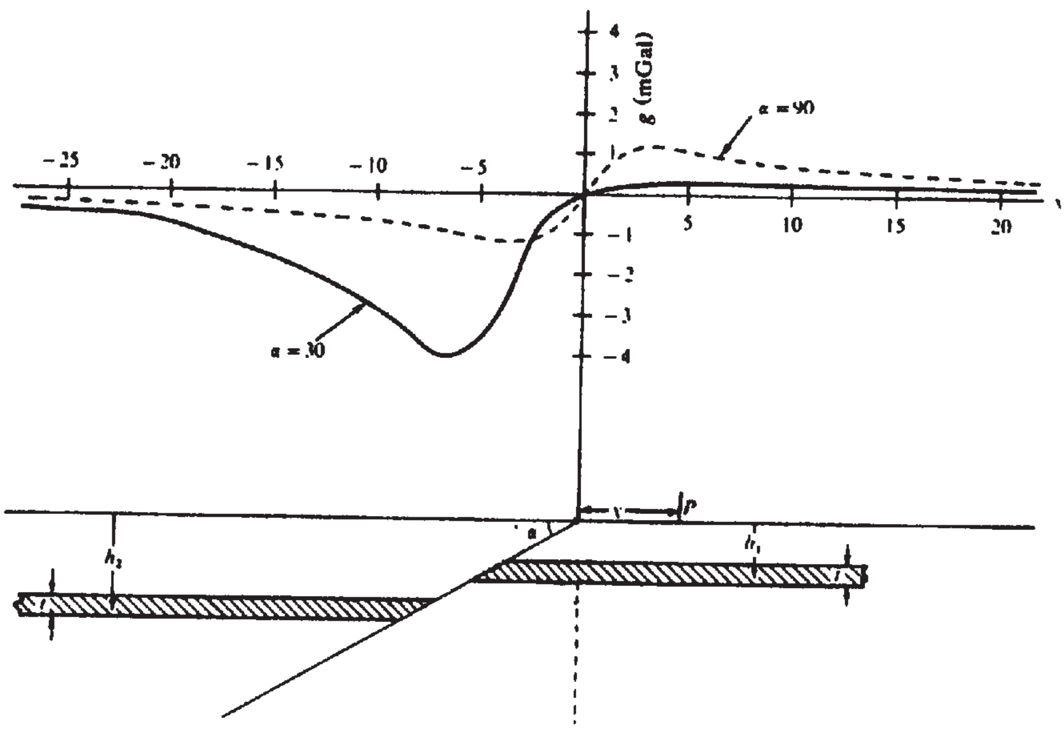

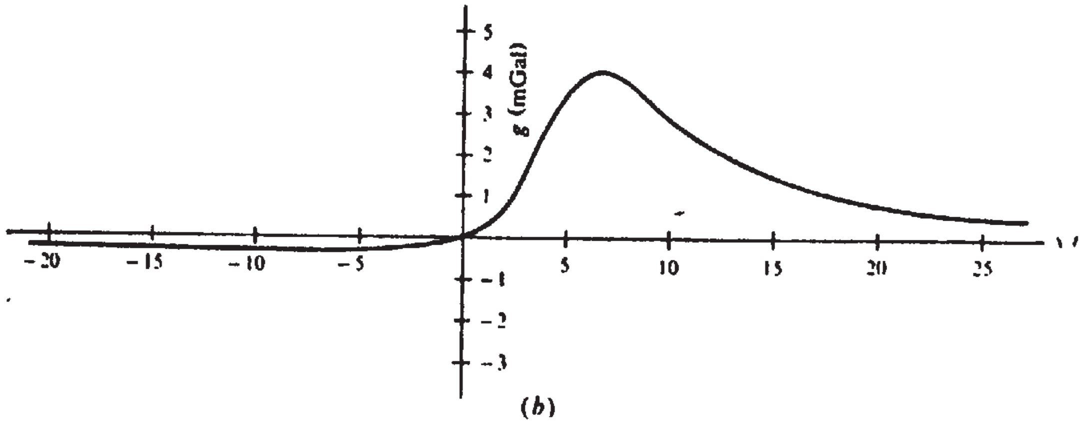

Figure 2.29. Gravity effect of a faulted horizontal sheet; $t = 300$ m, $h_1 = 750$ m, $h_2 = 1350$ m, and $\rho = 1\ g/cm^3$. (a) Normal fault dipping $\alpha = 30$ and 90°. (b) Reverse fault, $\alpha = -30°$.

where $\psi = \theta - \beta$. Substituting for dz, we get

$$g = 2\gamma\rho\left\{\pi t/2 - x\cos^2\beta\int_{\psi_1}^{\psi_2}(\psi + \beta)\, d\psi/\sin^2\psi\right\}$$

Using the relation $\int dx/\sin^2 x = -\cot x$, we can integrate the first term by parts, that is,

$$\int \psi\, d\psi/\sin^2\psi = -\psi\cot\psi + \int \cot\psi\, d\psi$$

$$= -\psi\cot\psi + \ln(\sin\psi)$$

Thus,

$$g = 2\gamma\rho\Big[\pi t/2 - x\cos^2\beta\{-\psi\cot\psi + \ln(\sin\psi) - \beta\cot\psi\}\Big|_{\psi_1}^{\psi_2}\Big]$$

$$= 2\gamma\rho\Big[\pi t/2 + x\cos^2\beta\{(\psi + \beta)\cot\psi - \ln(\sin\psi)\}\Big|_{\psi_1}^{\psi_2}\Big]$$

$$= 2\gamma\rho\Big[\pi t/2 + x\cos^2\beta\{(\psi_2 + \beta)\cot\psi_2 - (\psi_1 + \beta)\cot\psi_1 - \ln(\sin\psi_2/\sin\psi_1)\}\Big]$$

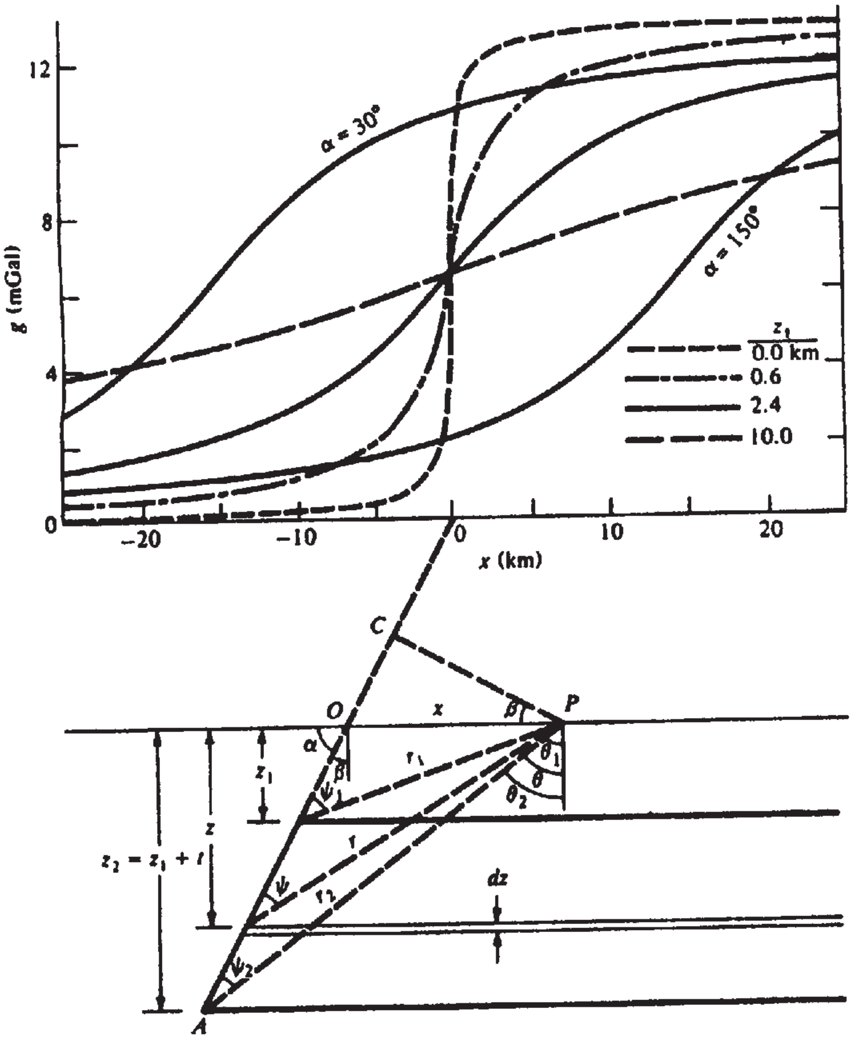

Figure 2.30. Gravity effect of a semiinfinite slab. $t = 300$ m, $\alpha = 90°$ except where otherwise noted on the curves, $\rho = 1$ gm/cm^3.

Figure 2.30 shows that

$$\begin{aligned}\beta(\cot\psi_2 - \cot\psi_1) &= \beta(AC/CP - BC/CP)\\ &= \beta(AB/CP)\\ &= \beta(t/\cos\beta)/(x\cos\beta)\\ &= \beta t/(x\cos^2\beta)\end{aligned}$$

so that we finally get

$$g = 2\gamma\rho\{(\pi/2 + \beta)t + x\cos^2\beta(F_2 - F_1)\} \tag{2.68}$$

where

$$F_i = \psi_i\cot\psi_i - \ln(\sin\psi_i) \qquad \psi_i = \theta_i - \beta_i$$
$$\theta_i = \tan^{-1}\{(x/z_i) + \tan\beta\}$$

Equation (2.68) is sometimes given in another form. From Figure 2.30 we have

$$x/\sin\psi_1 = r_1/\sin(\pi/2 + \beta) = r_1/\cos\beta$$
$$x/\sin\psi_2 = r_2/\cos\beta$$

so

$$(\sin\psi_2/\sin\psi_1) = r_1/r_2$$

Also,

$$\begin{aligned}\cot\psi_i &= \{(z_i/\cos\beta) + x\sin\beta\}/x\cos\beta\\ &= (z_i + x\sin\beta\cos\beta)/x\cos^2\beta\end{aligned}$$

Substituting in Equation (2.68) and noting that $t = (z_2 - z_1)$, we obtain

$$\begin{aligned}g &= 2\gamma\rho\{(\pi/2 + \beta)t + (\theta_2 - \beta)\\ &\qquad \times(z_2 + x\sin\beta\cos\beta)\\ &\qquad -(\theta_1 - \beta)(z_1 + x\sin\beta\cos\beta)\\ &\qquad + x\cos^2\beta\ln(r_2/r_1)\}\\ &= 2\gamma\rho\{(\pi t/2) + (z_2\theta_2 - z_1\theta_1)\\ &\qquad + x(\theta_2 - \theta_1)\sin\beta\cos\beta\\ &\qquad + x\cos^2\beta\ln(r_2/r_1)\}\end{aligned} \tag{2.69}$$

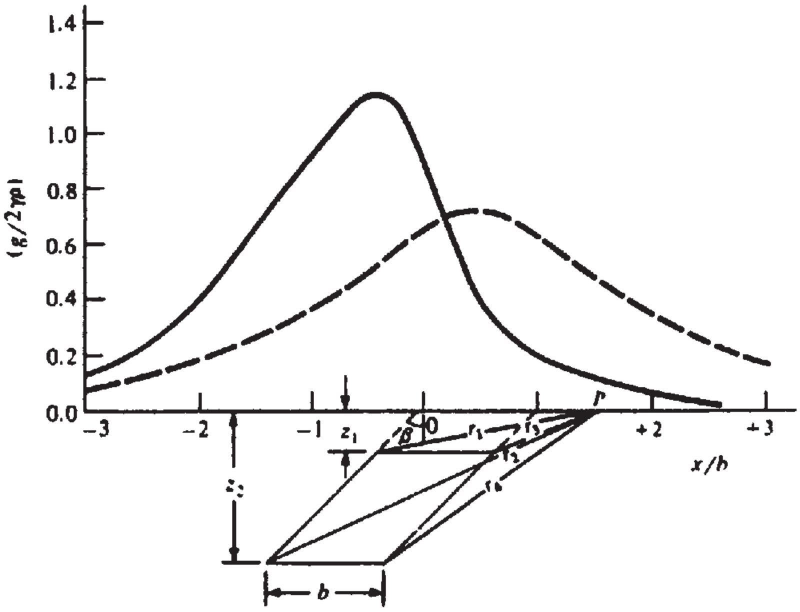

Figure 2.31. Gravity effect of a dike. Profiles are perpendicular to the dike. $L = \infty$, $b = 1$, $z_1 = 1/3$, $z_2 = 4/3$, $\beta = 45°$ (solid line), 0° (dashed line).

If the end of the slab is vertical, $\beta = 0$ and Equation (2.69) gives

$$g = 2\gamma\rho\{(\pi t/2) + (z_2\theta_2 - z_1\theta_1) + x\ln(r_2/r_1)\} \quad (2.70)$$

If the slab outcrops, $z_1 = 0$, $z_2 = t$, $\theta_1 = \pi/2$, $r_1 = x$, and

$$g = 2\gamma\rho\{(\pi t/2) + \theta_2 t + x(\theta_2 - \pi/2)\sin\beta\cos\beta + x\cos^2\beta\ln(r_2/x)\} \quad (2.71)$$

Figure 2.30 shows curves for a semiinfinite slab. The slope is quite sensitive to the depth of the slab but not to the dip of the end.

(c) Thick two-dimensional dike. The result for the dike in Figure 2.31 can be obtained by subtracting two slabs, one being displaced horizontally with respect to the other. The result is

$$g = 2\gamma\rho\cos^2\beta\{x(F_2 - F_1) - (x - b)(F_4 - F_3)\} \quad (2.72)$$

using Equation (2.68). In terms of Equation (2.69), this is

$$\begin{aligned} g = 2\gamma\rho\big[&z_2(\theta_2 - \theta_4) - z_1(\theta_1 - \theta_3) \\ &+ \sin\beta\cos\beta\{x(\theta_2 - \theta_1) - (x - b)(\theta_4 - \theta_3)\} \\ &+ \cos^2\beta\{x\ln(r_2/r_1) - (x - b)\ln(r_4/r_3)\}\big] \\ = 2\gamma\rho\big[&z_2(\theta_2 - \theta_4) - z_1(\theta_1 - \theta_3) \\ &+ \sin\beta\cos\beta\{x(\theta_2 + \theta_3 - \theta_4 - \theta_1) + b(\theta_4 - \theta_3)\} \\ &+ \cos^2\beta\{x\ln(r_2r_3/r_4r_1) + b\ln(r_4/r_3)\}\big] \end{aligned} \quad (2.73)$$

When the sides of the dike are vertical, $\beta = 0$ and

$$g = 2\gamma\rho\{z_2(\theta_2 - \theta_4) - z_1(\theta_1 - \theta_3) + x\ln(r_2r_3/r_4r_1) + b\ln(r_4/r_3)\} \quad (2.74)$$

If the dike outcrops, $z_1 = 0$, $r_1 = x$, $r_3 = (x - b)$, $\theta_1 = \pi/2 = \theta_3$, and the result is

$$\begin{aligned} g = 2\gamma\rho\big[&z_2(\theta_2 - \theta_4) + \sin\beta\cos\beta \\ &\times\{x(\theta_2 - \theta_4) - b(\pi/2 - \theta_4)\} \\ &+ x\cos^2\beta\ln\{r_2(x - b)/r_4x\} \\ &+ b\cos^2\beta\ln\{r_4/(x - b)\}\big] \end{aligned} \quad (2.75)$$