5	5	11
25.5–30.5	4	15
30.5–35.5	3	18
35.5–40.5	2	20
	20	

Solution

Step 1 Convert each frequency to a proportion or relative frequency by dividing the frequency for each class by the total number of observations.

For class 5.5–10.5, the relative frequency is $\frac{1}{20} = 0.05$; for class 10.5–15.5, the relative frequency is $\frac{2}{20} = 0.10$; for class 15.5–20.5, the relative frequency is $\frac{3}{20} = 0.15$; and so on.

Place these values in the column labeled Relative frequency.

Step 2 Find the cumulative relative frequencies. To do this, add the frequency in each class to the total frequency of the preceding class. In this case, $0 + 0.05 = 0.05$, $0.05 + 0.10 = 0.15$, $0.15 + 0.15 = 0.30$, $0.30 + 0.25 = 0.55$, etc. Place these values in the column labeled Cumulative relative frequency.

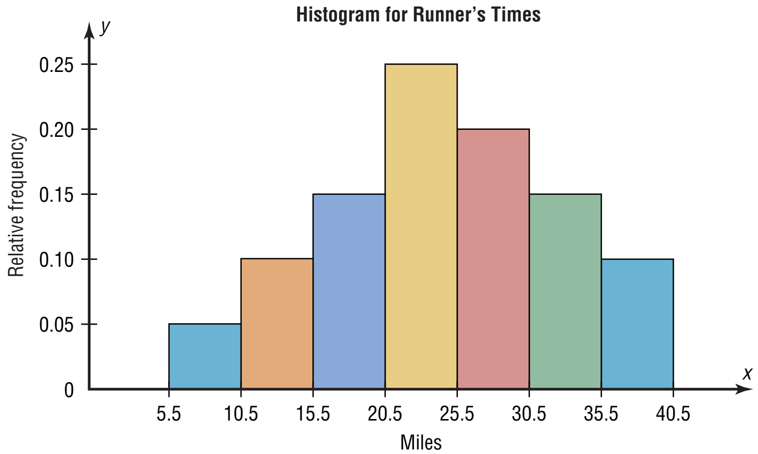
Using the same procedure, find the relative frequencies for the Cumulative frequency column. The relative frequencies are shown here.

Class boundaries	Midpoints	Relative frequency	Cumulative relative frequency
5.5–10.5	8	0.05	0.05
10.5–15.5	13	0.10	0.15
15.5–20.5	18	0.15	0.30
20.5–25.5	23	0.25	0.55
25.5–30.5	28	0.20	0.75
30.5–35.5	33	0.15	0.90
35.5–40.5	38	0.10	1.00
		1.00	

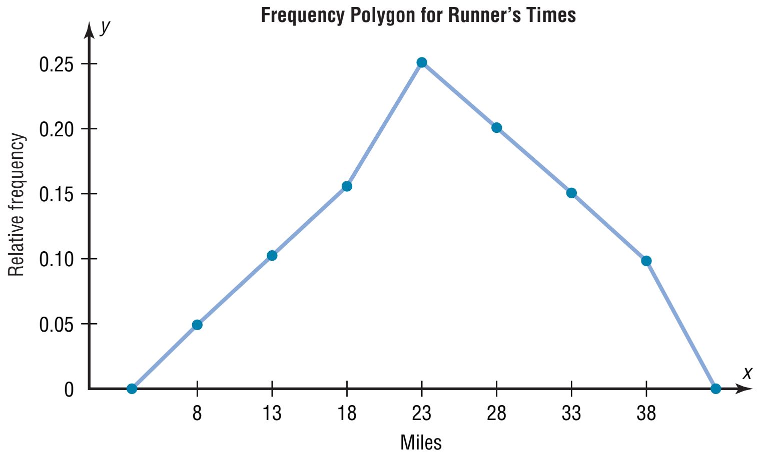
Step 3 Draw each graph as shown in Figure 2–7. For the histogram and ogive, use the class boundaries along the x axis. For the frequency polygon, use the midpoints on the x axis. The scale on the y axis uses proportions.

Figure 2–7

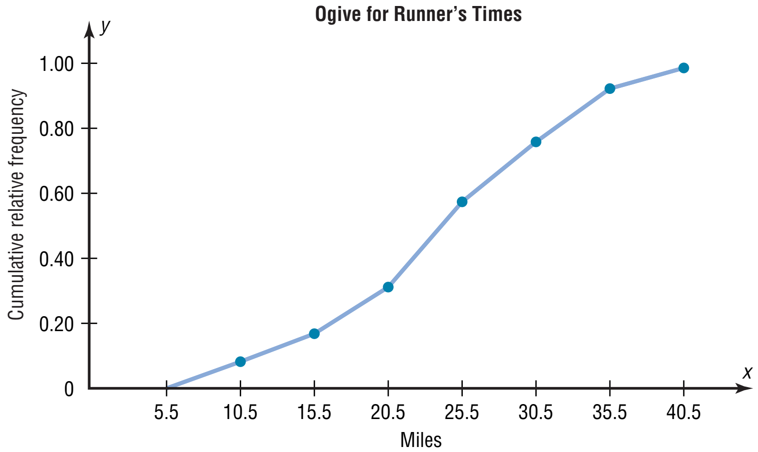
Graphs for Example 2–7



(a) Histogram



(b) Frequency polygon

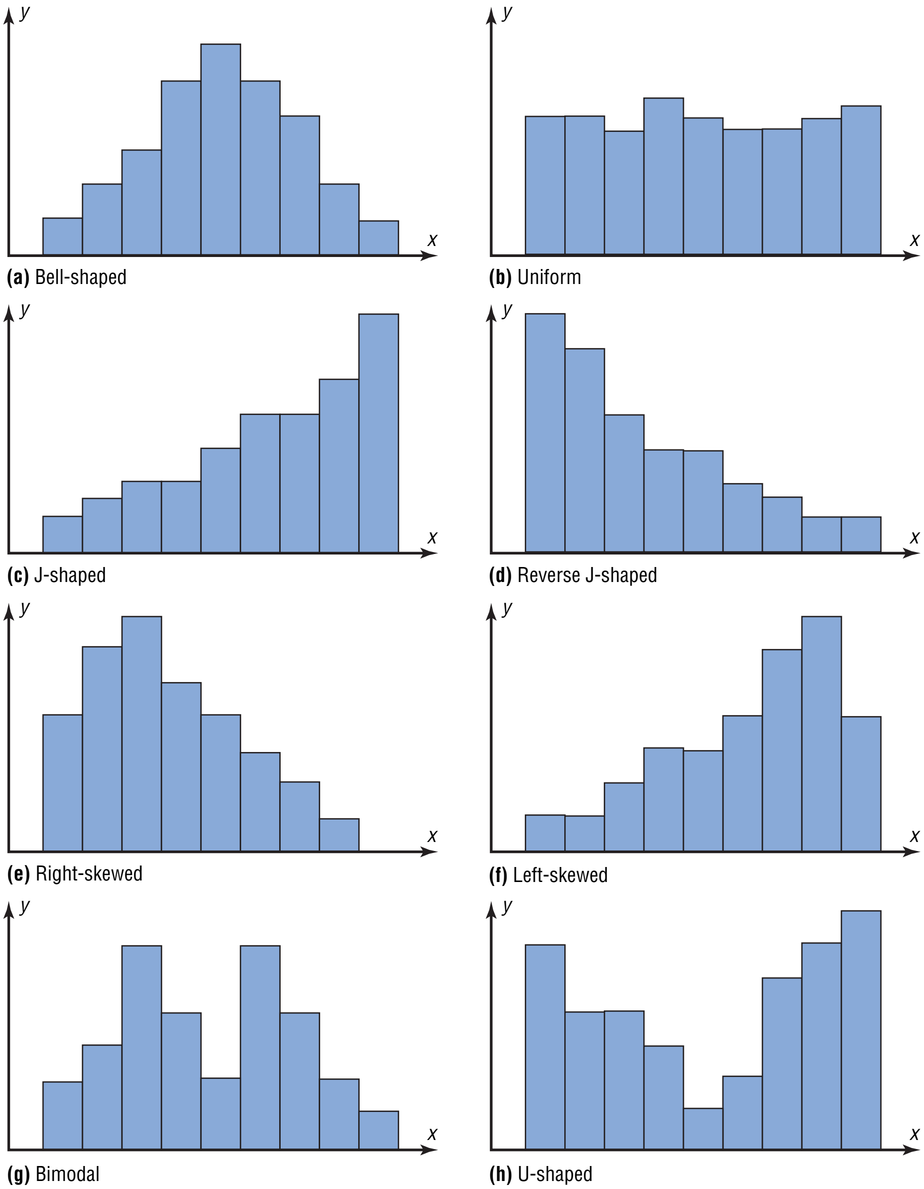


(c) Ogive

Distribution Shapes

When one is describing data, it is important to be able to recognize the shapes of the distribution values. In later chapters you will see that the shape of a distribution also determines the appropriate statistical methods used to analyze the data.

Figure 2–8
Distribution Shapes



A distribution can have many shapes, and one method of analyzing a distribution is to draw a histogram or frequency polygon for the distribution. Several of the most common shapes are shown in Figure 2–8: *the bell-shaped or mound-shaped, the uniform-shaped, the J-shaped, the reverse J-shaped, the positively or right-skewed shaped, the negatively or left-skewed shaped, the bimodal-shaped, and the U-shaped.*

Distributions are most often not perfectly shaped, so it is not necessary to have an exact shape but rather to identify an overall pattern.

A *bell-shaped distribution* shown in Figure 2–8(a) has a single peak and tapers off at either end. It is approximately symmetric; i.e., it is roughly the same on both sides of a line running through the center.

A *uniform distribution* is basically flat or rectangular. See Figure 2–8(b).

A *J-shaped distribution* is shown in Figure 2–8(c), and it has a few data values on the left side and increases as one moves to the right. A *reverse J-shaped distribution* is the opposite of the J-shaped distribution. See Figure 2–8(d).

When the peak of a distribution is to the left and the data values taper off to the right, a distribution is said to be *positively or right-skewed.* See Figure 2–8(e). When the data values are clustered to the right and taper off to the left, a distribution is said to be *negatively or left-skewed.* See Figure 2–8(f). Skewness will be explained in detail in Chapter 3, pages 108–109. Distributions with one peak, such as those shown in Figure 2–8(a), (e), and (f), are said to be *unimodal.* (The highest peak of a distribution indicates where the mode of the data values is. The mode is the data value that occurs more often than any other data value. Modes are explained in Chapter 3.) When a distribution has two peaks of the same height, it is said to be *bimodal.* See Figure 2–8(g). Finally, the graph shown in Figure 2–8(h) is a *U-shaped* distribution.

Distributions can have other shapes in addition to the ones shown here; however, these are some of the more common ones that you will encounter in analyzing data.

When you are analyzing histograms and frequency polygons, look at the shape of the curve. For example, does it have one peak or two peaks? Is it relatively flat, or is it U-shaped? Are the data values spread out on the graph, or are they clustered around the center? Are there data values in the extreme ends? These may be *outliers.* (See Section 3–4 for an explanation of outliers.) Are there any gaps in the histogram, or does the frequency polygon touch the x axis somewhere other than the ends? Finally, are the data clustered at one end or the other, indicating a *skewed distribution*?

For example, the histogram for the record high temperatures shown in Figure 2–2 shows a single peaked distribution, with the class 109.5–114.5 containing the largest number of temperatures. The distribution has no gaps, and there are fewer temperatures in the highest class than in the lowest class.

Applying the Concepts **2–3**

Selling Real Estate

Assume you are a realtor in Bradenton, Florida. You have recently obtained a listing of the selling prices of the homes that have sold in that area in the last 6 months. You wish to organize that data so you will be able to provide potential buyers with useful information. Use the following data to create a histogram, frequency polygon, and cumulative frequency polygon.

142,000	127,000	99,600	162,000	89,000	93,000	99,500
73,800	135,000	119,500	67,900	156,300	104,500	108,650
123,000	91,000	205,000	110,000	156,300	104,000	133,900
179,000	112,000	147,000	321,550	87,900	88,400	180,000
159,400	205,300	144,400	163,000	96,000	81,000	131,000
114,000	119,600	93,000	123,000	187,000	96,000	80,000
231,000	189,500	177,600	83,400	77,000	132,300	166,000

1. What questions could be answered more easily by looking at the histogram rather than the listing of home prices?
2. What different questions could be answered more easily by looking at the frequency polygon rather than the listing of home prices?
3. What different questions could be answered more easily by looking at the cumulative frequency polygon rather than the listing of home prices?
4. Are there any extremely large or extremely small data values compared to the other data values?
5. Which graph displays these extremes the best?
6. Is the distribution skewed?

See page 93 for the answers.

Exercises 2–3

1. For 108 randomly selected college applicants, the following frequency distribution for entrance exam scores was obtained. Construct a histogram, frequency polygon, and ogive for the data. (The data for this exercise will be used for Exercise 13 in this section.)

Class limits	Frequency
90–98	6
99–107	22
108–116	43
117–125	28
126–134	9

Applicants who score above 107 need not enroll in a summer developmental program. In this group, how many students do not have to enroll in the developmental program?

2. For 75 employees of a large department store, the following distribution for years of service was obtained. Construct a histogram, frequency polygon, and ogive for the data. (The data for this exercise will be used for Exercise 14 in this section.)

Class limits	Frequency
1–5	21
6–10	25
11–15	15
16–20	0
21–25	8
26–30	6

A majority of the employees have worked for how many years or less?

3. The scores for the 2002 LPGA—Giant Eagle are shown.

Score	Frequency
202–204	2
205–207	7
208–210	16
211–213	26
214–216	18
217–219	4

Source: LPGA.com.

Construct a histogram, frequency polygon, and ogive for the distribution. Comment on the skewness of the distribution.

4. The salaries (in millions of dollars) for 31 NFL teams for a specific season are given in this frequency distribution.

Class limits	Frequency
39.9–42.8	2
42.9–45.8	2
45.9–48.8	5
48.9–51.8	5
51.9–54.8	12
54.9–57.8	5

Source: NFL.com.

Construct a histogram, frequency polygon, and ogive for the data; and comment on the shape of the distribution.

5. Thirty automobiles were tested for fuel efficiency, in miles per gallon (mpg). The following frequency distribution was obtained. Construct a histogram, frequency polygon, and ogive for the data.

Class boundaries	Frequency
7.5–12.5	3
12.5–17.5	5
17.5–22.5	15
22.5–27.5	5
27.5–32.5	2

6. Construct a histogram, frequency polygon, and ogive for the data in Exercise 14 in Section 2–2, and analyze the results.

7. The air quality measured for selected cities in the United States for 1993 and 2002 is shown. The data are the number of days per year that the cities failed to meet acceptable standards. Construct a histogram for both years and see if there are any notable changes. If so, explain. (The data in this exercise will be used for Exercise 17 in this section.)

1993		2002	
Class	Frequency	Class	Frequency
0–27	20	0–27	19
28–55	4	28–55	6
56–83	3	56–83	2
84–111	1	84–111	0
112–139	1	112–139	0
140–167	0	140–167	3
168–195	1	168–195	0

Source: *World Almanac and Book of Facts.*

8. In a study of reaction times of dogs to a specific stimulus, an animal trainer obtained the following data, given in seconds. Construct a histogram, frequency polygon, and ogive for the data, and analyze the results.

(The histogram in this exercise will be used for Exercise 18 in this section, Exercise 16 in Section 3–2, and Exercise 26 in Section 3–3.)

Class limits	Frequency
2.3–2.9	10
3.0–3.6	12
3.7–4.3	6
4.4–5.0	8
5.1–5.7	4
5.8–6.4	2

9. Construct a histogram, frequency polygon, and ogive for the data in Exercise 15 of Section 2–2, and analyze the results.

10. The frequency distributions shown indicate the percentages of public school students in fourth-grade reading and mathematics who performed at or above the required proficiency levels for the 50 states in the United States. Draw histograms for each and decide if there is any difference in the performance of the students in the subjects.

Class	Reading Frequency	Math Frequency
17.5–22.5	7	5
22.5–27.5	6	9
27.5–32.5	14	11
32.5–37.5	19	16
37.5–42.5	3	8
42.5–47.5	1	1

Source: *National Center for Educational Statistics.*

11. Construct a histogram, frequency polygon, and ogive for the data in Exercise 16 in Section 2–2, and analyze the results.

12. For the data in Exercise 18 in Section 2–2, construct a histogram for the home run distances for each player and compare them. Are they basically the same, or are there any noticeable differences? Explain your answer.

13. For the data in Exercise 1 in this section, construct a histogram, frequency polygon, and ogive, using relative frequencies. What proportion of the applicants need to enroll in the summer developmental program?

14. For the data in Exercise 2 in this section, construct a histogram, frequency polygon, and ogive, using relative frequencies. What proportion of the employees have been with the store for more than 20 years?

15. The number of calories per serving for selected ready-to-eat cereals is listed here. Construct a frequency distribution using 7 classes. Draw a histogram, frequency polygon, and ogive for the data, using relative frequencies. Describe the shape of the histogram.

130	190	140	80	100	120	220	220	110	100
210	130	100	90	210	120	200	120	180	120
190	210	120	200	130	180	260	270	100	160
190	240	80	120	90	190	200	210	190	180
115	210	110	225	190	130				

Source: *The Doctor's Pocket Calorie, Fat, and Carbohydrate Counter.*

16. The amount of protein (in grams) for a variety of fast-food sandwiches is reported here. Construct a frequency distribution using 6 classes. Draw a histogram, frequency polygon, and ogive for the data, using relative frequencies. Describe the shape of the histogram.

23	30	20	27	44	26	35	20	29	29
25	15	18	27	19	22	12	26	34	15
27	35	26	43	35	14	24	12	23	31
40	35	38	57	22	42	24	21	27	33

Source: *The Doctor's Pocket Calorie, Fat, and Carbohydrate Counter.*

17. For the data for year 2002 in Exercise 7 in this section, construct a histogram, frequency polygon, and ogive, using relative frequencies.

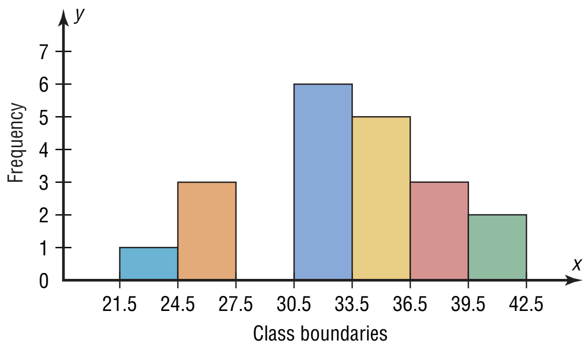
18. The animal trainer in Exercise 8 in this section selected another group of dogs who were much older than the first group and measured their reaction times to the same stimulus. Construct a histogram, frequency polygon, and ogive for the data.

Class limits	Frequency
2.3–2.9	1
3.0–3.6	3
3.7–4.3	4
4.4–5.0	16
5.1–5.7	14
5.8–6.4	4

Analyze the results and compare the histogram for this group with the one obtained in Exercise 8 in this section. Are there any differences in the histograms? (The data in this exercise will be used for Exercise 16 in Section 3–2 and Exercise 26 in Section 3–3.)

Extending the Concepts

19. Using the histogram shown here, do the following.



a. Construct a frequency distribution; include class limits, class frequencies, midpoints, and cumulative frequencies.
b. Construct a frequency polygon.
c. Construct an ogive.

20. Using the results from Exercise 19, answer these questions.

a. How many values are in the class 27.5–30.5?
b. How many values fall between 24.5 and 36.5?
c. How many values are below 33.5?
d. How many values are above 30.5?

Technology *Step by Step*

MINITAB
Step by Step

Construct a Histogram

1. Enter the data from Example 2–2, the high temperatures for the 50 states.

2. Select **Graph>Histogram.**

3. Select [Simple], then click [OK].

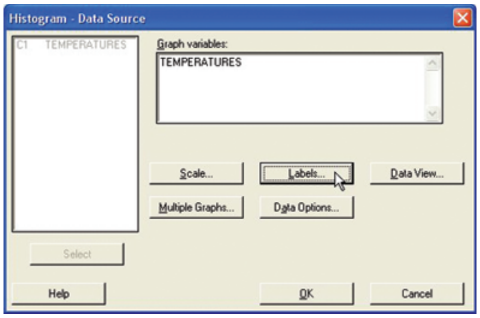
4. Click C1 TEMPERATURES in the Graph variables dialog box.

5. Click [Labels]. There are two tabs, Title/Footnote and Data Labels.

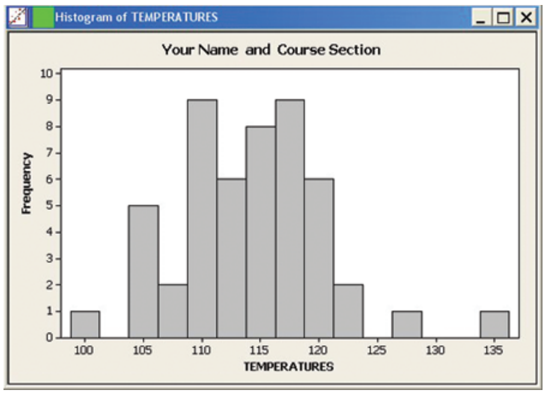
a) Click in the box for Title, and type in Your Name and Course Section.

b) Click [OK]. The Histogram dialog box is still open.

6. Click [OK]. A new graph window containing the histogram will open.



7. Click the **File** menu to print or save the graph.



8. Click **File>Exit.**
9. Save the project as **Ch2-3.mpj.**

TI-83 Plus or TI-84 Plus
Step by Step

Constructing a Histogram

To display the graphs on the screen, enter the appropriate values in the calculator, using the WINDOW menu. The default values are $X_{\text{min}} = -10$, $X_{\text{max}} = +10$, $Y_{\text{min}} = -10$, and $Y_{\text{max}} = +10$.

The X_{scl} changes the distance between the tick marks on the x axis and can be used to change the class width for the histogram.

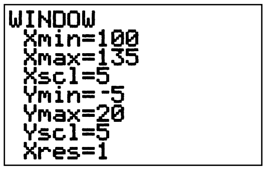
To change the values in the WINDOW:

1. Press **WINDOW.**
2. Move the cursor to the value that needs to be changed. Then type in the desired value and press **ENTER.**
3. Continue until all values are appropriate.
4. Press **[2nd] [QUIT]** to leave the WINDOW menu.

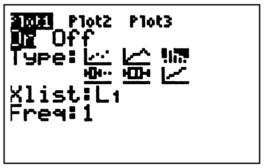
To plot the histogram from raw data:

1. Enter the data in L_1.
2. Make sure WINDOW values are appropriate for the histogram.
3. Press **[2nd] [STAT PLOT] ENTER.**
4. Press **ENTER** to turn the plot on, if necessary.
5. Move cursor to the Histogram symbol and press **ENTER,** if necessary.
6. Make sure Xlist is L_1.
7. Make sure Freq is 1.
8. Press **GRAPH** to display the histogram.
9. To obtain the number of data values in each class, press the **TRACE** key, followed by ◀ or ▶ keys.

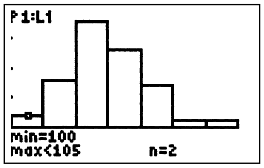
Input



Input



Output



Example TI2–1

Plot a histogram for the following data from Examples 2–2 and 2–4.

112	100	127	120	134	118	105	110	109	112
110	118	117	116	118	122	114	114	105	109
107	112	114	115	118	117	118	122	106	110
116	108	110	121	113	120	119	111	104	111
120	113	120	117	105	110	118	112	114	114

Press **TRACE** and use the arrow keys to determine the number of values in each group.

To graph a histogram from grouped data:

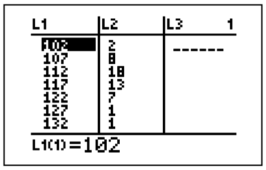
1. Enter the midpoints into L_1.
2. Enter the frequencies into L_2.
3. Make sure WINDOW values are appropriate for the histogram.
4. Press **[2nd] [STAT PLOT] ENTER.**
5. Press **ENTER** to turn the plot on, if necessary.
6. Move cursor to the histogram symbol, and press **ENTER,** if necessary.
7. Make sure Xlist is L_1.
8. Make sure Freq is L_2.
9. Press **GRAPH** to display the histogram.

Example TI2–2

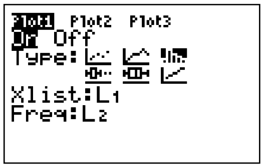
Plot a histogram for the data from Examples 2–4 and 2–5.

Class boundaries	Midpoints	Frequency
99.5–104.5	102	2
104.5–109.5	107	8
109.5–114.5	112	18
114.5–119.5	117	13
119.5–124.5	122	7
124.5–129.5	127	1
129.5–134.5	132	1

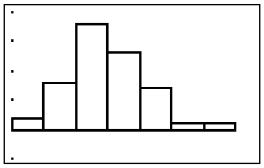
Input



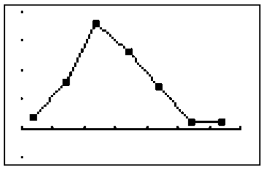
Input



Output

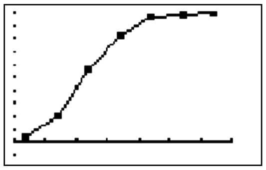


Output



To graph a frequency polygon from grouped data, follow the same steps as for the histogram except change the graph type from histogram (third graph) to a line graph (second graph).

Output



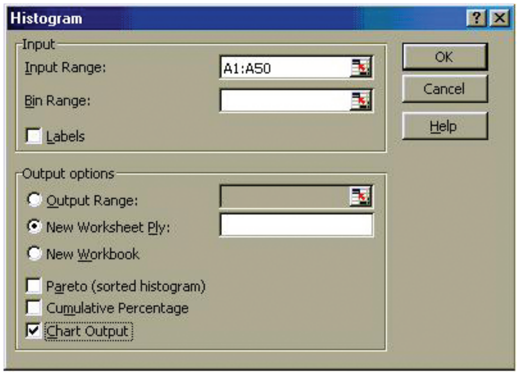
To graph an ogive from grouped data, modify the procedure for the histogram as follows:

1. Enter the upper class boundaries into L_1.
2. Enter the cumulative frequencies into L_2.
3. Change the graph type from histogram (third graph) to line (second graph).
4. Change the Y_{max} from the WINDOW menu to the sample size.

Excel
Step by Step

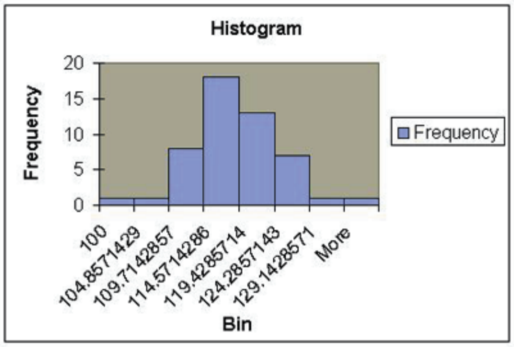
Constructing a Histogram

1. Press **[Ctrl]-N** for a new worksheet.
2. Enter the data from Examples 2–2 and 2–4 in column A, one number per cell.
3. Select **Tools>Data Analysis.**
4. In Data Analysis, select Histogram and click the [OK] button.
5. In the Histogram dialog box, type **A1:A50** as the Input Range.



6. Select New Worksheet Ply and Chart Output. Click [OK].

Excel presents both a table and a chart on the new worksheet ply. It decides "bins" for the histogram itself (here it picked a bin size of 7 units), but you can also define your own bin range on the data worksheet.



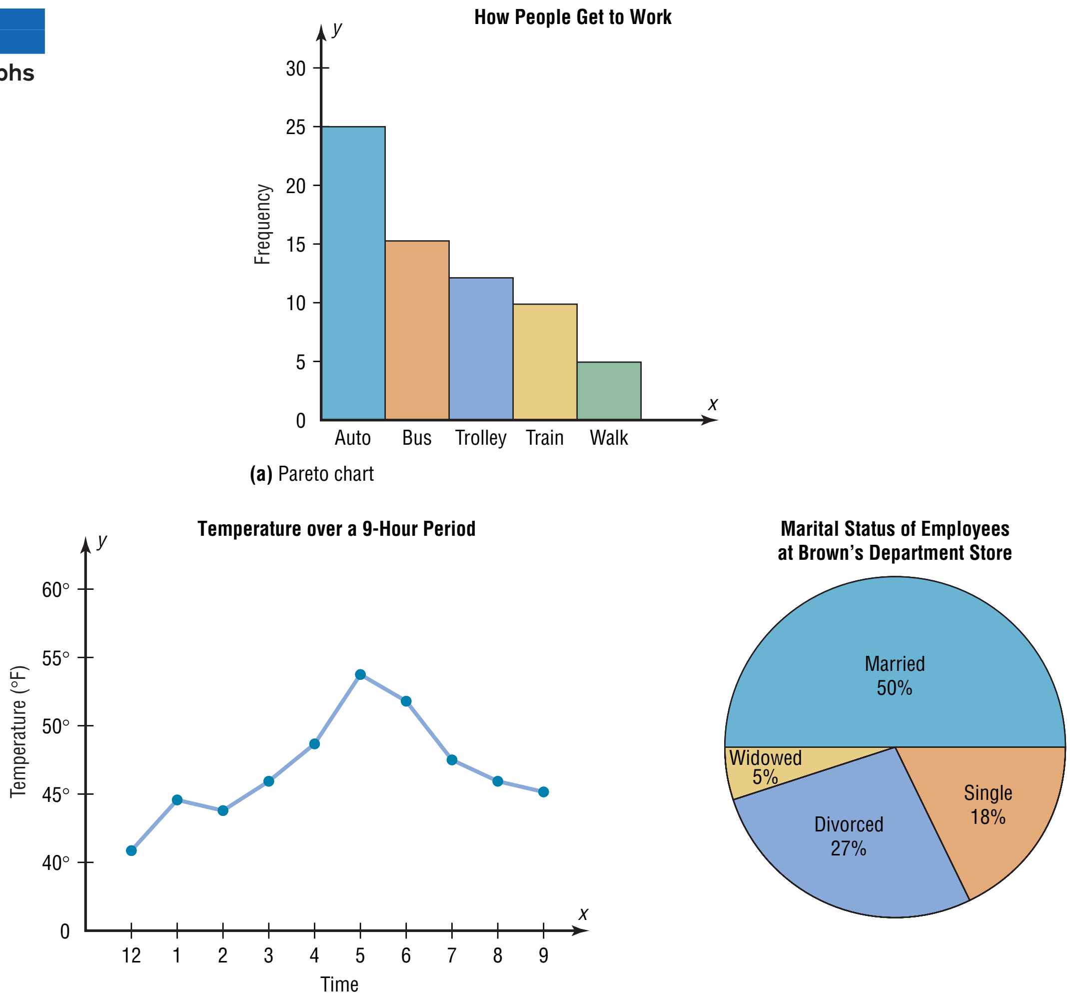
The vertical bars on the histogram can be made contiguous by right-clicking on one of the bars and selecting **Format Data Series.** Select the Options tab, then enter **0** in the Gap Width box.

2–4 Other Types of Graphs

In addition to the histogram, the frequency polygon, and the ogive, several other types of graphs are often used in statistics. They are the Pareto chart, the time series graph, and the pie graph. Figure 2–9 shows an example of each type of graph.

Figure 2–9

Other Types of Graphs Used in Statistics



(a) Pareto chart

(b) Time series graph

(c) Pie graph

Objective 3

Represent data using Pareto charts, time series graphs, and pie graphs.

Pareto Charts

In Section 2–3, graphs such as the histogram, frequency polygon, and ogive showed how data can be represented when the variable displayed on the horizontal axis is quantitative, such as heights and weights.

On the other hand, when the variable displayed on the horizontal axis is qualitative or categorical, a *Pareto chart* can be used.

A **Pareto chart** is used to represent a frequency distribution for a categorical variable, and the frequencies are displayed by the heights of vertical bars, which are arranged in order from highest to lowest.

Example 2–8

The table shown here is the average cost per mile for passenger vehicles on state turnpikes. Construct and analyze a Pareto chart for the data.

State	Number
Indiana	2.9¢
Oklahoma	4.3¢
Florida	6.0¢
Maine	3.8¢
Pennsylvania	5.8¢

Source: *Pittsburgh Tribune Review.*

Historical Note

Vilfredo Pareto (1848–1923) was an Italian scholar who developed theories in economics, statistics, and the social sciences. His contributions to statistics include the development of a mathematical function used in economics. This function has many statistical applications and is called the Pareto distribution. In addition, he researched income distribution, and his findings became known as Pareto's law.

Solution

Step 1 Arrange the data from the largest to smallest according to frequency.

State	Number
Florida	6.0¢
Pennsylvania	5.8¢
Oklahoma	4.3¢
Maine	3.8¢
Indiana	2.9¢

Step 2 Draw and label the x and y axes.

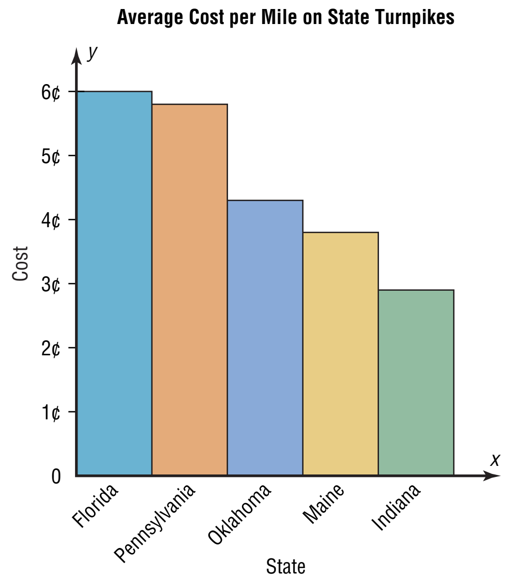
Step 3 Draw the bars corresponding to the frequencies. See Figure 2–10. The Pareto chart shows that Florida has the highest cost per mile. The cost is more than twice as high as the cost for Indiana.

Suggestions for Drawing Pareto Charts

1. Make the bars the same width.
2. Arrange the data from largest to smallest according to frequency.
3. Make the units that are used for the frequency equal in size.

When you analyze a Pareto chart, make comparisons by looking at the heights of the bars.

Figure 2–10
Pareto Chart for Example 2–8



The Time Series Graph

When data are collected over a period of time, they can be represented by a time series graph.

A **time series graph** represents data that occur over a specific period of time.

Example 2–9 shows the procedure for constructing a time series graph.

Example 2–9

The number (in millions) of vehicles, both passenger and commercial, that used the Pennsylvania Turnpike for the years 1999 through 2003 is shown. Construct and analyze a time series graph for the data.

Year	Number
1999	156.2
2000	160.1
2001	162.3
2002	172.8
2003	179.4

Source: *Tribune Review.*

Historical Note

Time series graphs are over 1000 years old. The first ones were used to chart the movements of the planets and the sun.

Solution

Step 1 Draw and label the x and y axes.

Step 2 Label the x axis for years and the y axis for the number of vehicles.

Step 3 Plot each point according to the table.

Step 4 Draw line segments connecting adjacent points. Do not try to fit a smooth curve through the data points. See Figure 2–11. The graph shows a steady increase over the 5-year period.

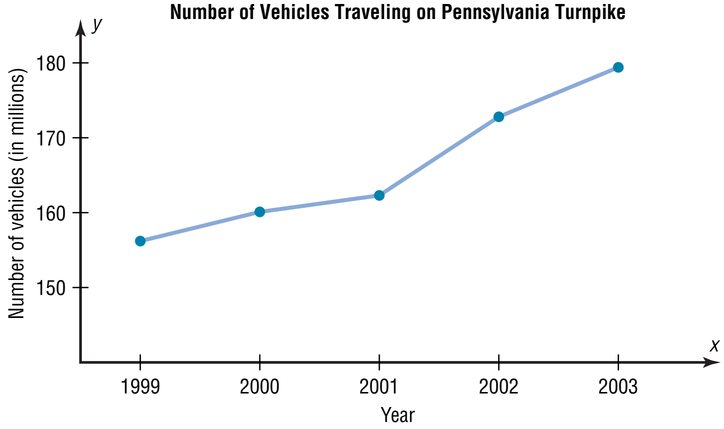


Figure 2–11

Time Series Graph for Example 2–9

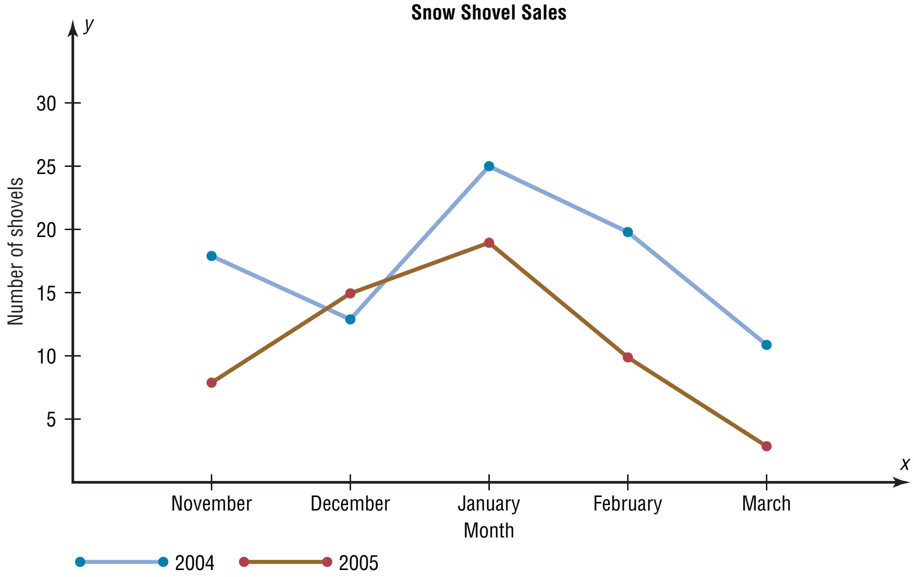


Figure 2–12

Two Time Series Graphs for Comparison

When you analyze a time series graph, look for a trend or pattern that occurs over the time period. For example, is the line ascending (indicating an increase over time) or descending (indicating a decrease over time)? Another thing to look for is the slope, or steepness, of the line. A line that is steep over a specific time period indicates a rapid increase or decrease over that period.

Two data sets can be compared on the same graph (called a *compound time series graph*) if two lines are used, as shown in Figure 2–12. This graph shows the number of snow shovels sold at a store for two seasons.

The Pie Graph

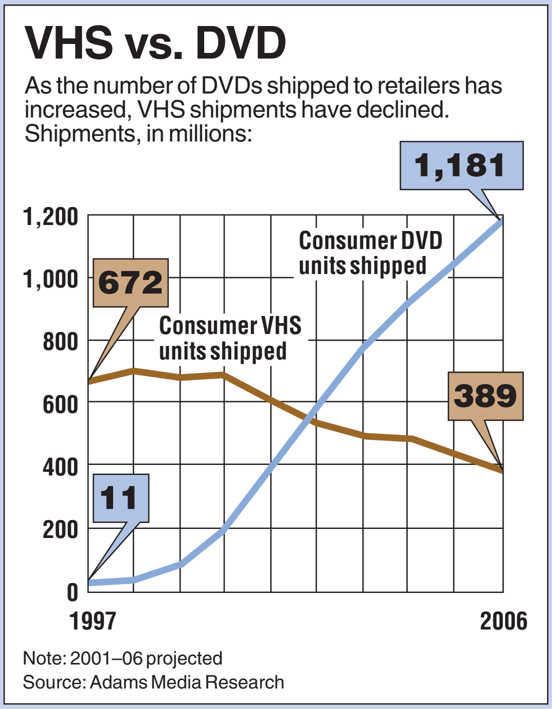
Pie graphs are used extensively in statistics. The purpose of the pie graph is to show the relationship of the parts to the whole by visually comparing the sizes of the sections. Percentages or proportions can be used. The variable is nominal or categorical.

A **pie graph** is a circle that is divided into sections or wedges according to the percentage of frequencies in each category of the distribution.

Example 2–10 shows the procedure for constructing a pie graph.

Speaking of Statistics

This time series graph compares the number of DVD units and the number of VHS units shipped to retailers, using a compound time series graph. Explain in your own words the information that is presented in the graph.



Source: Copyright 2001, *USA TODAY.* Reprinted with permission.

Example 2–10

This frequency distribution shows the number of pounds of each snack food eaten during the Super Bowl. Construct a pie graph for the data.

Snack	Pounds (frequency)
Potato chips	11.2 million
Tortilla chips	8.2 million
Pretzels	4.3 million
Popcorn	3.8 million
Snack nuts	2.5 million
Total	n = 30.0 million

Source: *USA TODAY Weekend.*

Solution

Step 1 Since there are 360° in a circle, the frequency for each class must be converted into a proportional part of the circle. This conversion is done by using the formula

$$\text{Degrees} = \frac{f}{n} \cdot 360°$$

where f = frequency for each class and n = sum of the frequencies. Hence, the following conversions are obtained. The degrees should sum to 360°.*

**Note:* The degrees column does not always sum to 360° due to rounding.

Potato chips	$\frac{11.2}{30} \cdot 360° = 134°$
Tortilla chips	$\frac{8.2}{30} \cdot 360° = 98°$
Pretzels	$\frac{4.3}{30} \cdot 360° = 52°$
Popcorn	$\frac{3.8}{30} \cdot 360° = 46°$
Snack nuts	$\frac{2.5}{30} \cdot 360° = 30°$
Total	$360°$

Step 2 Each frequency must also be converted to a percentage. Recall from Example 2–1 that this conversion is done by using the formula

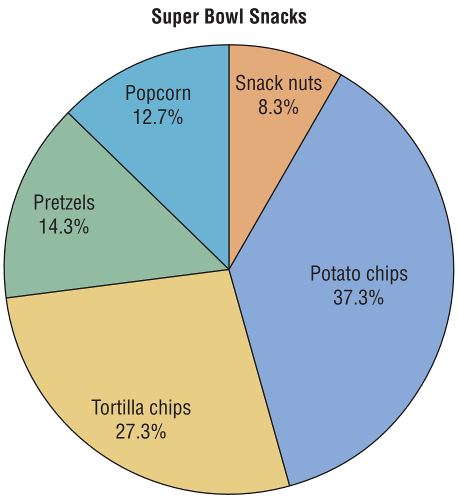
$$\% = \frac{f}{n} \cdot 100\%$$

Hence, the following percentages are obtained. The percentages should sum to 100%.*

Potato chips	$\frac{11.2}{30} \cdot 100\% = 37.3\%$
Tortilla chips	$\frac{8.2}{30} \cdot 100\% = 27.3\%$
Pretzels	$\frac{4.3}{30} \cdot 100\% = 14.3\%$
Popcorn	$\frac{3.8}{30} \cdot 100\% = 12.7\%$
Snack nuts	$\frac{2.5}{30} \cdot 100\% = 8.3\%$
Total	99.9%

Step 3 Next, using a protractor and a compass, draw the graph using the appropriate degree measures found in step 1, and label each section with the name and percentages, as shown in Figure 2–13.

Figure 2–13
Pie Graph for Example 2–10



*Note: The percent column does not always sum to 100% due to rounding.

Example 2–11

Construct a pie graph showing the blood types of the army inductees described in Example 2–1. The frequency distribution is repeated here.

Class	Frequency	Percent
A	5	20
B	7	28
O	9	36
AB	4	16
	25	100

Solution

Step 1 Find the number of degrees for each class, using the formula

$$\text{Degrees} = \frac{f}{n} \cdot 360^\circ$$

For each class, then, the following results are obtained.

A $\quad \frac{5}{25} \cdot 360^\circ = 72^\circ$

B $\quad \frac{7}{25} \cdot 360^\circ = 100.8^\circ$

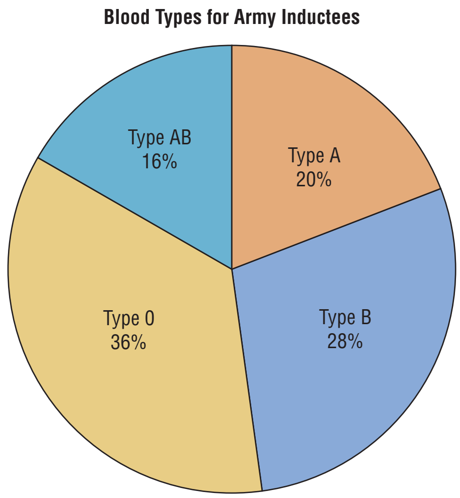
O $\quad \frac{9}{25} \cdot 360^\circ = 129.6^\circ$

AB $\quad \frac{4}{25} \cdot 360^\circ = 57.6^\circ$

Step 2 Find the percentages. (This was already done in Example 2–1.)

Step 3 Using a protractor, graph each section and write its name and corresponding percentage, as shown in Figure 2–14.

Figure 2–14
Pie Graph for Example 2–11



The graph in Figure 2–14 shows that in this case the most common blood type is type O.

To analyze the nature of the data shown in the pie graph, compare the sections. For example, are any sections relatively large compared to the rest?

Figure 2–14 shows that among the inductees, type O blood is more prevalent than any other type. People who have type AB blood are in the minority. More than twice as many people have type O blood as type AB.

Misleading Graphs

Graphs give a visual representation that enables readers to analyze and interpret data more easily than they could simply by looking at numbers. However, inappropriately drawn graphs can misrepresent the data and lead the reader to false conclusions. For example, a car manufacturer's ad stated that 98% of the vehicles it had sold in the past 10 years were still on the road. The ad then showed a graph similar to the one in Figure 2–15. The graph shows the percentage of the manufacturer's automobiles still on the road and the percentage of its competitors' automobiles still on the road. Is there a large difference? Not necessarily.

Notice the scale on the vertical axis in Figure 2–15. It has been cut off (or truncated) and starts at 95%. When the graph is redrawn using a scale that goes from 0 to 100%, as in Figure 2–16, there is hardly a noticeable difference in the percentages. Thus, changing the units at the starting point on the *y* axis can convey a very different visual representation of the data.

It is not wrong to truncate an axis of the graph; many times it is necessary to do so (see Example 2–9). However, the reader should be aware of this fact and interpret the graph accordingly. Do not be misled if an inappropriate impression is given.

Let's consider another example. The percentage of the world's total motor vehicles produced by manufacturers in the United States declined from 24% in 1998 to 21.5% in 2000, as shown by the data on the next page.

Figure 2–15

Graph of Automaker's Claim Using a Scale from 95 to 100%

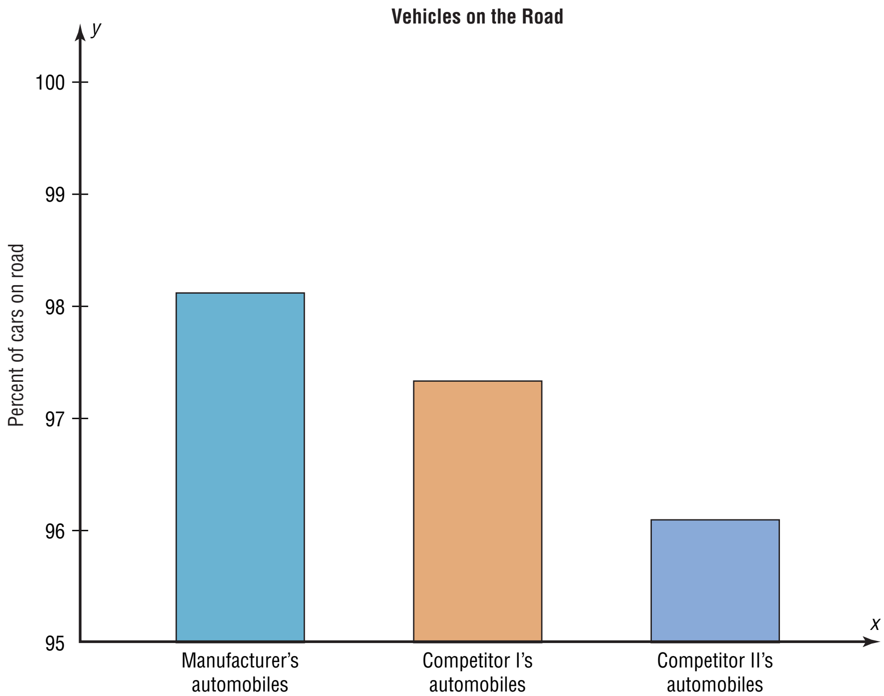
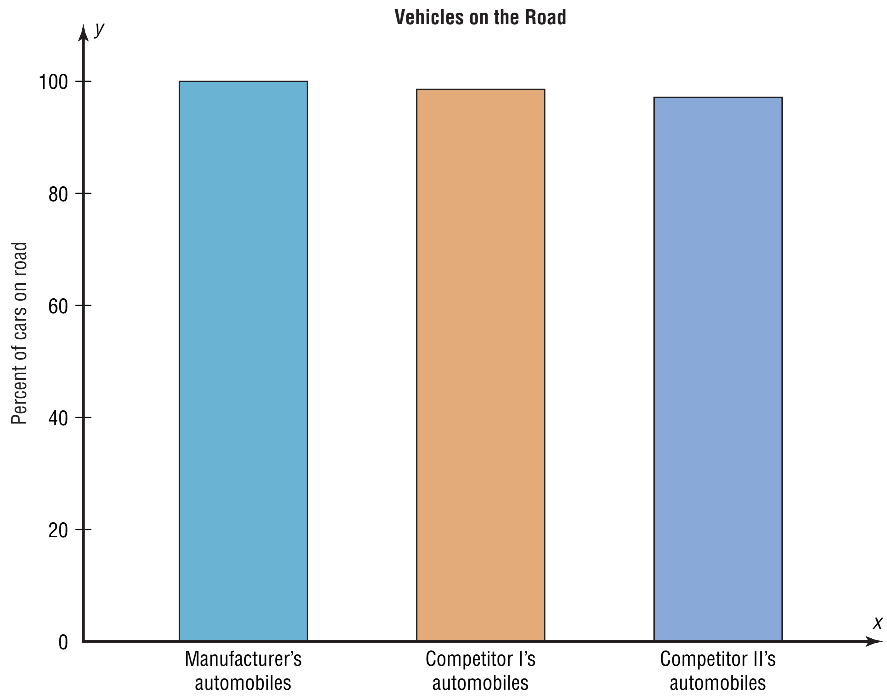


Figure 2–16
Graph in Figure 2–15 Redrawn Using a Scale from 0 to 100%



Year	1995	1996	1997	1998	1999	2000
Percent produced in United States	24.0	23.0	22.7	22.4	22.7	21.5

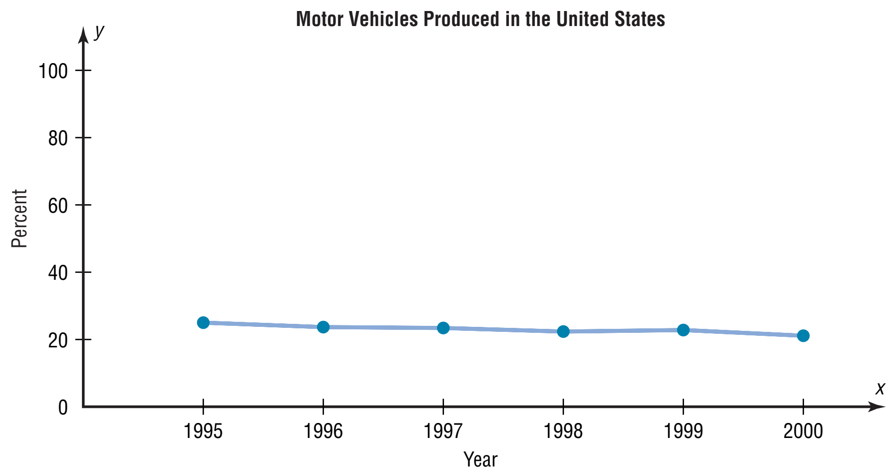
Source: *The World Almanac and Book of Facts.*

Interesting Fact

The most popular flavor of ice cream is vanilla, and about one-fourth of the ice cream sold is vanilla.

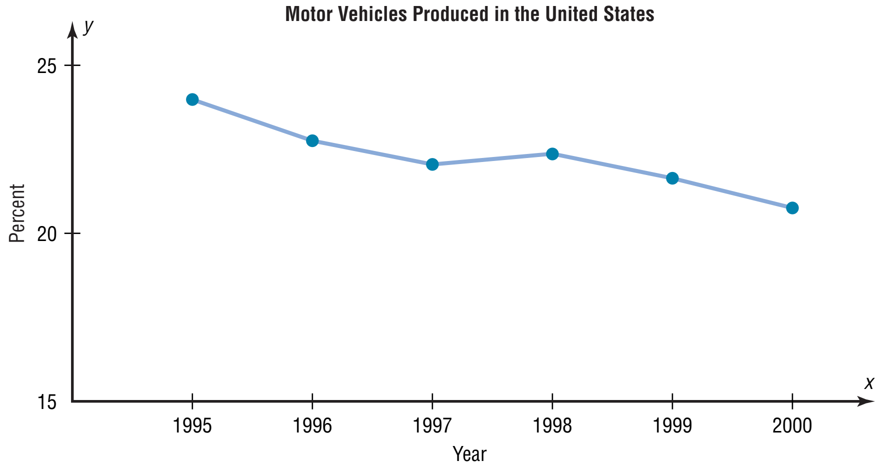
When one draws the graph, as shown in Figure 2–17(a), a scale ranging from 0 to 100% shows a slight decrease. However, this decrease can be emphasized by using a scale that ranges from 15 to 25%, as shown in Figure 2–17(b). Again, by changing the units or the starting point on the *y* axis, one can change the visual message.

Figure 2–17
Percent of World's Motor Vehicles Produced by Manufacturers in the United States



(a) Using a scale from 0% to 100%

Figure 2–17
(continued)



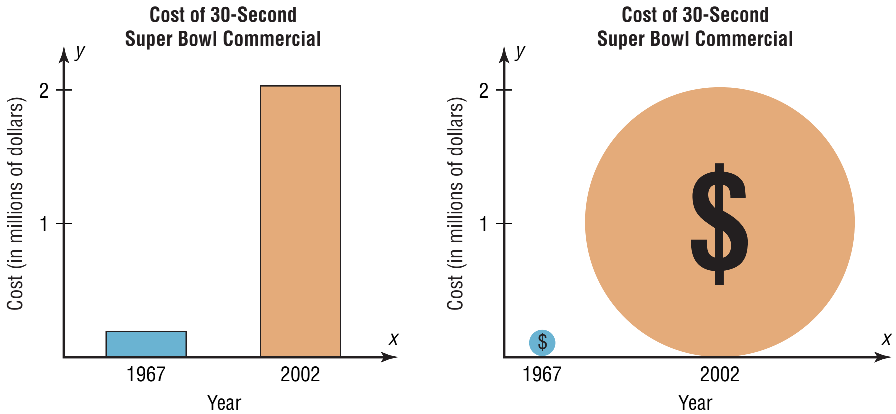
(b) Using a scale from 15% to 25%

Another misleading graphing technique sometimes used involves exaggerating a one-dimensional increase by showing it in two dimensions. For example, the average cost of a 30-second Super Bowl commercial has increased from $42,000 in 1967 to $1.9 million in 2002 (Source: *USA TODAY*).

The increase shown by the graph in Figure 2–18(a) represents the change by a comparison of the heights of the two bars in one dimension. The same data are shown two-dimensionally with circles in Figure 2–18(b). Notice that the difference seems much larger because the eye is comparing the areas of the circles rather than the lengths of the diameters.

Note that it is not wrong to use the graphing techniques of truncating the scales or representing data by two-dimensional pictures. But when these techniques are used, the reader should be cautious of the conclusion drawn on the basis of the graphs.

Figure 2–18
Comparison of Costs for a 30-Second Super Bowl Commercial

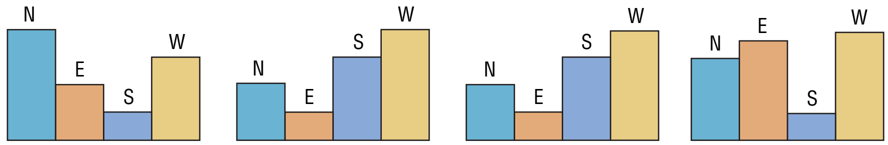


(a) Graph using bars **(b)** Graph using circles

Another way to misrepresent data on a graph is by omitting labels or units on the axes of the graph. The graph shown in Figure 2–19 compares the cost of living, economic growth, population growth, etc., of four main geographic areas in the United States. However, since there are no numbers on the *y* axis, very little information can be gained from this graph, except a crude ranking of each factor. There is no way to decide the actual magnitude of the differences.

Figure 2–19

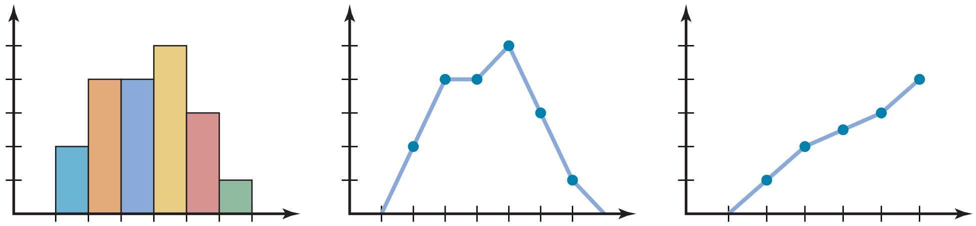
A Graph with No Units on the *y* Axis



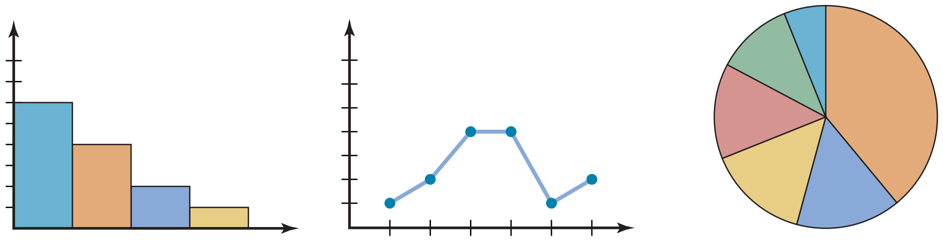
Finally, all graphs should contain a source for the information presented. The inclusion of a source for the data will enable you to check the reliability of the organization presenting the data. A summary of the types of graphs and their uses is shown in Figure 2–20.

Figure 2–20

Summary of Graphs and Uses of Each



(a) Histogram; frequency polygon; ogive
Used when the data are contained in a grouped frequency distribution.



(b) Pareto chart
Used to show frequencies for nominal or qualitative variables.

(c) Time series graph
Used to show a pattern or trend that occurs over a period of time.

(d) Pie graph
Used to show the relationship between the parts and the whole. (Most often uses percentages.)

Stem and Leaf Plots

The stem and leaf plot is a method of organizing data and is a combination of sorting and graphing. It has the advantage over a grouped frequency distribution of retaining the actual data while showing them in graphical form.

Objective 4

Draw and interpret a stem and leaf plot.

A **stem and leaf plot** is a data plot that uses part of the data value as the stem and part of the data value as the leaf to form groups or classes.

Example 2–12 shows the procedure for constructing a stem and leaf plot.

Example 2–12

At an outpatient testing center, the number of cardiograms performed each day for 20 days is shown. Construct a stem and leaf plot for the data.

25	31	20	32	13
14	43	02	57	23
36	32	33	32	44
32	52	44	51	45

Speaking of **Statistics**

How Much Paper Money Is in Circulation Today?

The Federal Reserve estimated that during a recent year, there were 22 billion bills in circulation. About 35% of them were $1 bills, 3% were $2 bills, 8% were $5 bills, 7% were $10 bills, 23% were $20 bills, 5% were $50 bills, and 19% were $100 bills. It costs about 3¢ to print each $1 bill.

The average life of a $1 bill is 22 months, a $10 bill 3 years, a $20 bill 4 years, a $50 bill 9 years, and a $100 bill 9 years. What type of graph would you use to represent the average lifetimes of the bills?

Solution

Step 1 Arrange the data in order:

02, 13, 14, 20, 23, 25, 31, 32, 32, 32,
32, 33, 36, 43, 44, 44, 45, 51, 52, 57

Note: Arranging the data in order is not essential and can be cumbersome when the data set is large; however, it is helpful in constructing a stem and leaf plot. The leaves in the final stem and leaf plot should be arranged in order.

Step 2 Separate the data according to the first digit, as shown.

02   13, 14   20, 23, 25   31, 32, 32, 32, 32, 33, 36
43, 44, 44, 45   51, 52, 57

Step 3 A display can be made by using the leading digit as the *stem* and the trailing digit as the *leaf.* For example, for the value 32, the leading digit, 3, is the stem and the trailing digit, 2, is the leaf. For the value 14, the 1 is the stem and the 4 is the leaf. Now a plot can be constructed as shown in Figure 2–21.

Leading digit (stem)	Trailing digit (leaf)
0	2
1	3 4
2	0 3 5
3	1 2 2 2 2 3 6
4	3 4 4 5
5	1 2 7

Figure 2–21
Stem and Leaf Plot for Example 2–12

0	2
1	3 4
2	0 3 5
3	1 2 2 2 2 3 6
4	3 4 4 5
5	1 2 7

Figure 2–21 shows that the distribution peaks in the center and that there are no gaps in the data. For 7 of the 20 days, the number of patients receiving cardiograms was between 31 and 36. The plot also shows that the testing center treated from a minimum of 2 patients to a maximum of 57 patients in any one day.

If there are no data values in a class, you should write the stem number and leave the leaf row blank. Do not put a zero in the leaf row.

Example 2–13

An insurance company researcher conducted a survey on the number of car thefts in a large city for a period of 30 days last summer. The raw data are shown. Construct a stem and leaf plot by using classes 50–54, 55–59, 60–64, 65–69, 70–74, and 75–79.

52	62	51	50	69
58	77	66	53	57
75	56	55	67	73
79	59	68	65	72
57	51	63	69	75
65	53	78	66	55

Solution

Step 1 Arrange the data in order.

50, 51, 51, 52, 53, 53, 55, 55, 56, 57, 57, 58, 59, 62, 63, 65, 65, 66, 66, 67, 68, 69, 69, 72, 73, 75, 75, 77, 78, 79

Step 2 Separate the data according to the classes.

50, 51, 51, 52, 53, 53 55, 55, 56, 57, 57, 58, 59
62, 63 65, 65, 66, 66, 67, 68, 69, 69 72, 73
75, 75, 77, 78, 79

Step 3 Plot the data as shown here.

Leading digit (stem)	Trailing digit (leaf)
5	0 1 1 2 3 3
5	5 5 6 7 7 8 9
6	2 3
6	5 5 6 6 7 8 9 9
7	2 3
7	5 5 7 8 9

The graph for this plot is shown in Figure 2–22.

Figure 2–22
Stem and Leaf Plot for Example 2–13

5	0 1 1 2 3 3
5	5 5 6 7 7 8 9
6	2 3
6	5 5 6 6 7 8 9 9
7	2 3
7	5 5 7 8 9

When the data values are in the hundreds, such as 325, the stem is 32 and the leaf is 5. For example, the stem and leaf plot for the data values 325, 327, 330, 332, 335, 341, 345, and 347 looks like this.

32	5 7
33	0 2 5
34	1 5 7

When you analyze a stem and leaf plot, look for peaks and gaps in the distribution. See if the distribution is symmetric or skewed. Check the variability of the data by looking at the spread.

Interesting Fact

The average number of pencils and index cards David Letterman tosses over his shoulder during one show is 4.

Related distributions can be compared by using a back-to-back stem and leaf plot. The back-to-back stem and leaf plot uses the same digits for the stems of both distributions, but the digits that are used for the leaves are arranged in order out from the stems on both sides. Example 2–14 shows a back-to-back stem and leaf plot.

Example 2–14

The number of stories in two selected samples of tall buildings in Atlanta and Philadelphia are shown. Construct a back-to-back stem and leaf plot, and compare the distributions.

Atlanta					Philadelphia				
55	70	44	36	40	61	40	38	32	30
63	40	44	34	38	58	40	40	25	30
60	47	52	32	32	54	40	36	30	30
50	53	32	28	31	53	39	36	34	33
52	32	34	32	50	50	38	36	39	32
26	29								

Source: *The World Almanac and Book of Facts.*

Solution

Step 1 Arrange the data for both data sets in order.

Step 2 Construct a stem and leaf plot using the same digits as stems. Place the digits for the leaves for Atlanta on the left side of the stem and the digits for the leaves for Philadelphia on the right side, as shown. See Figure 2–23.

Figure 2–23

Back-to-Back Stem and Leaf Plot for Example 2–14

Atlanta		Philadelphia
9 8 6	2	5
8 6 4 4 2 2 2 2 2 1	3	0 0 0 0 2 2 3 4 6 6 6 8 8 9 9
7 4 4 0 0	4	0 0 0 0
5 3 2 2 0 0	5	0 3 4 8
3 0	6	1
0	7	

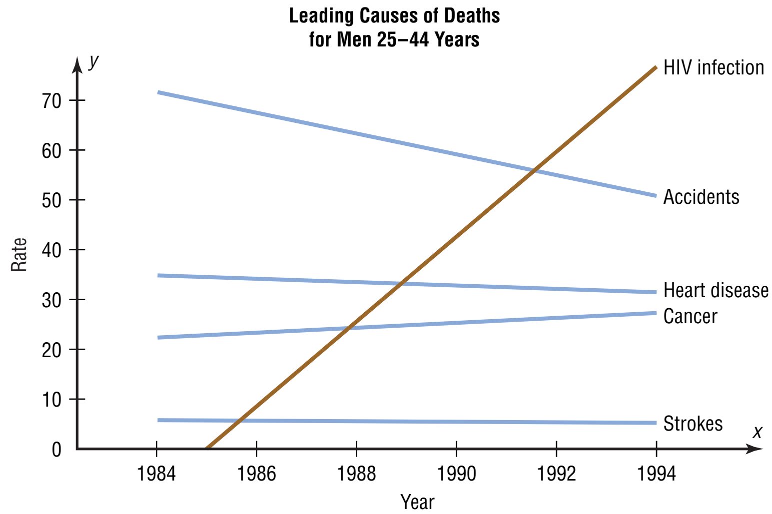
Step 3 Compare the distributions. The buildings in Atlanta have a large variation in the number of stories per building. Although both distributions are peaked in the 30- to 39-story class, Philadelphia has more buildings in this class. Atlanta has more buildings that have 40 or more stories than Philadelphia does.

Stem and leaf plots are part of the techniques called *exploratory data analysis.* More information on this topic is presented in Chapter 3.

Applying the Concepts 2–4

Leading Cause of Death

The following shows approximations of the leading causes of death among men ages 25–44 years. The rates are per 100,000 men. Answer the following questions about the graph.



1. What are the variables in the graph?
2. Are the variables qualitative or quantitative?
3. Are the variables discrete or continuous?
4. What type of graph was used to display the data?
5. Could a Pareto chart be used to display the data?
6. Could a pie chart be used to display the data?
7. List some typical uses for the Pareto chart.
8. List some typical uses for the time series chart.

See page 93 for the answers.

Exercises 2–4

1. The population of federal prisons, according to the most serious offenses, consists of the following. Make a Pareto chart of the population. Based on the Pareto chart, where should most of the money for rehabilitation be spent?

Violent offenses	12.6%
Property offenses	8.5
Drug offenses	60.2
Public order offenses	
Weapons	8.2
Immigration	4.9
Other	5.6

Source: *N.Y. Times Almanac.*

2. Construct a Pareto chart for the number of homicides (rate per 100,000 population) reported for the following states.

State	Number of homicides
Connecticut	4.1
Maine	2.0
New Jersey	4.0
Pennsylvania	5.3
New York	5.1

Source: *FBI Uniform Crime Report.*

3. The following data represent the estimated number (in millions) of computers connected to the Internet worldwide. Construct a Pareto chart for the data. Based on the data, suggest the best place to market appropriate Internet products.

Location	Number of computers
Homes	240
Small companies	102
Large companies	148
Government agencies	33
Schools	47

Source: IDC.

4. The World Roller Coaster Census Report lists the following number of roller coasters on each continent. Represent the data graphically, using a Pareto chart.

Africa	17
Asia	315
Australia	22
Europe	413
North America	643
South America	45

Source: www.rcdb.com.

5. The following percentages indicate the source of energy used worldwide. Construct a Pareto chart for the energy used.

Petroleum	39.8%
Coal	23.2
Dry natural gas	22.4
Hydroelectric	7.0
Nuclear	6.4
Other (wind, solar, etc.)	1.2

Source: *N.Y. Times Almanac.*

6. Draw a time series graph to represent the data for the number of airline departures (in millions) for the given years. Over the years, is the number of departures increasing, decreasing, or about the same?

Year	1996	1997	1998	1999	2000	2001	2002
Number of departures	7.9	9.9	10.5	10.9	11.0	9.8	10.1

Source: *The World Almanac and Book of Facts.*

7. The data represent the personal consumption (in billions of dollars) for tobacco in the United States. Draw a time series graph for the data and explain the trend.

Year	1995	1996	1997	1998	1999	2000	2001	2002
Amount	8.5	8.7	9.0	9.3	9.6	9.9	10.2	10.4

Source: *The World Almanac and Book of Facts.*

8. Draw a time series graph for the data shown and comment on the trend. The data represent the number of active nuclear reactors.

Year	1992	1994	1996	1998	2000	2002
Number	109	109	109	104	104	104

Source: *The World Almanac and Book of Facts.*

9. The percentages of voters voting in 10 presidential elections are shown here. Construct a time series graph and analyze the results.

1964	95.83%	1984	74.63%
1968	89.65	1988	72.48
1972	79.85	1992	78.01
1976	77.64	1996	65.97
1980	76.53	2000	67.50

Source: *N.Y. Times Almanac.*

10. The following data are based on a survey from American Travel Survey on why people travel. Construct a pie graph for the data and analyze the results.

Purpose	**Number**
Personal business	146
Visit friends or relatives	330
Work-related	225
Leisure	299

Source: *USA TODAY.*

11. The assets of the richest 1% of Americans are distributed as follows. Make a pie graph for the percentages.

Principal residence	7.8%
Liquid assets	5.0
Pension accounts	6.9
Stock, mutual funds, and personal trusts	31.6
Businesses and other real estate	46.9
Miscellaneous	1.8

Source: *The New York Times.*

12. The following elements comprise the earth's crust, the outermost solid layer. Illustrate the composition of the earth's crust with a pie graph.

Oxygen	45.6%
Silicon	27.3
Aluminum	8.4
Iron	6.2
Calcium	4.7
Other	7.8

Source: *N.Y. Times Almanac.*

13. In a recent survey, 3 in 10 people indicated that they are likely to leave their jobs when the economy improves. Of those surveyed, 34% indicated that they would make a career change, 29% want a new job in the same industry, 21% are going to start a business, and 16% are going to retire. Make a pie chart and a Pareto chart for the data. Which chart do you think better represents the data?

Source: *National Survey Institute.*

14. State which graph (Pareto chart, time series graph, or pie graph) would most appropriately represent the given situation.

a. The number of students enrolled at a local college for each year during the last 5 years.

b. The budget for the student activities department at a certain college for each year during the last 5 years.

c. The means of transportation the students use to get to school.

d. The percentage of votes each of the four candidates received in the last election.

e. The record temperatures of a city for the last 30 years.

f. The frequency of each type of crime committed in a city during the year.

15. The age at inauguration for each U.S. President is shown. Construct a stem and leaf plot and analyze the data.

57	54	52	55	51	56
61	68	56	55	54	61
57	51	46	54	51	52
57	49	54	42	60	69
58	64	49	51	62	64
57	48	50	56	43	46
61	65	47	55	55	54

Source: *N.Y. Times Almanac.*

16. The National Insurance Crime Bureau reported that these data represent the number of registered vehicles per car stolen for 35 selected cities in the United States. For example, in Miami, 1 automobile is stolen for every 38 registered vehicles in the city. Construct a stem and leaf plot for the data and analyze the distribution. (The data have been rounded to the nearest whole number.)

38	53	53	56	69	89	94
41	58	68	66	69	89	52
50	70	83	81	80	90	74
50	70	83	59	75	78	73
92	84	87	84	85	84	89

Source: *USA TODAY.*

17. The growth (in centimeters) of two varieties of plant after 20 days is shown in this table. Construct a back-to-back stem and leaf plot for the data, and compare the distributions.

Variety 1				Variety 2			
20	12	39	38	18	45	62	59
41	43	51	52	53	25	13	57
59	55	53	59	42	55	56	38
50	58	35	38	41	36	50	62
23	32	43	53	45	55		

18. The data shown represent the percentage of unemployed males and females in 1995 for a sample of countries of the world. Using the whole numbers as stems and the decimals as leaves, construct a back-to-back stem and leaf plot and compare the distributions of the two groups.

Females					Males				
8.0	3.7	8.6	5.0	7.0	8.8	1.9	5.6	4.6	1.5
3.3	8.6	3.2	8.8	6.8	2.2	5.6	3.1	5.9	6.6
9.2	5.9	7.2	4.6	5.6	9.8	8.7	6.0	5.2	5.6
5.3	7.7	8.0	8.7	0.5	4.4	9.6	6.6	6.0	0.3
6.5	3.4	3.0	9.4		4.6	3.1	4.1	7.7	

Source: *N.Y. Times Almanac.*

19. These data represent the numbers of cities served on nonstop flights by Southwest Airlines's largest airports. Construct a stem and leaf plot.

38	41	25	32	13
19	18	28	14	29

Source: *Southwest Airlines.*

Extending the Concepts

20. The number of successful space launches by the United States and Japan for the years 1993–1997 is shown here. Construct a compound time series graph for the data. What comparison can be made regarding the launches?

Year	1993	1994	1995	1996	1997
United States	29	27	24	32	37
Japan	1	4	2	1	2

Source: *The World Almanac and Book of Facts.*

21. Meat production for veal and lamb for the years 1960–2000 is shown here. (Data are in millions of pounds.) Construct a compound time series graph for the data. What comparison can be made regarding meat production?

Year	1960	1970	1980	1990	2000
Veal	1109	588	400	327	225
Lamb	769	551	318	358	234

Source: *The World Almanac and Book of Facts.*

22. The top 10 airlines with the most aircraft are listed. Represent these data with an appropriate graph.

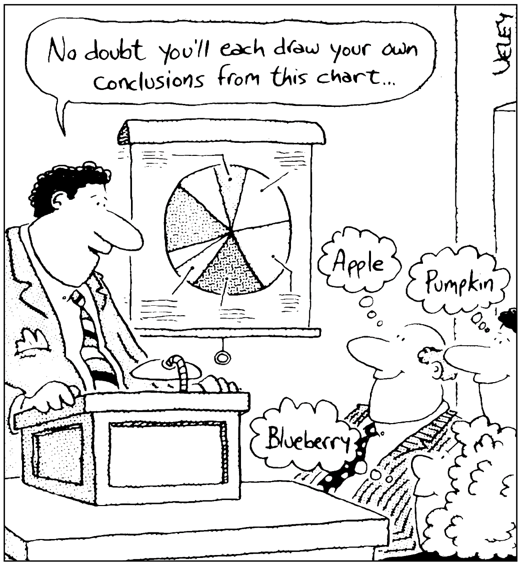
American	714	Continental	364
United	603	Southwest	327
Delta	600	British Airways	268
Northwest	424	American Eagle	245
U.S. Airways	384	Lufthansa (Ger.)	233

Source: *Top 10 of Everything.*

23. The top prize-winning countries for Nobel Prizes in Physiology or Medicine are listed here. Represent the data with an appropriate graph.

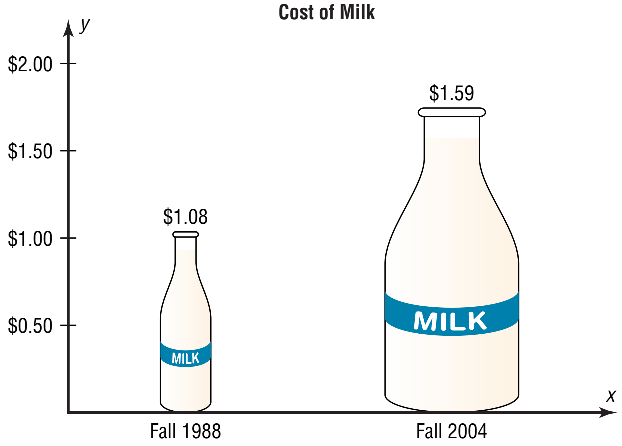
United States	80	Denmark	5
United Kingdom	24	Austria	4
Germany	16	Belgium	4
Sweden	8	Italy	3
France	7	Australia	3
Switzerland	6		

Source: *Top 10 of Everything.*

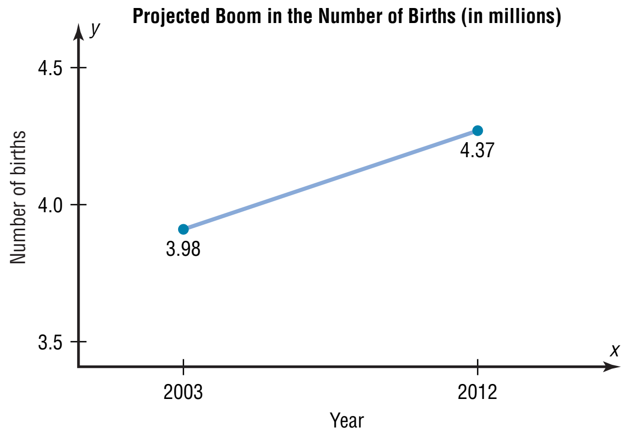


Source: Cartoon by Bradford Veley, Marquette, Michigan. Used with permission.

24. The graph shows the increase in the price of a quart of milk. Why might the increase appear to be larger than it really is?



25. The graph shows the projected boom (in millions) in the number of births. Cite several reasons why the graph might be misleading.

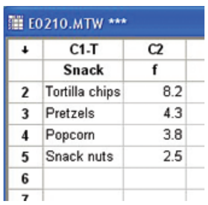


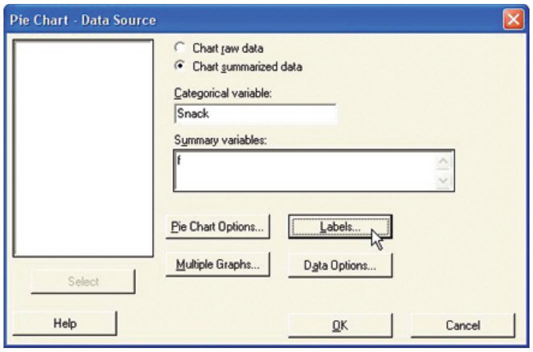
Technology *Step by Step*

MINITAB
Step by Step

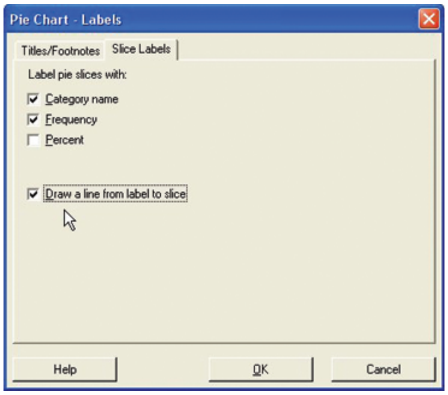
Construct a Pie Chart

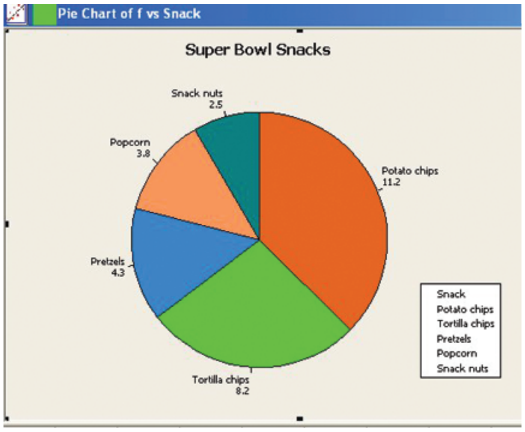
1. Enter the summary data for snack foods and frequencies from Example 2–10 into C1 and C2.





2. Name them **Snack** and **f.**
3. Select **Graph>Pie Chart.**
   a) Click the option for Chart summarized data.
   b) Press [Tab] to move to Categorical variable, then double-click C1 to select it.
   c) Press [Tab] to move to Summary variables, and select the column with the frequencies f.



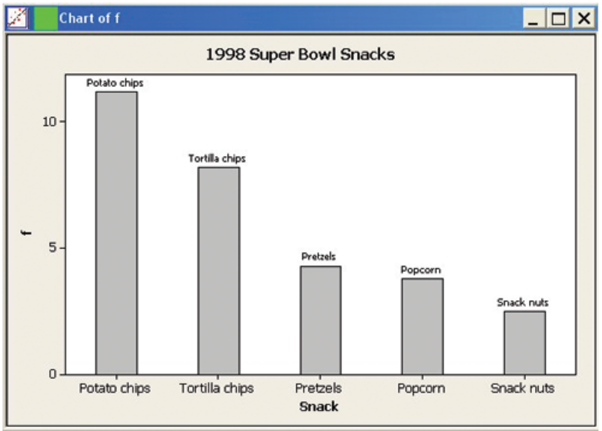


4. Click the [Labels] tab, then Titles/Footnotes.
   a) Type in the title: **Super Bowl Snacks.**
   b) Click the Slice Labels tab, then the options for Category name and Frequency.
   c) Click the option to Draw a line from label to slice.
   d) Click [OK] twice to create the chart.

Construct a Bar Chart

The procedure for constructing a bar chart is similar to that for the pie chart.

1. Select **Graph>Bar Chart.**
   a) Click on the drop-down list in Bars Represent: then select values from a table.
   b) Click on the Simple chart, then click [OK]. The dialog box will be similar to the Pie Chart Dialog Box.
2. Select the frequency column C2 f for Graph variables: and Snack for the Categorical variable.

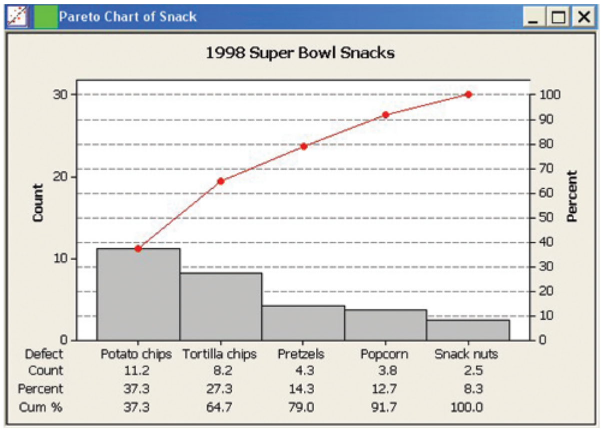


3. Click on [Labels], then type the title in the Titles/Footnote tab: **1998 Super Bowl Snacks.**
4. Click the tab for Data Labels, then click the option to Use labels from column: and select C1 Snacks.
5. Click [OK] twice.

Construct a Pareto Chart

Pareto charts are a quality control tool. They are similar to a bar chart with no gaps between the bars, and the bars are arranged by frequency.

1. Select **Stat>Quality Tools>Pareto.**
2. Click the option to Chart defects table.
3. Click in the box for the Labels in: and select Snack.
4. Click on the frequencies column C2 f.



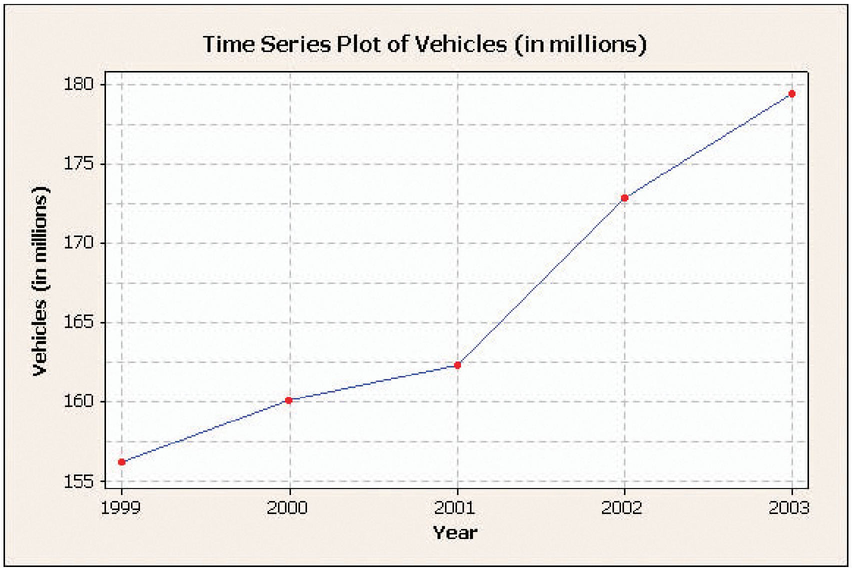
5. Click on [Options].
   a) Check the box for Cumulative percents.
   b) Type in the title, **1998 Super Bowl Snacks.**
6. Click [OK] twice. The chart is completed.

Construct a Time Series Plot

The data used are from Example 2–9, the number of vehicles that used the Pennsylvania Turnpike.

1. Add a blank worksheet to the project by selecting **File>New>New Worksheet.**
2. To enter the dates from 1999 to 2003 in C1, select **Calc>Make Patterned Data>Simple Set of Numbers.**
   a) Type **Year** in the text box for Store patterned data in.
   b) From first value: should be **1999.**
   c) To Last value: should be **2003.**
   d) In steps of should be **1** (for every other year). The last two boxes should be 1, the default value.
   e) Click [OK]. The sequence from 1999 to 2003 will be entered in C1 whose label will be Year.
3. Type **Vehicles (in millions)** for the label row above row 1 in C2.

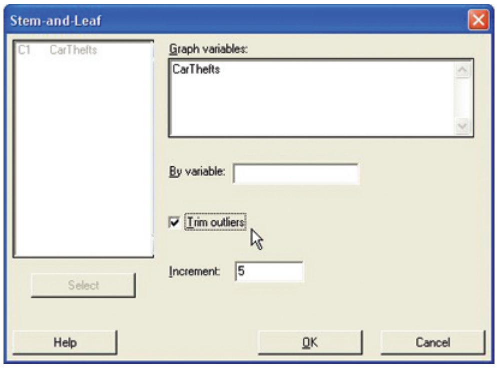
4. Type **156.2** for the first number, then press [Enter]. Never enter the commas for large numbers!
5. Continue entering the value in each row of C2.



6. To make the graph, select **Graph>Time series plot,** then Simple, and press [OK].
   a) For Series select Vehicles (in millions), then click [Time/scale].
   b) Click the Stamp option and select Year for the Stamp column.
   c) Click the Gridlines tab and select all three boxes, Y major, Y minor, and X major.
   d) Click [OK] twice. A new window will open that contains the graph.
   e) To change the title, double-click the title in the graph window. A dialog box will open, allowing you to edit the text.

Construct a Stem and Leaf Plot

1. Type in the data for Example 2–13. Label the column **CarThefts.**
2. Select **STAT>EDA>Stem-and-Leaf.** This is the same as **Graph>Stem-and-Leaf.**
3. Double-click on C1 CarThefts in the column list.
4. Click in the Increment text box, and enter the class width of **5.**
5. Click [OK]. This character graph will be displayed in the session window.



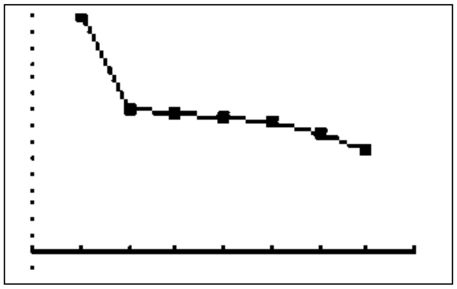
Stem-and-Leaf Display: CarThefts
Stem-and-leaf of CarThefts N = 30
Leaf Unit = 1.0

6	5	011233
13	5	5567789
15	6	23
15	6	55667899
7	7	23
5	7	55789

TI-83 Plus or TI-84 Plus Step by Step

To graph a time series, follow the procedure for a frequency polygon from Section 2–3, using the data from Example 2–9.

Output

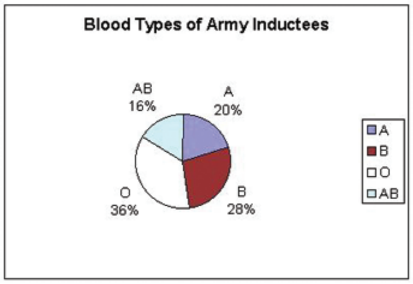


Excel Step by Step

Constructing a Pie Chart

To make a pie (or bar) chart:

1. Enter the blood types from Example 2–11 into column A of a new worksheet.
2. Enter the frequencies corresponding to each blood type in column B.
3. Go to the **Chart Wizard.**
4. Select Pie (or Bar), and select the first subtype.
5. Click the Data Range tab. Enter both columns as the Data Range.
6. Check column for the Series in option, then click Next.
7. Create a title for your chart, such as **Blood Types of Army Inductees.**
8. Click the Data Labels tab and check the Show percent option.
9. Place chart as a new sheet or as an embedded sheet in the active worksheet, then select Finish.

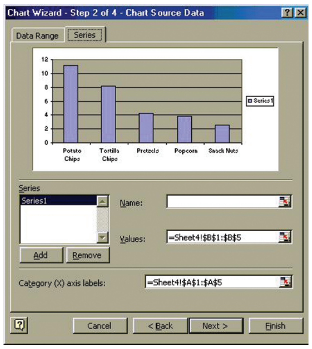


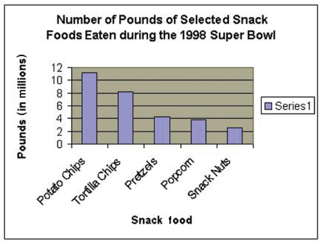
Constructing a Pareto Chart

To make a Pareto chart using the data from Example 2–10:

1. On a new worksheet, enter the snack food categories in column A and the corresponding frequencies (in pounds) in column B. The order is important here. Make sure to enter the data in the order shown in Example 2–10.
2. Go to the **Chart Wizard.**
3. Select the Column chart type, select the first chart subtype, and then click Next.
4. Click the Data Range tab. Select both columns as the Data Range.

5. Check column for the Series in option.
6. Click the Series tab. Select B1:B5 for Values, select A1:A5 for the Category (X) axis labels, and then click Next.



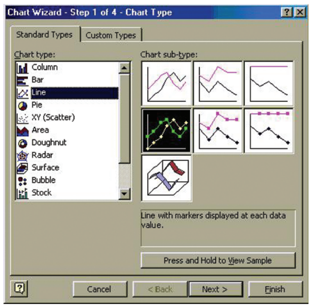


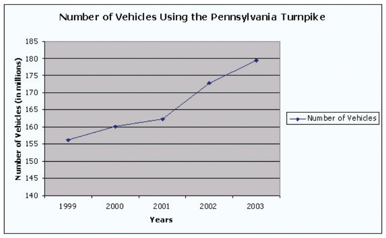
7. Create a title for your chart, such as **Number of Pounds of Selected Snack Foods Eaten During the 1998 Super Bowl.** Enter **Snack Food** as the Category (X) axis and **Pounds in Millions** as the Value (Y) axis. Click Finish.

Constructing a Time Series Plot

To make a time series plot using the data from Example 2–9:

1. On a new worksheet, enter the Years in column A and the corresponding Number (of vehicles) in column B.
2. Go to the **Chart Wizard.**
3. Select the Line chart type, select the fourth subtype, and then click Next.





4. Click the Data Range tab. Select B1:B6 as the Data Range.
5. Check column for the Series in option.
6. Click the Series tab. Select A2:A6 for the Category (X) axis labels, and then click Next.
7. Create a title for your chart, such as **Number of Vehicles Using the Pennsylvania Turnpike Between 1999 and 2003.** Enter **Year** as the Category (X) axis and **Number of Vehicles (in millions)** as the Value (Y) axis. Click Finish.

2–5 Summary

When data are collected, they are called raw data. Since very little knowledge can be obtained from raw data, they must be organized in some meaningful way. A frequency distribution using classes is the solution. Once a frequency distribution is constructed, the representation of the data by graphs is a simple task. The most commonly used graphs in research statistics are the histogram, frequency polygon, and ogive. Other graphs, such as the Pareto chart, time series graph, and pie graph, can also be used. Some of these graphs are seen frequently in newspapers, magazines, and various statistical reports.

Finally, a stem and leaf plot uses part of the data values as stems and part of the data values as leaves. This graph has the advantages of a frequency distribution and a histogram.

Important Terms

categorical frequency distribution 36
class 35
class boundaries 37
class midpoint 38
class width 37
cumulative frequency 51
frequency 35
frequency distribution 35
frequency polygon 50
grouped frequency distribution 37
histogram 48
lower class limit 37
ogive 51
open-ended distribution 39
Pareto chart 64
pie graph 66
raw data 35
relative frequency graph 53
stem and leaf plot 73
time series graph 65
ungrouped frequency distribution 41
upper class limit 37

Important Formulas

Formula for the percentage of values in each class:

$$\% = \frac{f}{n} \cdot 100\%$$

where

f = frequency of the class
n = total number of values

Formula for the range:

$$R = \text{highest value} - \text{lowest value}$$

Formula for the class width:

$$\text{Class width} = \text{upper boundary} - \text{lower boundary}$$

Formula for the class midpoint:

$$X_m = \frac{\text{lower boundary} + \text{upper boundary}}{2}$$

or

$$X_m = \frac{\text{lower limit} + \text{upper limit}}{2}$$

Formula for the degrees for each section of a pie graph:

$$\text{Degrees} = \frac{f}{n} \cdot 360°$$

Review Exercises

1. The Brunswick Research Organization surveyed 50 randomly selected individuals and asked them the primary way they received the daily news. Their choices were via newspaper (N), television (T), radio (R), or Internet (I). Construct a categorical frequency distribution for the data and interpret the results. The data in this exercise will be used for Exercise 2 in this section.

N	N	T	T	T	I	R	R	I	T
I	N	R	R	I	N	N	I	T	N
I	R	T	T	T	T	N	R	R	I
R	R	I	N	T	R	T	I	I	T
T	I	N	T	T	I	R	N	R	T

2. Construct a pie graph for the data in Exercise 1, and analyze the results.

3. A sporting goods store kept a record of sales of five items for one randomly selected hour during a recent sale. Construct a frequency distribution for the data (B = baseballs, G = golf balls, T = tennis balls, S = soccer balls, F = footballs). (The data for this exercise will be used for Exercise 4 in this section.)

F	B	B	B	G	T	F
G	G	F	S	G	T	
F	T	T	T	S	T	
F	S	S	G	S	B	

4. Draw a pie graph for the data in Exercise 3 showing the sales of each item, and analyze the results.

5. The blood urea nitrogen (BUN) count of 20 randomly selected patients is given here in milligrams per deciliter (mg/dl). Construct an ungrouped frequency distribution for the data. (The data for this exercise will be used for Exercise 6.)

17	18	13	14
12	17	11	20
13	18	19	17
14	16	17	12
16	15	19	22

6. Construct a histogram, frequency polygon, and ogive for the data in Exercise 5 in this section, and analyze the results.

7. The data show the estimated added cost per vehicle use due to bad roads. Construct a frequency distribution using 6 classes. (The data for this exercise will be used for Exercises 8 and 11 in this section.)

165	186	122	172	140	153	208	169
156	114	113	135	131	125	177	136
136	127	112	188	171	179	152	155
116	90	187	136	159	97	141	85
91	170	111	147	165	163	159	150

Source: Federal Highway Administration.

8. Construct a histogram, frequency polygon, and ogive for the data in Exercise 7 in this section, and analyze the results.

9. The data shown (in millions of dollars) are the values of the 30 National Football League franchises. Construct a frequency distribution for the data using 8 classes. (The data for this exercise will be used for Exercises 10 and 12 in this section.)

170	191	171	235	173	187	181	191
200	218	243	200	182	320	184	239
186	199	186	210	209	240	204	193
211	186	197	204	188	242		

Source: Pittsburgh Post-Gazette.

10. Construct a histogram, frequency polygon, and ogive for the data in Exercise 9 in this section, and analyze the results.

11. Construct a histogram, frequency polygon, and ogive by using relative frequencies for the data in Exercise 7 in this section.

12. Construct a histogram, frequency polygon, and ogive by using relative frequencies for the data in Exercise 9 in this section.

13. Construct a Pareto chart for the number of homicides reported for the following cities.

City	Number of homicides
New Orleans	363
Washington, D.C.	352
Chicago	824
Baltimore	323
Atlanta	184

Source: USA TODAY.

14. Construct a Pareto chart for the number of trial-ready civil action and equity cases decided in less than 6 months for the selected counties in southwestern Pennsylvania.

County	Number of cases
Westmoreland	427
Washington	298
Green	151
Fayette	106
Somerset	87

Source: Pittsburgh Tribune-Review.

15. The given data represent the federal minimum hourly wage in the years shown. Draw a time series graph to represent the data and analyze the results.

Year	Wage
1960	$1.00
1965	1.25
1970	1.60
1975	2.10
1980	3.10
1985	3.35
1990	3.80
1995	4.25
2000	5.15
2005	5.15

Source: The World Almanac and Book of Facts.

16. The number of bank failures in the United States during the years 1989–2000 is shown. Draw a time series graph to represent the data and analyze the results.

Year	Number of failures
1989	207
1990	169
1991	127
1992	122
1993	41
1994	13
1995	6
1996	5
1997	1
1998	3
1999	8
2000	7
2001	4
2002	11

Source: The World Almanac and Book of Facts.

17. The data show the number (in millions) of viewers who watched the first and second presidential debates. Construct two time series graphs and compare the results.

Year	1992	1996	2000	2004
First debate	62.4	36.1	46.6	62.5
Second debate	69.9	36.3	37.6	46.7

Source: Nielson Media Research.

18. In a study of 100 women, the numbers shown here indicate the major reason why each woman surveyed worked outside the home. Construct a pie graph for the data and analyze the results.

Reason	Number of women
To support self/family	62
For extra money	18
For something different to do	12
Other	8

19. A survey asked if people would like to spend the rest of their careers with their present employers. The results are shown. Construct a pie graph for the data and analyze the results.

Answer	Number of people
Yes	660
No	260
Undecided	80

20. The number of visitors to the Railroad Museum during 24 randomly selected hours is shown here. Construct a stem and leaf plot for the data.

67	62	38	73	34	43	72	35
53	55	58	63	47	42	51	62
32	29	47	62	29	38	36	41

21. The data set shown here represents the number of hours that 25 part-time employees worked at the Sea Side Amusement Park during a randomly selected week in June. Construct a stem and leaf plot for the data and summarize the results.

16	25	18	39	25	17	29	14	37
22	18	12	23	32	35	24	26	
20	19	25	26	38	38	33	29	

22. A special aptitude test is given to job applicants. The data shown here represent the scores of 30 applicants. Construct a stem and leaf plot for the data and summarize the results.

204	210	227	218	254
256	238	242	253	227
251	243	233	251	241
237	247	211	222	231
218	212	217	227	209
260	230	228	242	200

Data Analysis

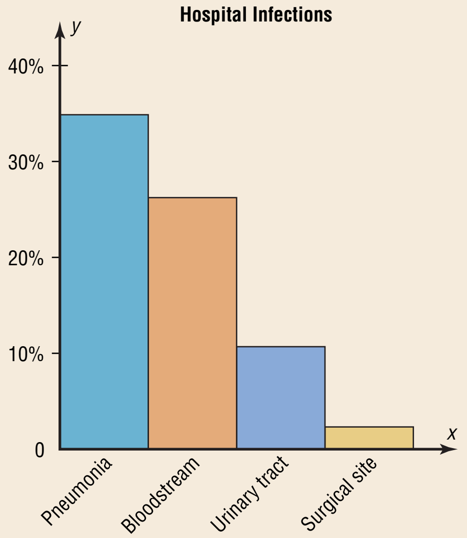
A Data Bank is found in Appendix D, or on the World Wide Web by following links from www.mhhe.com/math/stat/bluman

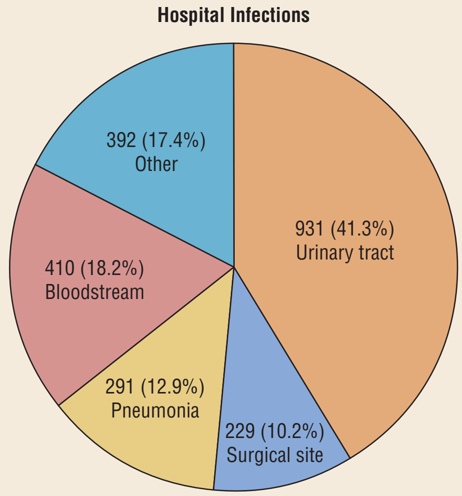
1. From the Data Bank located in Appendix D, choose one of the following variables: age, weight, cholesterol level, systolic pressure, IQ, or sodium level. Select at least 30 values. For these values, construct a grouped frequency distribution. Draw a histogram, frequency polygon, and ogive for the distribution. Describe briefly the shape of the distribution.
2. From the Data Bank, choose one of the following variables: educational level, smoking status, or exercise.

Statistics Today

How Serious Are Hospital Infections?—Revisited

The data presented in numerical form do not convey an easy-to-interpret conclusion; however, when data are presented in graphical form, readers can see the visual impact of the numbers. In this case, a Pareto graph shows that infections from pneumonia are the most common, followed by bloodstream infections. The pie graph also shows that urinary tract infections occur with the highest frequency.





Select at least 20 values. Construct an ungrouped frequency distribution for the data. For the distribution, draw a Pareto chart and describe briefly the nature of the chart.

3. From the Data Bank, select at least 30 subjects and construct a categorical distribution for their marital status. Draw a pie graph and describe briefly the findings.
4. Using the data from Data Set IV in Appendix D, construct a frequency distribution and draw a histogram. Describe briefly the shape of the distribution of the tallest buildings in New York City.
5. Using the data from Data Set XI in Appendix D, construct a frequency distribution and draw a frequency polygon. Describe briefly the shape of the distribution for the number of pages in statistics books.
6. Using the data from Data Set IX in Appendix D, divide the United States into four regions, as follows:

Northeast	CT ME MA NH NJ NY PA RI VT
Midwest	IL IN IA KS MI MN MS NE ND OH SD WI
South	AL AR DE DC FL GA KY LA MD NC OK SC TN TX VA WV
West	AK AZ CA CO HI ID MT NV NM OR UT WA WY

Find the total population for each region, and draw a Pareto chart and a pie graph for the data. Analyze the results. Explain which chart might be a better representation for the data.

7. Using the data from Data Set I in Appendix D, make a stem and leaf plot for the record low temperatures in the United States. Describe the nature of the plot.

Chapter Quiz

Determine whether each statement is true or false. If the statement is false, explain why.

1. In the construction of a frequency distribution, it is a good idea to have overlapping class limits, such as 10–20, 20–30, 30–40.
2. Histograms can be drawn by using vertical or horizontal bars.
3. It is not important to keep the width of each class the same in a frequency distribution.

4. Frequency distributions can aid the researcher in drawing charts and graphs.
5. The type of graph used to represent data is determined by the type of data collected and by the researcher's purpose.
6. In construction of a frequency polygon, the class limits are used for the *x* axis.
7. Data collected over a period of time can be graphed by using a pie graph.

Select the best answer.

8. What is another name for the ogive?
   a. Histogram
   b. Frequency polygon
   c. Cumulative frequency graph
   d. Pareto chart
9. What are the boundaries for 8.6–8.8?
   a. 8–9
   b. 8.5–8.9
   c. 8.55–8.85
   d. 8.65–8.75
10. What graph should be used to show the relationship between the parts and the whole?
   a. Histogram
   b. Pie graph
   c. Pareto chart
   d. Ogive
11. Except for rounding errors, relative frequencies should add up to what sum?
   a. 0
   b. 1
   c. 50
   d. 100

Complete these statements with the best answers.

12. The three types of frequency distributions are ______, ______, and ______.
13. In a frequency distribution, the number of classes should be between ______ and ______.
14. Data such as blood types (A, B, AB, O) can be organized into a(n) ______ frequency distribution.
15. Data collected over a period of time can be graphed using a(n) ______ graph.
16. A statistical device used in exploratory data analysis that is a combination of a frequency distribution and a histogram is called a(n) ______.
17. On a Pareto chart, the frequencies should be represented on the ______ axis.
18. A questionnaire on housing arrangements showed this information obtained from 25 respondents. Construct a frequency distribution for the data (H = house, A = apartment, M = mobile home, C = condominium).

H	C	H	M	H	A	C	A	M
C	M	C	A	M	A	C	C	M
C	C	H	A	H	H	M		

19. Construct a pie graph for the data in Problem 18.
20. When 30 randomly selected customers left a convenience store, each was asked the number of items he or she purchased. Construct an ungrouped frequency distribution for the data.

2	9	4	3	6
6	2	8	6	5
7	5	3	8	6
6	2	3	2	4
6	9	9	8	9
4	2	1	7	4

21. Construct a histogram, a frequency polygon, and an ogive for the data in Problem 20.
22. For a recent year, the number of murders in 25 selected cities is shown. Construct a frequency distribution using 9 classes, and analyze the nature of the data in terms of shape, extreme values, etc. (The information in this exercise will be used for Exercise 23 in this section.

248	348	74	514	597
270	71	226	41	39
366	73	241	46	34
149	68	73	63	65
109	598	278	69	27

Source: *Pittsburgh Tribune Review.*

23. Construct a histogram, frequency polygon, and ogive for the data in Problem 22. Analyze the histogram.
24. Construct a Pareto chart for the number of tons (in millions) of trash recycled per year by Americans based on an Environmental Protection Agency study.

Type	**Amount**
Paper	320.0
Iron/steel	292.0
Aluminum	276.0
Yard waste	242.4
Glass	196.0
Plastics	41.6

Source: *USA TODAY.*

25. The data show the number of fatal trespasser casualties on railroad property in the United States. Draw a time series graph and explain any trend.

Year	1998	1999	2000	2001
Number	536	463	511	540

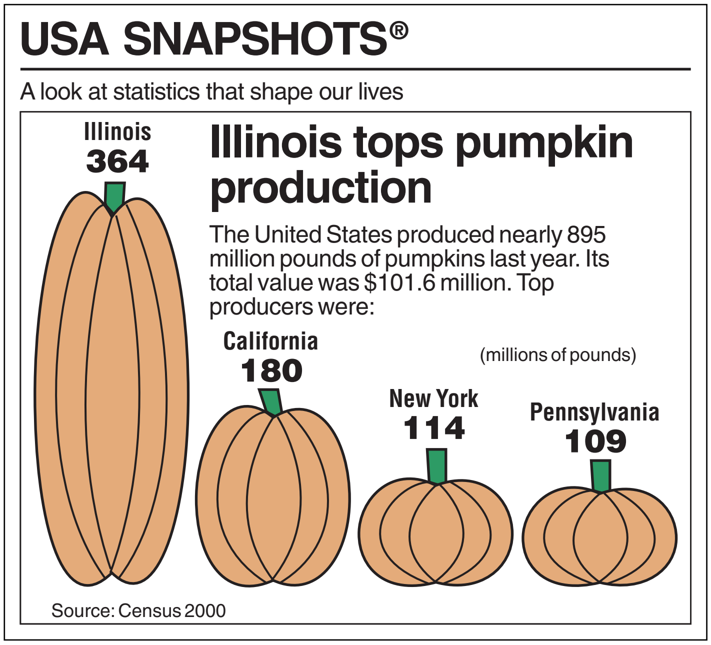
Source: *Federal Railroad Administration.*

26. The number of visitors to the Historic Museum for 25 randomly selected hours is shown. Construct a stem and leaf plot for the data.

15	53	48	19	38
86	63	98	79	38
62	89	67	39	26
28	35	54	88	76
31	47	53	41	68

Critical Thinking Challenges

1. The *USA TODAY* Snapshot shows pumpkin production. Can you see anything misleading about the way the graph is drawn?



Source: Copyright 2001, *USA TODAY.* Reprinted with permission.

2. Shown are various statistics about the Great Lakes. Using appropriate graphs (your choice) and summary statements, write a report analyzing the data.

	Superior	Michigan	Huron	Erie	Ontario
Length (miles)	350	307	206	241	193
Breadth (miles)	160	118	183	57	53
Depth (feet)	1,330	923	750	210	802
Volume (cubic miles)	2,900	1,180	850	116	393
Area (square miles)	31,700	22,300	23,000	9,910	7,550
Shoreline (U.S., miles)	863	1,400	580	431	300

Source: *The World Almanac and Book of Facts.*

3. A compound time series graph is shown for the percentage of sales of CDs, cassettes, and vinyl records. Using the graph, answer these questions.

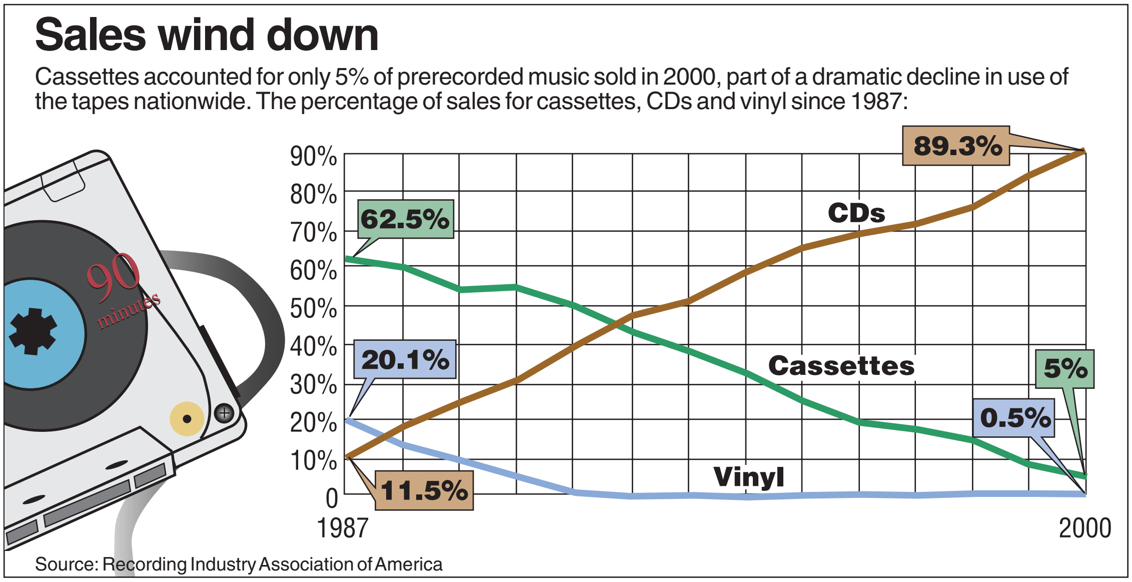
   a. In what year did cassette sales match CD sales?

   b. In what year did cassette sales account for 50% of the sales?

   c. What was the percentage difference in sales between CDs and cassettes for the year 1996?

   d. In what year did CDs account for 30% of the sales?

   e. What was the change in percentage of sales for cassettes from 1987 to 2000?



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Data Projects

Where appropriate, use MINITAB, the TI-83 Plus, the TI-84 Plus, Excel, or a computer program of your choice to complete the following exercises.

1. Select a categorical (nominal) variable, such as the colors of cars in the school's parking lot or the major fields of the students in statistics class, and collect data on this variable.

a. State the purpose of the project.
b. Define the population.
c. State how the sample was selected.
d. Show the raw data.
e. Construct a frequency distribution for the variable.
f. Draw some appropriate graphs (pie, Pareto, etc.) for the data.
g. Analyze the results.

2. Using an almanac, select a variable that varies over a period of several years (e.g., silver production), and draw a time series graph for the data. Write a short paragraph interpreting the findings.

3. Select a variable (interval or ratio) and collect at least 30 values. For example, you may ask the students in your class how many hours they study per week or how old they are.

a. State the purpose of the project.
b. Define the population.
c. State how the sample was selected.
d. Show the raw data.
e. Construct a frequency distribution for the data.
f. Draw a histogram, frequency polygon, and ogive for the data.
g. Analyze the results.

You may use these websites to obtain raw data:

Visit the data sets at the book's website found at
http://www.mhhe.com/math/stat/bluman
Click on the 6th edition.

http://lib.stat.cmu.edu/DASL

http://www.statcan.ca

Answers to Applying the Concepts

Section 2–2 Ages of Presidents at Inauguration

1. The data were obtained from the population of all presidents at the time this text was written.
2. The oldest inauguration age was 69 years old.
3. The youngest inauguration age was 42 years old.
4. Answers will vary. One possible answer is

Age at inauguration	Frequency
42–45	2
46–49	6
50–53	7
54–57	16
58–61	5
62–65	4
66–69	2

5. Answers will vary. For the frequency distribution given in Exercise 4, there is a peak for the 54–57 bin.
6. Answers will vary. This frequency distribution shows no outliers. However, if we had split our frequency into 14 bins instead of 7, then the ages 42, 43, 68, and 69 might appear as outliers.
7. Answers will vary. The data appear to be unimodal and fairly symmetric, centering around 55 years of age.

Section 2–3 Selling Real Estate

1. A histogram of the data gives price ranges and the counts of homes in each price range. We can also talk about how the data are distributed by looking at a histogram.
2. A frequency polygon shows increases or decreases in the number of home prices around values.
3. A cumulative frequency polygon shows the number of homes sold at or below a given price.
4. The house that sold for $321,550 is an extreme value in this data set.
5. Answers will vary. One possible answer is that the histogram displays the outlier well since there is a gap in the prices of the homes sold.
6. The distribution of the data is skewed to the right.

Section 2–4 Leading Cause of Death

1. The variables in the graph are the year, cause of death, and rate of death per 100,000 men.
2. The cause of death is qualitative, while the year and death rates are quantitative.
3. Year is a discrete variable, and death rate is continuous. Since cause of death is qualitative, it is neither discrete nor continuous.
4. A line graph was used to display the data.
5. No, a Pareto chart could not be used to display the data, since we can only have one quantitative variable and one categorical variable in a Pareto chart.
6. We cannot use a pie chart for the same reasons as given for the Pareto chart.
7. A Pareto chart is typically used to show a categorical variable listed from the highest-frequency category to the category with the lowest frequency.
8. A time series chart is used to see trends in the data. It can also be used for forecasting and predicting.