2.4.2.4 *Sodin*

This is very comparable to the Worden and operates in similar ways. Further details are given by Robinson and Coruh (1988).

2.4.2.5 *Vibrating string*

If a mass is suspended on a fibre which is forced to oscillate by an a.c. circuit, then the frequency of vibration, which can be measured electronically, will vary with changes in gravity. For a fibre of length L, and mass per unit length m_s, from which a mass M is suspended, by measuring the frequency of vibration (f), gravity can be determined (Box 2.8). However, the technology is not sufficiently developed to provide the same resolution and accuracy as other gravimeters, but it does give the impression that even more compact and lightweight gravimeters may be forthcoming in the near future. There is potential for use in airborne survey systems.

Box 2.8 Determination of *g* using a vibrating string

$$\text{Gravity} = \frac{4 \times \text{string length}^2 \times \text{frequency}^2 \times \text{string mass}}{\text{suspended mass}};$$

$$g = \frac{4L^2 f^2 m_s}{M}$$

2.5 CORRECTIONS TO GRAVITY OBSERVATIONS

Gravimeters do not give direct measurements of gravity. Rather, a meter reading is taken which is then multiplied by an instrumental calibration factor to produce a value of observed gravity (g_{obs}). Before the results of the survey can be interpreted in geological terms, these raw gravity data have to be corrected to a common datum, such as sea level (geoid), in order to remove the effects of features that are only of indirect geological interest. The correction process is known as *gravity data reduction* or *reduction to the geoid*. The difference between the value of observed gravity (g_{obs}) and that determined either from the International Gravity Formula/Geodetic Reference System 67 for the same location, or relative to a local base station, is known as the *gravity anomaly*. The various corrections that can be applied are listed in Table 2.7 with the sections of this book in which each one is discussed.

Table 2.7 Corrections to gravity data

Correction	Book sections
Instrument drift	2.5.1
Earth tides	2.5.2
Eötvös	2.5.7
Latitude	2.2.3 and 2.5.3
Elevation	
Free-air correction	2.5.4
Bouguer correction	2.5.5
Terrain	2.5.6
Isostatic	2.5.8

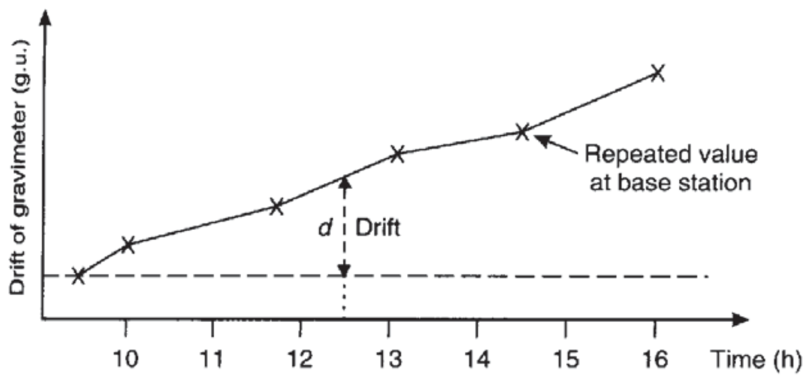

Figure 2.14 An instrumental drift curve

2.5.1 Instrumental drift

Gravimeter readings change (drift) with time as a result of elastic creep in the springs, producing an apparent change in gravity at a given station. The instrumental drift can be determined simply by repeating measurements at the same stations at different times of the day, typically every 1–2 hours. The differences between successive measurements at the same station are plotted to produce a *drift curve* (Figure 2.14). Observed gravity values from intervening stations can be corrected by subtracting the amount of drift from the observed gravity value. For example, in Figure 2.14 the value of gravity measured at an outlying station at 12.30 hours should be reduced by the amount of drift *d*. The range of drift of gravimeters is from a small fraction of one g.u. to about ten g.u. per hour. If the rate of drift is found to be irregular, return the instrument to the supplier – it is probably faulty!

2.5.2 Tides

Just as the water in the oceans responds to the gravitational pull of the Moon, and to a lesser extent of the Sun, so too does the solid earth.

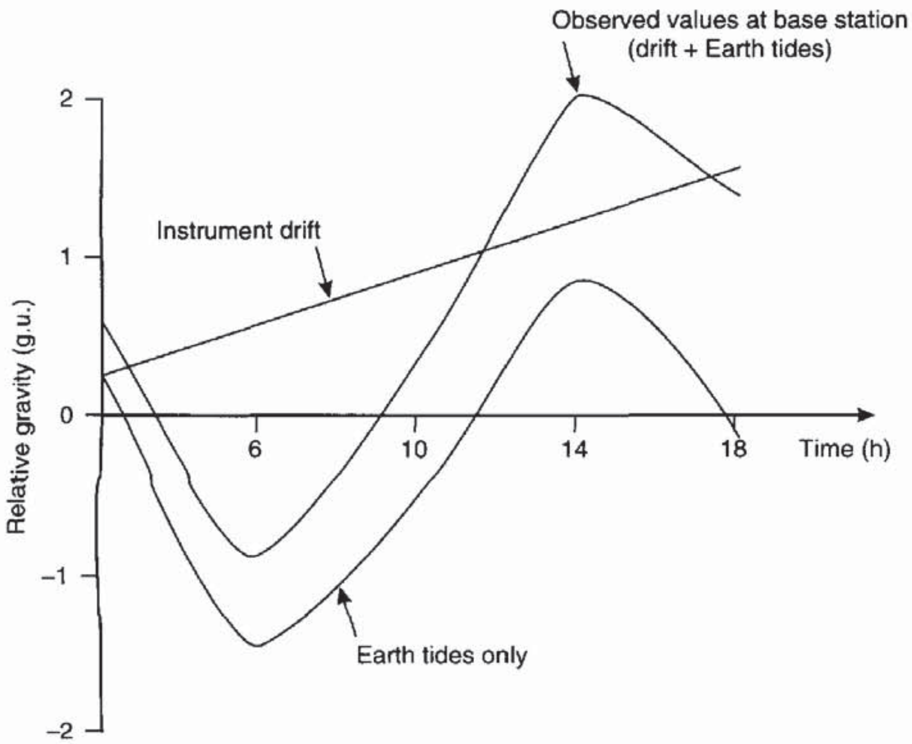

Figure 2.15 Graph of the effects of Earth tides and instrumental drift on the acceleration due to gravity

Earth tides give rise to a change in gravity of up to three g.u. with a minimum period of about 12 hours. Repeated measurements at the same stations permit estimation of the necessary corrections for tidal effects over short intervals, in addition to determination of the instrumental drift for a gravimeter (Figure 2.15). Alternatively, recourse can be made to published tide tables which are published periodically (e.g. *Tidal Gravity Corrections for 1991*, European Association of Exploration Geophysicists, The Hague).

2.5.3 Latitude

The latitude correction is usually made by subtracting the theoretical gravity calculated using the International Gravity Formula (g_ϕ) (Section 2.2.3) from the observed value (g_{obs}). For small-scale surveys which extend over a total latitude range of less than one degree, and not tied into the absolute gravity network, a simpler correction for latitude can be made. A local base station is selected for which the horizontal gravity gradient (δg_L) can be determined at a given degree of latitude (ϕ) by the expression in Box 2.9.

Box 2.9 Local latitude correction

$$\delta g_L = -8.108 \sin 2\phi \quad \text{g.u. per km N}$$

Note that the correction is negative with distance northwards in the northern hemisphere or with distance southwards in the southern hemisphere. This is to compensate for the increase in the gravity field from the equator towards the poles. For a latitude of 51°N, the local latitude correction is about 8 g.u./km. For gravity surveys conducted with an accuracy of ± 0.1 g.u., the latitudinal position of the gravity station needs to be known to within ± 10 m, which is well within the capability of modern position-fixing.

2.5.4 Free-air correction

The basis of this correction is that it makes allowance for the reduction in magnitude of gravity with height above the geoid (see Figure 2.16 and Box 2.10), irrespective of the nature of the rock below. It is analogous to measuring gravity in the basket of a hot-air balloon in flight – hence the term *free-air correction*. The free-air correction is the difference between gravity measured at sea level and at an elevation of h metres with no rock in between. A value of 3.086 g.u./m

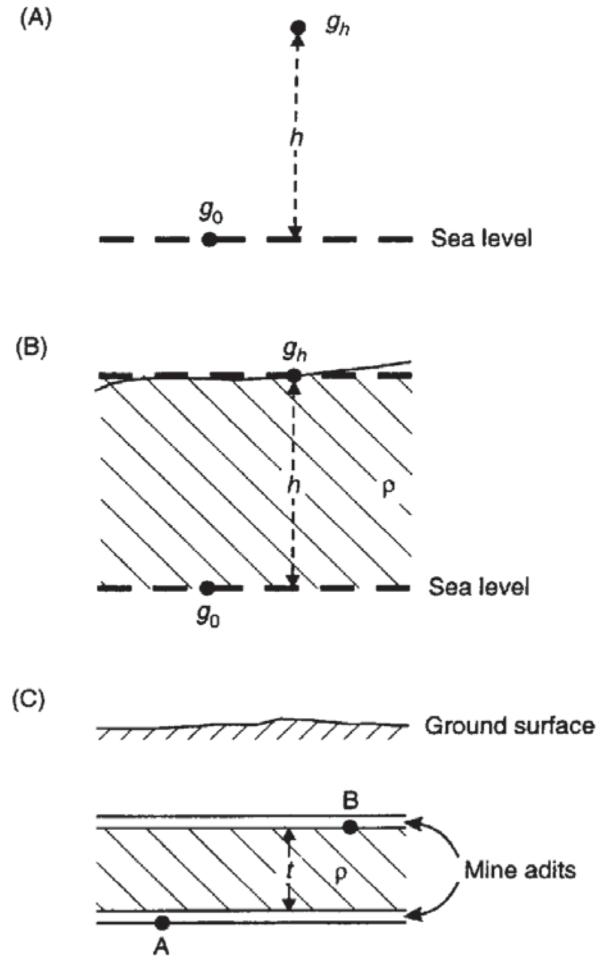

Figure 2.16 Schematic showing (A) the free-air correction, (B) the Bouguer correction, and (C) the Bouguer correction for measurements made underground

is accepted for most practical applications and is positive at elevations above sea level, and negative below. The free-air correction term varies slightly with latitude from 3.083 g.u./m at the equator to 3.088 g.u./m at the poles. With the normal measuring precision of modern gravimeters being around 0.1 g.u., elevations must be known to within 3–5 cm.

Box 2.10 Free-air correction (see also Figure 2.16)

Taking the Earth to be a sphere (rather than an oblate spheroid) with its mass concentrated at its centre of mass, then the value of gravity at sea level is:

$$g_0 = GM/R^2.$$

The value of gravity at a station at an elevation of h metres above sea level is:

$$g_h = GM/(R+h)^2 = \frac{GM}{R^2}\left(\frac{1-2h}{R}\cdots\right).$$

The difference in gravity between sea level and at h metres is the free-air correction:

$$\delta g_F = g_0 - g_h = \frac{2g_0 h}{R}.$$

With $g_0 = 9\,817\,855$ g.u., $R = 6\,371\,000$ M, and with h in metres,

$$\delta g_F = 3.082h \text{ g.u.}$$

Taking into account that the Earth is an oblate spheroid, rather than a sphere, the normally accepted value of the free-air correction is:

$$\boldsymbol{\delta g_F = 3.086\,h} \textbf{ g.u.}$$

The reduction in g with increasing height above the ground is important in airborne gravimetry. Anomalies detected by helicopter-mounted gravimeters will have decreased amplitudes and lengthened wavelengths compared with those obtained from land-based surveys. To compare land gravity survey data with airborne, it is necessary to correct for the free-air attenuation of the gravity anomaly by using *upward continuation*, which is discussed in Section 2.6.

The quantity calculated by applying both the latitude and the free-air corrections is called the *free-air anomaly* and is commonly used to display corrected gravity data for oceans and continental shelves (see, e.g., Talwani *et al.* 1965).

2.5.5 Bouguer correction

Whereas the free-air correction compensates for the reduction in that part of gravity due only to increased distance from the centre of mass, the Bouguer correction (δg_B) is used to account for the rock mass between the measuring station and sea level (Figure 2.16).

The Bouguer correction calculates the extra gravitational pull exerted by a rock slab of thickness h metres and mean density ρ (Mg/m^3) which results in measurements of gravity (g_{obs}) being overestimated by an amount equal to $0.4192\rho h$ g.u. (Box 2.11). The Bouguer correction should be subtracted from the observed gravity value for stations above sea level. For an average rock density of 2.65 Mg/m^3, the Bouguer correction amounts to 1.12 g.u./m. For marine surveys, the Bouguer correction is slightly different in that the low density of sea water is effectively replaced by an equivalent thickness of rock of a specified density (Box 2.11).

Box 2.11 Bouguer correction

Bouguer correction (δg_B) $= 2\pi G\rho h = \beta\rho h$ (g.u.), where:

$$\beta = 2\pi G = 0.4192 \text{ g.u. m}^2\,\text{Mg}^{-1}$$
$$G = 6.67 \times 10^{-8}\,\text{m}^3\,\text{Mg}^{-1}\,\text{s}^{-2}.$$

Density (ρ) is in Mg m^{-3} and height (h) is in metres.

For marine surveys, the Bouguer correction is given by:

$$\delta g_B = \beta(\rho_r - \rho_w)h_w \text{ (g.u.)}.$$

where ρ_r and ρ_w are the densities of rock and sea water respectively, and h_w is the water depth in metres.

A further development of this correction has to be made for gravity measurements made underground (Figure 2.16C). In this case, the Bouguer correction has to allow for the extra gravitational pull ($=0.4191\rho t$ g.u.) on Station A caused by the slab of thickness t metres between the two stations A and B, whereas the value of gravity at Station A is underestimated by the equal but upward attraction of the same slab. The difference in gravity between the two stations is *twice* the normal Bouguer correction ($=0.8384\rho t$ g.u.). Allowances also have to be made for underground machinery, mine layout and the

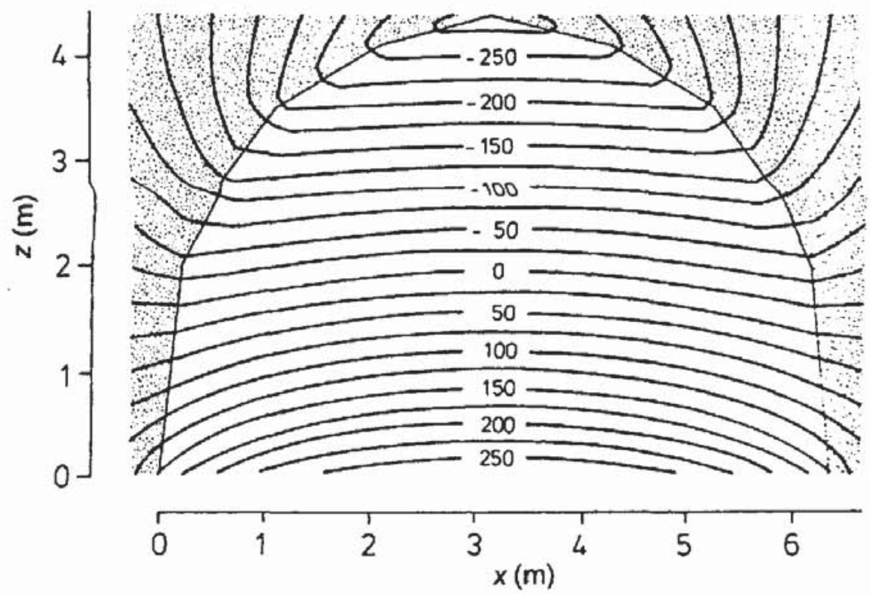

Figure 2.17 Micro-isogals for a typical gallery in a deep coal mine; the contour interval is 25 μGal and the density of the host rock is 2.65 Mg/m^3. From Casten and Gram (1989), by permission

variable density of local rocks, and this is sometimes referred to as the *Gallery correction* (Figure 2.17).

The Bouguer correction on land has to be modified by allowances for terrain roughness (see Section 2.5.6) in areas where there is a marked topographic change over a short distance, such as an escarpment of cliff. In such a situation the approximation of a semi-infinite horizontal slab of rock no longer holds true and more detailed calculations are necessary (Parasnis 1986; p. 72).

The free-air and Bouguer corrections are commonly combined into one *elevation correction* (δg_E) to simplify data handling (Box 2.12). It should be noted that in some cases, the resulting gravity anomaly may be misleading and the combined calculation should be used judiciously. For a density of 2.60 Mg/m^3, the total elevation correction is 2 g.u./m, which requires elevations to be known to an accuracy within 5 cm if gravity readings are to be made to within 0.1 g.u.

Box 2.12 Elevation correction

Elevation correction (δg_E) = (Free-air − Bouguer) corrections:

$$\delta g_E = \delta g_F - \delta g_B.$$

Substituting in the terms $\delta g_F = 3.086h$ and $\delta g_B = 0.4192\rho h$:

$$\boldsymbol{\delta g_E = (3.086 - 0.4192\rho)h \text{ (g.u.)}}$$

where ρ is the average rock density in Mg/m^3.

One of the main problems with the Bouguer correction is knowing which density to use. For example, a difference of 0.1 Mg/m^3 in density for a gravity measurement made at an elevation of 250 m will result in a discrepancy of more than 10 g.u. in the Bouguer correction. In many cases, it may be possible to obtain an estimate of rock densities from appropriate surface samples, or from borehole samples, if available. Caution should be used in the latter case as rock-core samples will relax mechanically, producing many cracks, and expanding slightly in response to the reduced pressure at the surface, giving rise to an underestimate of *in situ* density.

Nettleton (1939, 1940) found a very simple way of determining the appropriateness of the chosen density using a graphical method. Corrected gravity data should show no correlation with topography as all such effects should have been removed through the data reduction process. If a range of densities is chosen and the resulting elevation corrections computed along one gravity profile, the density that shows least correlation with topography is taken as the 'correct' one (Figure 2.18). It is known, however, that this method becomes less accurate if there is any topographic expression due to dipping beds with a significant density contrast to those above and below. Examples of where this might occur are in association with an inclined dense igneous intrusion, or in a marked structural feature with significant variations in density.

A generalised Nettleton method has been proposed by Rimbert *et al.* (1987) which allows for density to vary over a geographic area. The topographic data of an area are smoothed and reduced to produce a surface that lies just below the low topographic points on the survey, and which they refer to as the 'regional topography'. The density between the 'regional' and the actual topography is considered as constant (ρ_0) for a fixed radius around each station. The density below the 'regional' topography is taken as uniform throughout the area of the survey. The variations in density above the 'regional' topography are accounted for statistically but can be plotted in map form to demonstrate the areal variation in density which can be correlated independently with maps of the known geology. Rimbert *et al.* (1987) have achieved accuracies to within 3–4 g.u. with this method in an area in the south of France in which the observed gravity anomalies ranged from 40 to 150 g.u.

Another method to calculate density is to plot a graph of the latitude-corrected gravity data from an area without marked gravity anomalies, against station elevation (Reeves and MacLeod 1986). The resulting straight-line graph yields a gradient that is numerically equal to the elevation correction. Using the result in Box 2.16 and rearranging the equation to make density the subject of the equation, it is possible to solve and find a value for the density. Reeves and MacLeod used this method on data for a survey in Belgium, and produced a graphical elevation correction of 2.35 g.u./m which gave

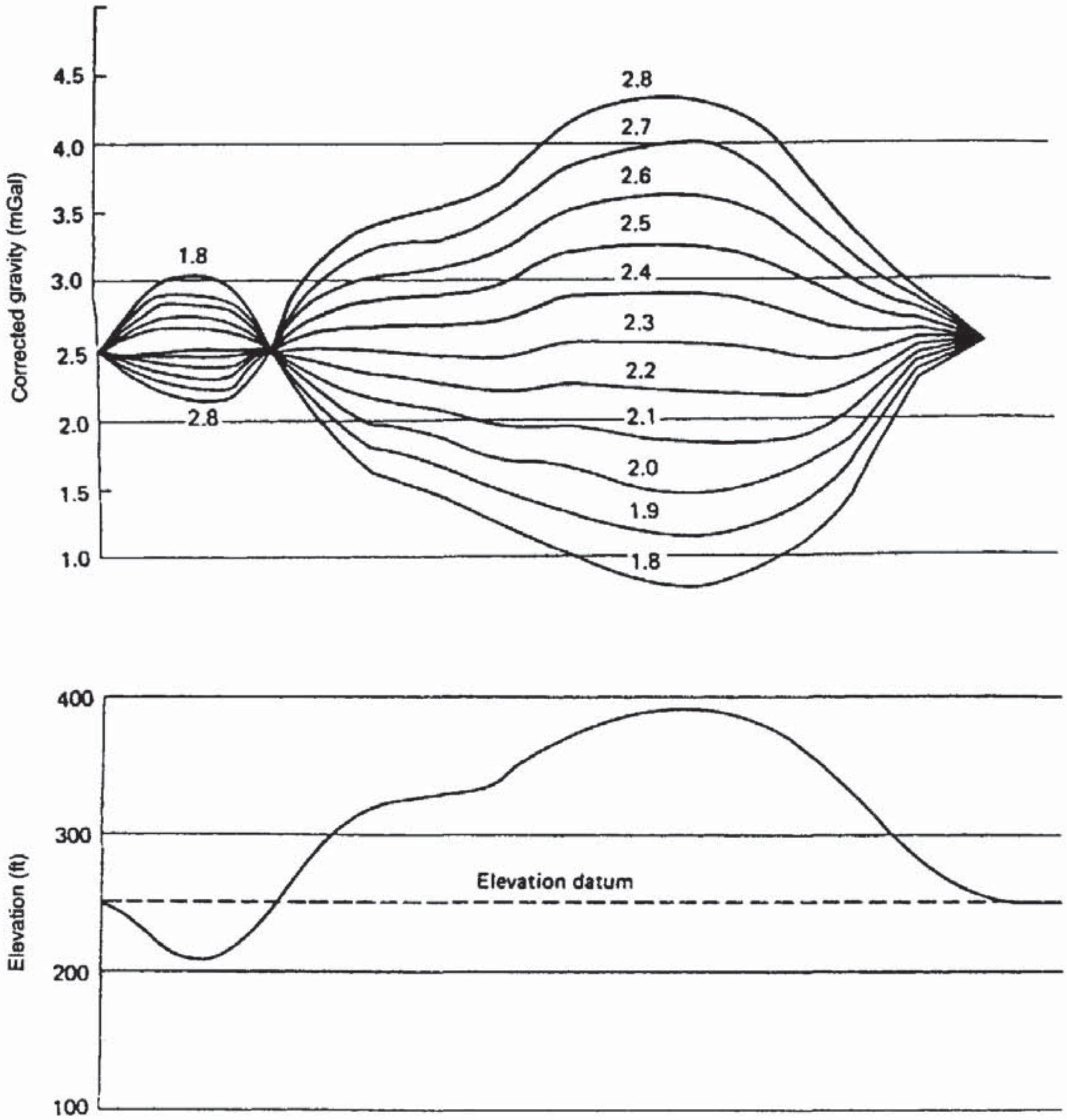

Figure 2.18 The Nettleton method for determining the density of near surface rock formations with the topography as shown. The most appropriate density is about 2.3 Mg/m³ (least correlation with the topography). Corrections are referred to a height of 76 m. From Dobrin (1976), by permission

a density estimate of 1.74 Mg/m³, compared with 1.6 Mg/m³ determined using the Nettleton method.

The danger with a graphical method – even if linear regression statistics are used to determine the gradient of the best straight-line fit through the data – is that a small difference in the graphical gradient can result in an unacceptably large density discrepancy, and this makes the method rather insensitive to small changes in density. For example, the data presented by Reeves and MacLeod, rather than showing a single trend with elevation, indicate two: data up to an elevation of 215 m yield a density of 1.43 Mg/m³ and the remaining data up to 280 m yield a density of 1.79 Mg/m³, a difference of 0.36 Mg/m³. Densities should be determined to better than 0.1 Mg/m³ if possible.

In underground gravity surveys, a similar method is employed (Hussein 1983). The vertical gravity gradient (i.e. δg_E) is obtained by measuring g at two or more elevations separated by only 1.5–3 m at the same location within an underground chamber. The density can then be calculated as described above. As the measurements of g are strictly controlled, the error in density can be minimised routinely to within 0.08 Mg/m^3 (Casten and Gram 1989).

Rock densities for depths below which it is not possible to sample can also be estimated using the relationship of density with P-wave velocities as described, for example, by Nafe and Drake (1963), Woollard (1950, 1959, 1975), Birch (1960, 1961), and Christensen and Fountain (1975).

2.5.6 Terrain correction

In flat countryside, the elevation correction (the combined free-air and Bouguer correction) is normally adequate to cope with slight topographic effects on the acceleration due to gravity. However, in areas where there are considerable variations in elevation, particularly close to any gravity station, a special *terrain correction* must be applied. The Bouguer correction assumes an approximation to a semi-infinite horizontal slab of rock between the measuring station and sea level. It makes no allowance for hills and valleys and this is why the terrain correction is necessary.

The effect of topography on g is illustrated in Figure 2.19. Consider a gravity station beside a hill as in Figure 2.19A. The slab of rock which comprises the hill (mass M) has its centre of mass above the plane on which the gravimeter is situated. There is a force of attraction between the two masses. If the force is resolved into horizontal and vertical components and the latter only is considered, then it can be seen that the measurement of g at the gravity station will be underestimated by an amount δg. Conversely, if the gravity station is adjacent to a valley, as indicated in Figure 2.19B, then the valley represents a mass deficiency which can be represented by a negative mass ($-M$). The lack of mass results in the measurement of g to be underestimated by an amount δg. Consequently, a gravity measurement made next to either a hill or a valley requires a correction to be added to it to make allowance for the variable distribution of mass. The correction effectively removes the effects of the topography to fulfil the Bouguer approximation of a semi-infinite rock slab.

Physical computation of the terrain correction is extremely laborious as it has to be carried out for each and every station in an entire survey. A special transparent template, known as a *Hammer chart* after its originator Sigmund Hammer (1939), consists of a series of segmented concentric rings (Figure 2.20). This is superimposed over a topographic map and the average elevation of each segment of the

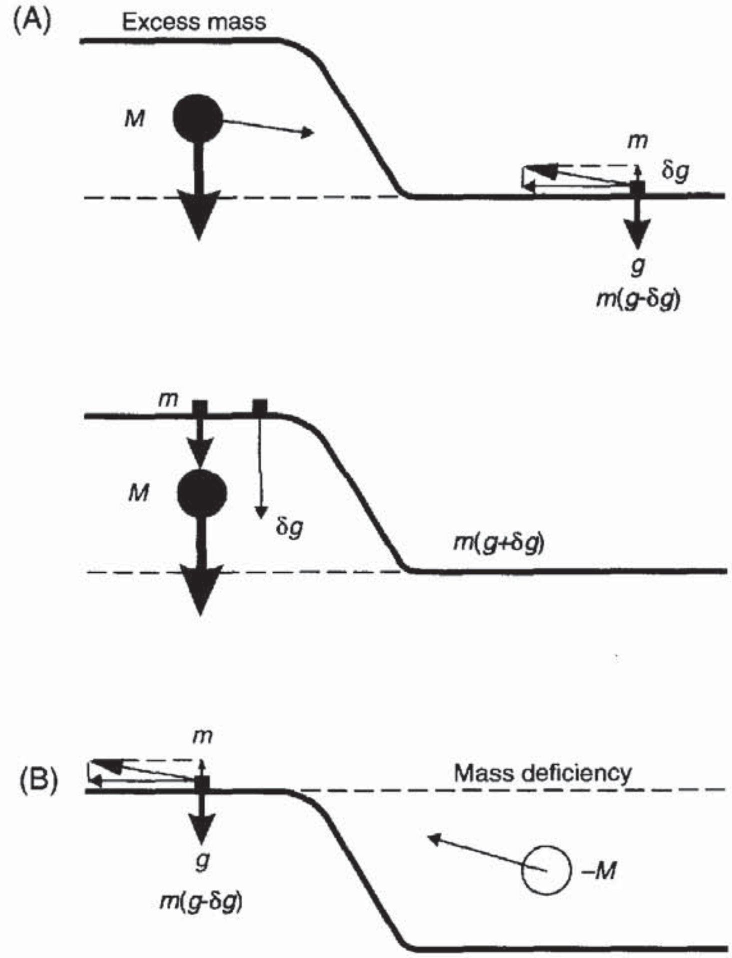

Figure 2.19 The effects of a hill and a valley on the measurement of gravity, illustrating the need for terrain corrections

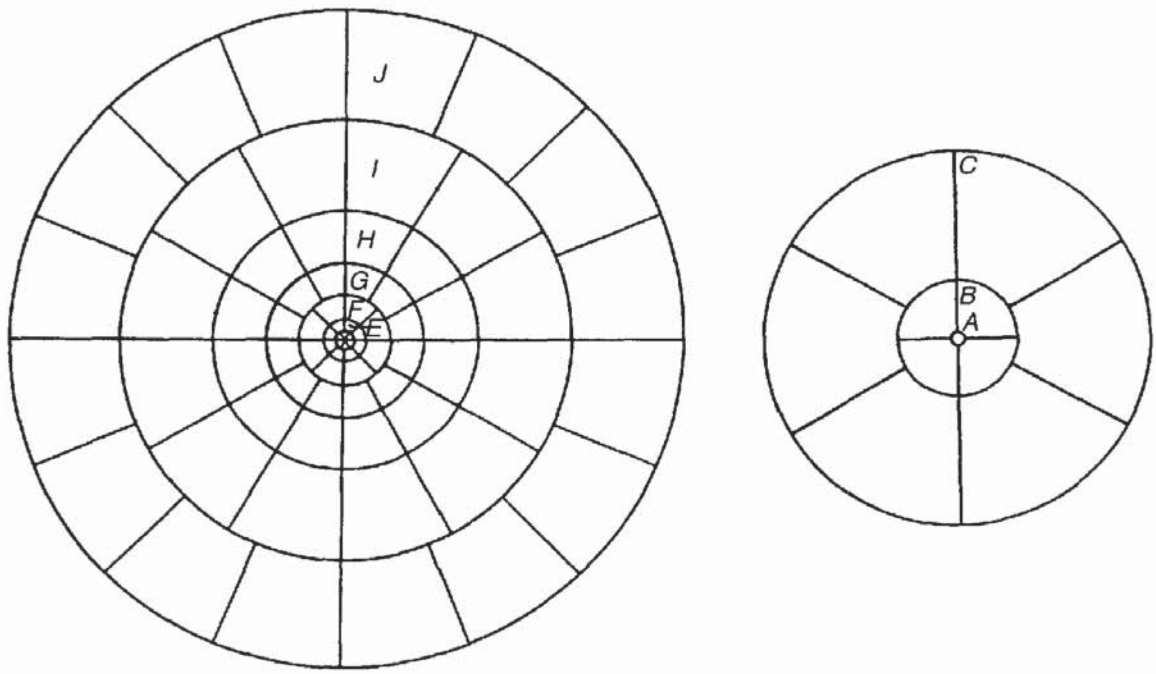

Figure 2.20 Hammer terrain correction chart with inner rings A–C shown expanded for clarity. After Dobrin (1976) and Milsom (1989), by permission