Since $(h/R_S) = \cos\theta$ in Figure 2.22b, $(g/2\pi)$ can be regarded as a surface density of mass replacing the mass below the xy plane [compare with Eq. (2.14)]. Equation (2.48) is the *upward continuation* equation that allows us to calculate the gravitational acceleration anywhere in free space from a knowledge of its values over the surface. Upward continuation is effectively smoothing. Although upward continuation is not done much in gravity analysis, it is used in magnetic interpretation to compare measurements made at different flight elevations.

If we can calculate the gravity field over a surface closer to the anomaly sources, the anomaly should be sharper and less confused by the effects of deeper features. This process, called *downward continuation*, was described by Peters (1949). It involves calculating a gravity value at depth from gravity values and derivatives on a shallower surface. The derivatives are usually evaluated by averaging over circles of different radii as described in Section 2.6.5. The main theoretical limitation on the method is singularities associated with masses through which the continuation process is carried. The main practical limitation is imposed by uncertainty in the measured field; because derivatives involve differences, their calculation magnifies uncertainties. The result is that minor noise is increased in the downward-continued field and this noise may outweigh the benefits of sharpening anomalies.

We begin with Laplace's equation (2.11b) (thus implicitly assuming that we will not continue through any masses) and the expressions for second derivatives calculated by finite differences [Eq. (2.45)]. For the point $(x_0, y_0, 0)$ and station spacing s, we write

$$\partial^2 g/\partial x^2 = \{g(x_0+s, y_0, 0) - 2g(x_0, y_0, 0) + g(x_0-s, y_0, 0)\}/s^2$$

$$\partial^2 g/\partial y^2 = \{g(x_0, y_0+s, 0) - 2g(x_0, y_0, 0) + g(x_0, y_0-s, 0)\}/s^2$$

$$\partial^2 g/\partial z^2 = \{g(x_0, y_0, +s) - 2g(x_0, y_0, 0) + g(x_0, y_0, -s)\}/s^2$$

If we take z to be positive downward, then $g(x_0, y_0, +s)$ is the gravity value a distance s below the station $g(x_0, y_0, 0)$. Substituting into Laplace's equation, we get

$$\begin{aligned} g(x_0, y_0, +s) = 6g(x_0, y_0, 0) & - \{g(x_0+s, y_0, 0) \\ & + g(x_0-s, y_0, 0) \\ & + g(x_0, y_0+s, 0) \\ & + g(x_0, y_0-s, 0) \\ & + g(x_0, y_0, -s)\} \end{aligned} \tag{2.49}$$

All of these terms can be found from the gravity values read from a grid except for the last term, which can be found from Equation (2.48). Similar but more complicated procedures use concentric circles passing through grid stations. Other methods employ Fourier transform theory (see Grant and West, 1965, p. 218).

2.7. GRAVITY INTERPRETATION

2.7.1. General

After the camouflaging interference effects of other features have been removed to the best of our ability, the interpretation problem usually is finding the mass distribution responsible for the residual anomaly. This often is done by *iterative modeling* (Bhattacharyya, 1978). The field of a model mass distribution is calculated and subtracted from the residual anomaly to determine the effects for which the model cannot account. Then the model is changed and the calculations repeated until the remaining effects become smaller than some value considered to be "close enough." To limit the number of possible changes, we include some predetermined constraints, for example, we might change only the upper surface of the mass distribution.

Before iterative modeling became practical, interpreters generally compared residual anomalies to anomalies associated with simple shapes, and this procedure is still useful in many situations. Simple shapes can be modeled with a microcomputer (Reeves and MacLeod, 1983). A gravity anomaly is not especially sensitive to minor variations in the shape of the anomalous mass, so that simple shapes often yield results that are close enough to be useful. Study of the gravity effect of simple shapes also helps in understanding the types of information that can be

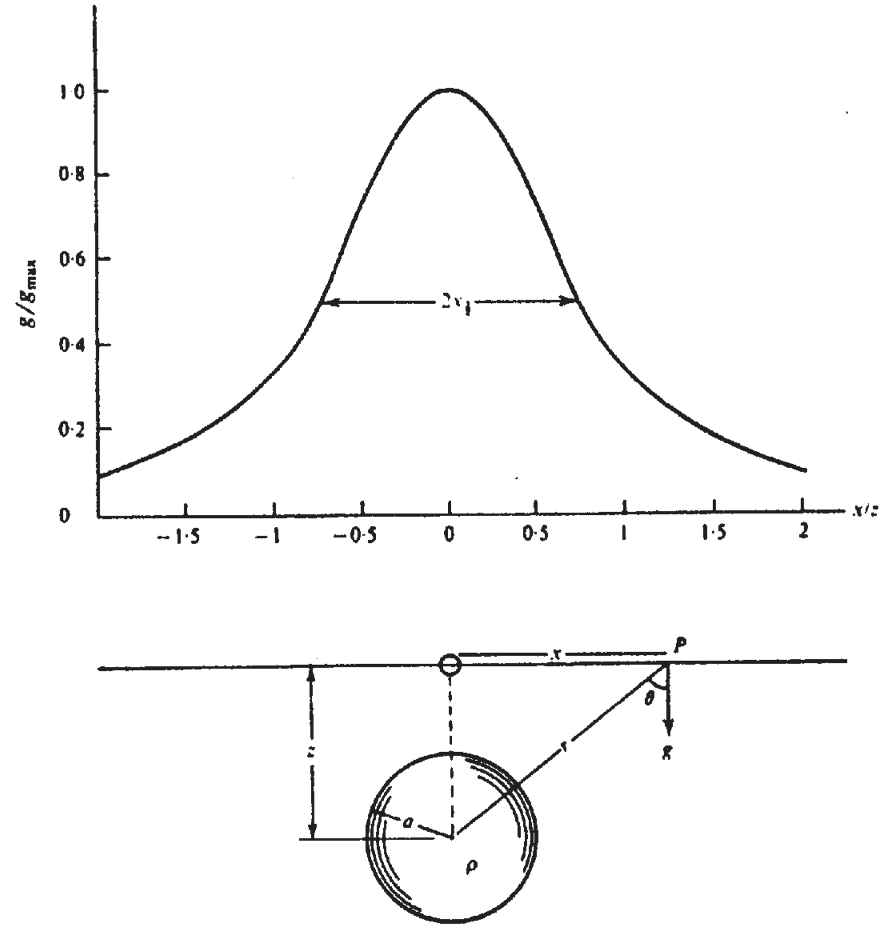

Figure 2.23. Gravity effect of a sphere.

learned, for example, in determining what aspects of an anomaly indicate the depth, shape, density contrast, total mass, and so forth.

In the following examples, the density symbol ρ is the density contrast with respect to the laterally equivalent material (in numerical relations, ρ is the difference in specific gravity because density is usually given in grams per cubic centimeters even where linear dimensions are given in English units).

2.7.2. Gravity Effect of a Sphere

The gravity effect of a sphere at a point P (Fig. 2.23), directed along r, is $g_r = \gamma M/r^2$. The vertical component is

$$g = g_r \cos\theta = \gamma M z/r^3$$

$$= k\rho a^3 z/(x^2 + z^2)^{3/2} \text{ mGal} \qquad (2.50)$$

where

$$k = 4\pi\gamma/3$$

$$= 27.9 \times 10^{-3} \quad \text{when } a, x, z \text{ are in meters}$$

$$= 8.52 \times 10^{-3} \quad \text{when } a, x, z \text{ are in feet}$$

Note that z is the depth to the sphere center rather than to the top of the sphere and that the profile is symmetrical about the origin taken directly above the center. The maximum value of g is

$$g_{max} = 27.9 \times 10^{-3}\rho a^3/z^2 \quad \text{when } a, z \text{ in meters} \qquad (2.51a)$$

$$= 8.52 \times 10^{-3}\rho a^3/z^2 \quad \text{when } a, z \text{ in feet} \qquad (2.51b)$$

The depth of the center of the sphere, z, can be found from a profile. When $g = g_{max}/2$, $z = 1.3x_{1/2}$,

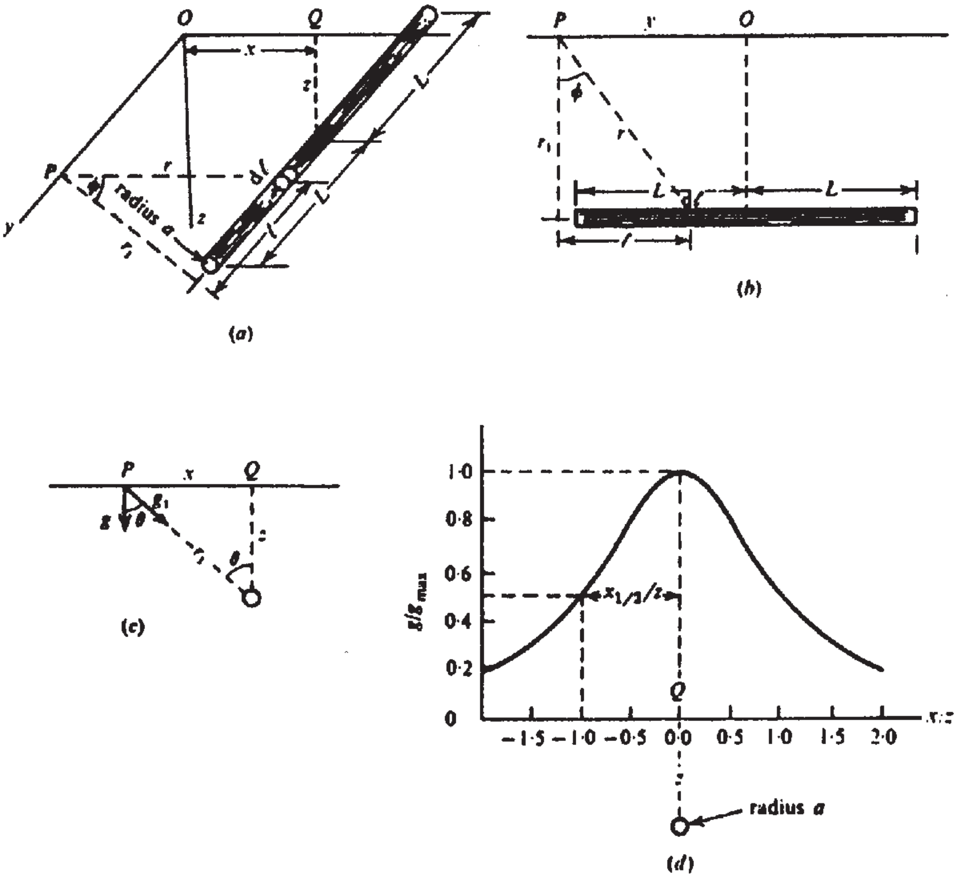

Figure 2.24. Gravity effect of a horizontal rod. (a) Three-dimensional view. (b) Projection on the plane containing the rod and the y axis. (c) Projection on the xz plane. (d) Gravity profile along the x axis ($L = \infty$).

where $x_{1/2}$ is the *half-width* of the profile, that is, half the width at the half-maximum value. We can also express the mass of the sphere, M, in terms of $x_{1/2}$ and g_{max}:

$$M = 25.5 g_{max}(x_{1/2})^2 \text{ tonnes} \qquad (2.52a)$$

where $x_{1/2}$ is in meters, or, where $x'_{1/2}$ is in feet,

$$M = 2.61 g_{max}(x'_{1/2})^2 \text{ short tons} \qquad (2.52b)$$

The spherical shape is particularly useful as a first approximation in the interpretation of three-dimensional anomalies that are approximately symmetrical.

2.7.3. Gravity Effect of a Horizontal Rod

The effect at $P(x, y, 0)$ of a segment of length dl of a horizontal rod perpendicular to the x axis at a depth z (Fig. 2.24) with mass m per unit length is

$$dg_r = \gamma m\, dl/r^2 = \gamma m(r_1\, d\phi/\cos^2\phi)/r^2 = \gamma m\, d\phi/r_1$$

where $dl = (r_1\, d\phi/\cos^2\phi)$. The component along r_1 is

$$dg_1 = dg_r \cos\phi = \gamma m \cos\phi\, d\phi/r_1$$

and the vertical component is

$$dg = dg_1 \cos\theta = dg_1(z/r_1) = \gamma m z \cos\phi\, d\phi/r_1^2$$

Integrating from $\tan^{-1}\{(y - L)/r_1\}$ to $\tan^{-1}\{(y + L)/r_1\}$, we get

$$g = \left(\frac{\gamma m z}{r_1^2}\right)\left[\frac{y + L}{\{(y + L)^2 + r_1^2\}^{1/2}} - \frac{y - L}{\{(y - L)^2 + r_1^2\}^{1/2}}\right]$$

$$= \frac{\gamma m}{z(1 + x^2/z^2)}\left[\frac{1}{\{1 + (x^2 + z^2)/(y + L)^2\}^{1/2}} - \frac{1}{\{1 + (x^2 + z^2)/(y - L)^2\}^{1/2}}\right] \qquad (2.53)$$

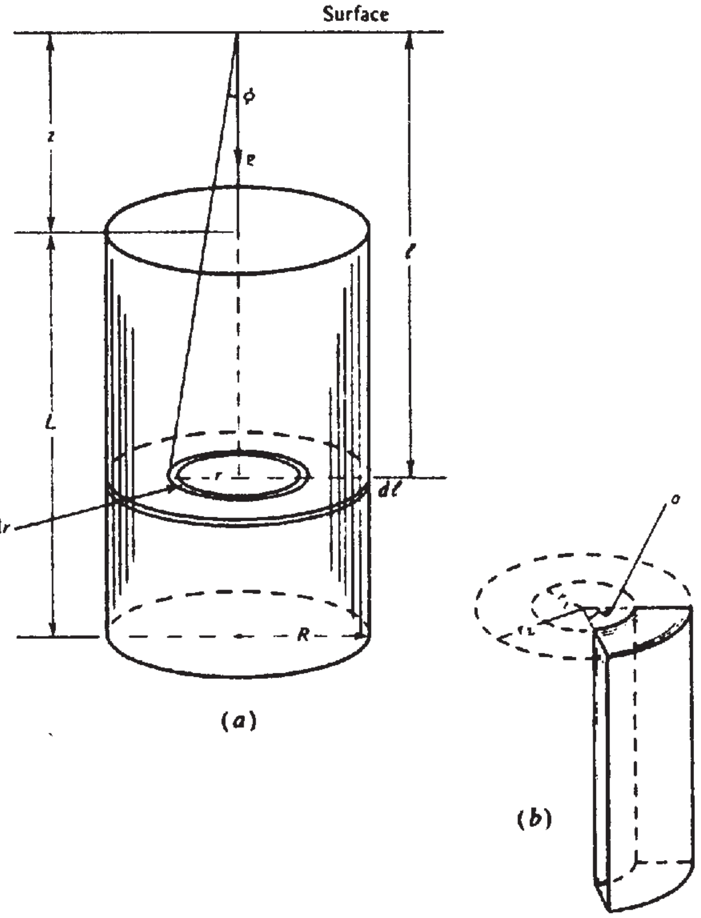

Figure 2.25. Gravity effect of a vertical cylinder. (a) Calculation of gravity over the axis. (b) Geometry of a cylindrical slice.

If the rod is infinite in length, the limits of integration would have been $\pm\pi/2$ and the result would be

$$g = 2\gamma m/z(1 + x^2/z^2) \qquad (2.54)$$

This is usually a good approximation when $L > 10z$. The depth z to the center of the rod in Equation (2.54) can be found from the half-width $x_{1/2}$:

$$z = x_{1/2} \qquad (2.55)$$

If the rod is expanded into a cylinder of radius a, the only change in Equations (2.53) and (2.54) is that $m = \pi a^2 \rho$.

2.7.4. Gravity Effect of a Vertical Cylinder

The gravity effect on the axis of a vertical cylinder (which is the maximum value) can easily be calculated. First we find g on the axis for a disk of thickness $d\ell$ (Fig. 2.25a). We start with an elementary ring of width dr whose mass is $\delta m = 2\pi\rho r\, dr\, d\ell$, so that its gravity effect is

$$\begin{aligned} \delta g &= \gamma\, \delta m \cos\phi/(r^2 + \ell^2) \\ &= (2\pi\rho\gamma\, d\ell)\, r\, dr \cos\phi/(r^2 + \ell^2) \\ &= 2\pi\gamma\rho\, d\ell \sin\phi\, d\phi \end{aligned}$$

on eliminating r. Integrating first from $\phi = 0$ to $\tan^{-1}(R/\ell)$ for the disk and then from $\ell = z$ to $z + L$, we get, for the whole cylinder,

$$\begin{aligned} g &= 2\pi\gamma\rho \int_z^{z+L} \left\{1 - \ell/(\ell^2 + R^2)^{1/2}\right\} d\ell \\ &= 2\pi\gamma\rho \Big[L + (z^2 + R^2)^{1/2} \\ &\qquad - \left\{(z + L)^2 + R^2\right\}^{1/2} \Big] \qquad (2.56) \end{aligned}$$

where

$$\begin{aligned} 2\pi\gamma &= 41.9 \times 10^{-3} \quad \text{when } z, R, L \text{ are in meters} \\ &= 12.77 \times 10^{-3} \quad \text{when } z, R, L \text{ are in feet} \end{aligned}$$

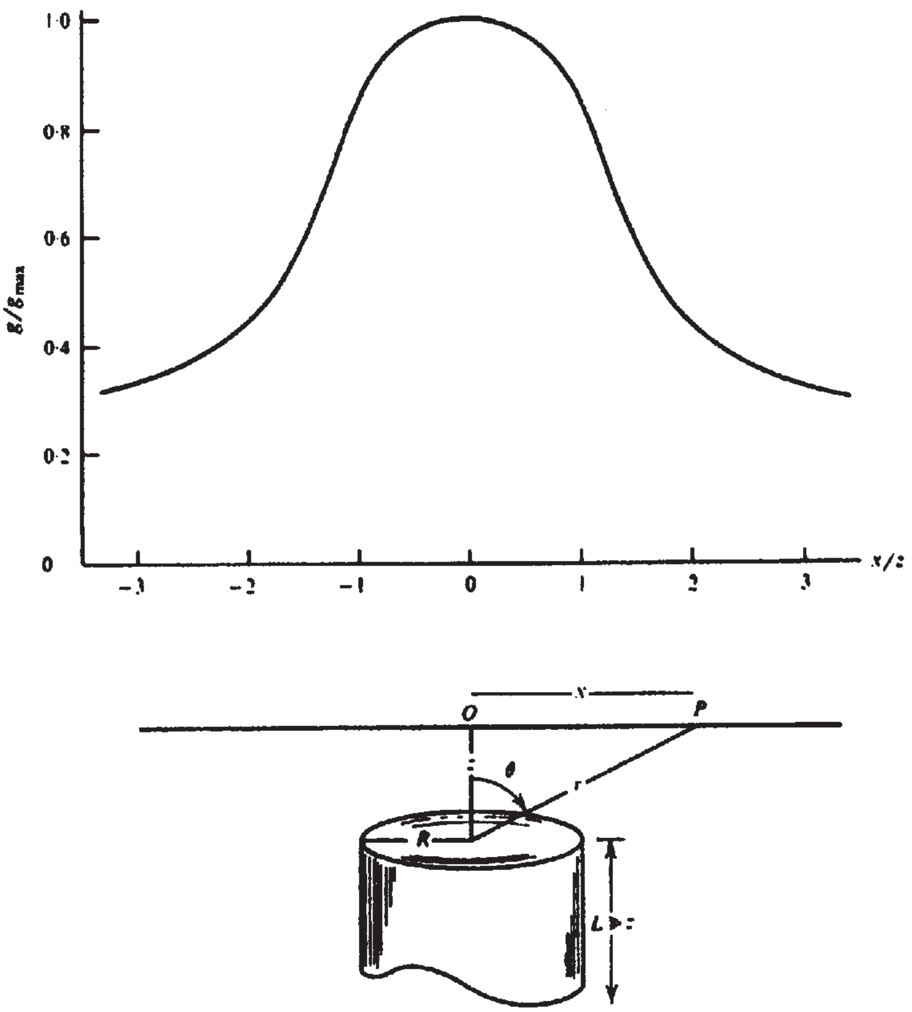

Figure 2.26. Gravity effect off the axis of a vertical cylinder.

There are several cases of special significance:

1. If $R \to \infty$, we have an infinite horizontal slab and

$$g = 2\pi\gamma\rho L \tag{2.57}$$

This is the Bouguer correction given in Section 2.3.2d. Note that g is independent of the depth of the slab and varies only with its thickness.

2. The terrain correction can be obtained using a sector of the cylinder as shown in Figure 2.25b. We have $\delta m = \rho(r\theta)\,dr\,d\ell$ so that

$$\delta g = \gamma(\rho r\theta\,dr\,d\ell)\cos\phi/(r^2 + \ell^2)$$

$$= \gamma\rho\theta\,d\ell\,\sin\phi\,d\phi$$

on eliminating r. We integrate from $\phi = \tan^{-1}(r_1/\ell)$ to $\tan^{-1}(r_2/\ell)$ and from $\ell = 0$ to L. The result is

$$\delta g_T = \gamma\rho\theta\left\{(r_2 - r_1) + \left(r_1^2 + L^2\right)^{1/2} - \left(r_2^2 + L^2\right)^{1/2}\right\} \tag{2.58}$$

which is Equation (2.26) with L replacing Δz.

3. When $z = 0$, the cylinder outcrops and we get

$$g = 2\pi\gamma\rho\left\{L + R - \left(L^2 + R^2\right)^{1/2}\right\} \tag{2.59}$$

4. If $L \to \infty$, we have

$$g = 2\pi\gamma\rho\left\{\left(z^2 + R^2\right)^{1/2} - z\right\} \tag{2.60}$$

If, in addition, $z = 0$, we have

$$g = 2\pi\gamma\rho R \tag{2.61}$$

When $L \gg z$, we can use Equation (2.60) to get the gravity off-axis (see MacRobert, 1948:151–5 or Pipes and Harvill, 1970:348–9). Because g satisfies Laplace's equation, we can express it in a series of

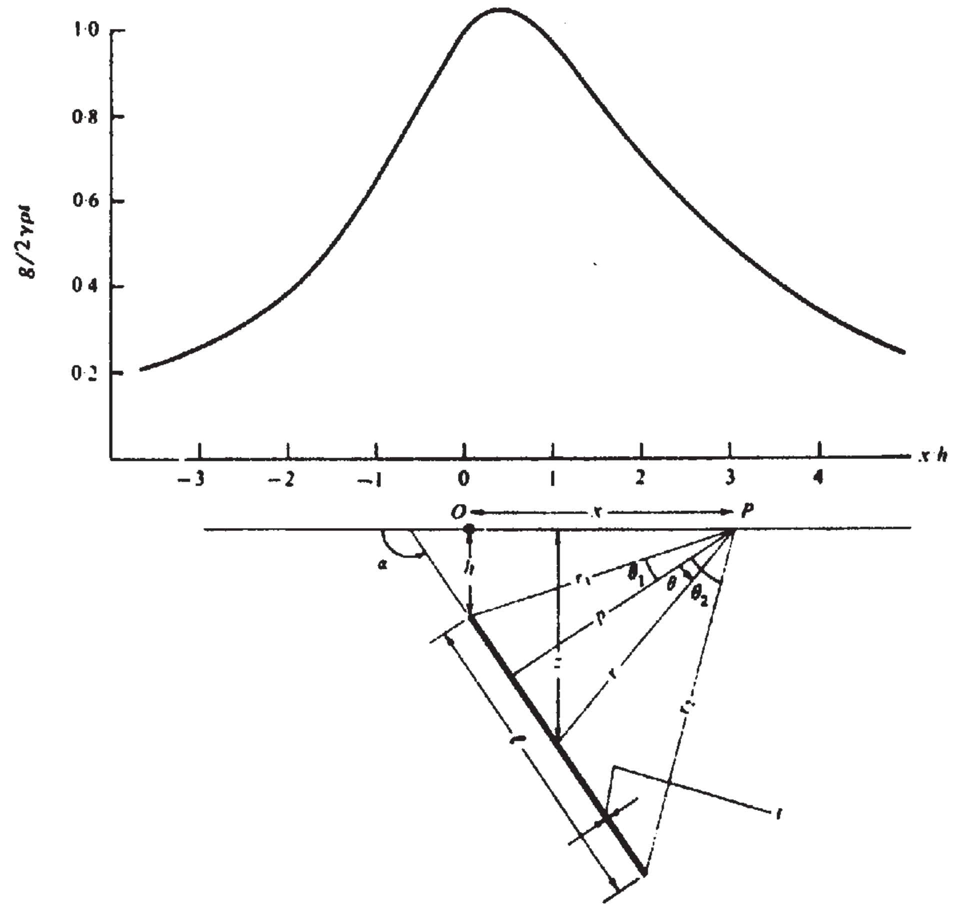

Figure 2.27. Gravity effect of a thin sheet of infinite strike length.

Legendre polynomials $P_n(\mu)$ where $\mu = \cos\theta$ (Pipes and Harvill, 1970:799–805). Taking $r > z$ in Figure 2.26, we have three cases to consider: $r > z > R$, $R > r > z$, and $r > R > z$. For the first case, we get (see problem 3)

$$g(r,\theta) = 2\pi\gamma\rho R\{(R/2r) - (R/2r)^3 P_2(\mu) + 2(R/2r)^5 P_4(\mu) - \cdots\} \quad (2.62)$$

For the second case, $R > r > z$, the result is

$$g(r,\theta) = 2\pi\gamma\rho R\{1 - 2(r/2R)P_1(\mu) + 2(r/2R)^2 P_2(\mu) - 2(r/2R)^4 P_4(\mu) + \cdots\} \quad (2.63)$$

The result for the third case, $r > R > z$, is the same as Equation (2.62), showing that Equation (2.62) is valid whenever $r > R$. From Equations (2.62) and (2.63) we get the curve in Figure 2.26.

2.7.5. Gravity Effect of a Thin Dipping Sheet

Considerable simplification can be effected when a body can be considered two-dimensional. In general, this holds when the strike length is about 20 times the other dimensions (including depth).

Referring to Figure 2.27, we have the following relations:

$$p = (x - h\cot\alpha)\sin\alpha = x\sin\alpha - h\cos\alpha,$$

$$r = p\sec\theta$$

$$z = r\sin(\alpha + \theta - \pi/2) = p(\sin\alpha\tan\theta - \cos\alpha)$$

$$dz = p\sin\alpha\sec^2\theta\, d\theta$$

$$r_1 = (x^2 + h^2)^{1/2},$$

$$r_2 = \{(x + \ell\cos\alpha)^2 + (h + \ell\sin\alpha)^2\}^{1/2}$$

Now we apply Equation (2.9) for a two-dimensional structure. The product $dx\,dz$ in Equation (2.9) represents an element of area of the cross section, that is,

$$dx\,dz = t\,d\ell = t\csc\alpha\, dz = tp\sec^2\theta\, d\theta$$

Equation (2.9) now gives (note that r' is the same as

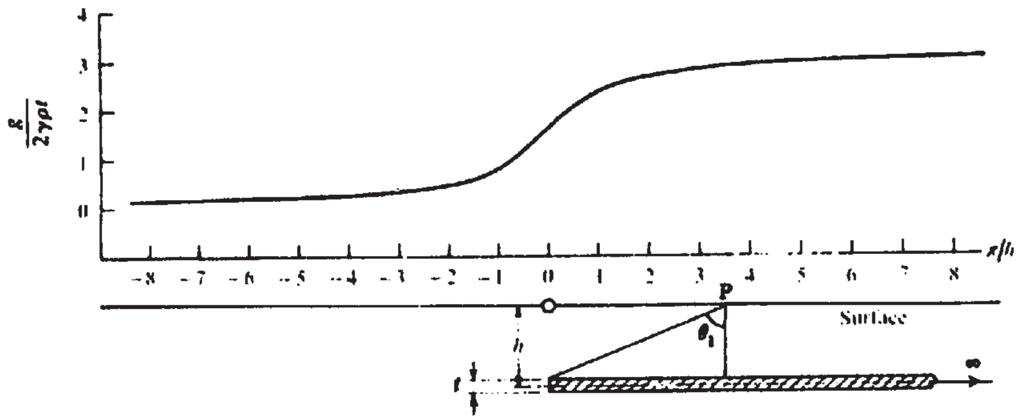

Figure 2.28. Gravity effect of a semiinfinite horizontal sheet.

r here)

$$g = 2\gamma\rho t p\int_{-\theta_1}^{\theta_2}(z/r^2)\sec^2\theta\, d\theta$$

$$= 2\gamma\rho t\int_{-\theta_1}^{\theta_2}(\sin\alpha\tan\theta - \cos\alpha)\, d\theta$$

$$= 2\gamma\rho t\{\sin\alpha\ln(\cos\theta_1/\cos\theta_2) - (\theta_2 + \theta_1)\cos\alpha\}$$

$$= 2\gamma\rho t\{\sin\alpha\ln(r_2/r_1) - (\theta_2 + \theta_1)\cos\alpha\} \tag{2.64}$$

If the sheet is vertical, Equation (2.64) simplifies to

$$g = 2\gamma\rho t\ln\left[\{(h + \ell)^2 + x^2\}/(x^2 + h^2)\right] \tag{2.65}$$

The thin sheet is a good approximation to a prism unless the thickness of the prism is somewhat greater than h, the depth to the top. When the dip is steep (> 60°), the depth can be roughly estimated from the half-width, for example, when $h \approx \ell$, $h \approx 0.7x_{1/2}$. However, when ℓ is large or when the dip is small it is not possible to get a reliable estimate.

2.7.6. Gravity Effect of Horizontal Sheets, Slabs, Dikes, and Faults

(a) Horizontal thin sheet. When the sheet in Equation (2.64) is horizontal, $\alpha = \pi$ and we have

$$g = 2\gamma\rho t(\theta_1 + \theta_2)$$

$$= 2\gamma\rho t\left[\tan^{-1}\{(\ell - x)/h\} + \tan^{-1}(x/h)\right] \tag{2.66}$$

If, in addition, $\ell \to \infty$, we have, for a semiinfinite horizontal sheet,

$$g = 2\gamma\rho t\{\pi/2 + \tan^{-1}(x/h)\} \tag{2.67}$$

and if the sheet extends to infinity in the other direction (that is, x goes to infinity as well) we have the Bouguer correction as in Equation (2.57) with t replacing L.

The profile for a semiinfinite horizontal sheet is shown in Figure 2.28. The thin sheet result can be used to approximate a horizontal slab with an error less than 2% when $h > 2t$. A fault often can be approximated by two semiinfinite horizontal sheets, one displaced above the other as in Figure 2.29.

(b) Horizontal slab. Equation (2.67) can be used to find the gravity effect of a semiinfinite horizontal slab terminating at a plane dipping at the angle α (Fig. 2.30). We use Equation (2.67) to get the effect of the thin sheet of thickness dz and then integrate to find the result for the slab (Geldart, Gill, and Sharma, 1966).

We must replace x in Equation (2.67) with $(x + z\tan\beta)$, so $\tan^{-1}(x/h)$ becomes $\tan^{-1}\{(x + z\tan\beta)/z\} = \theta$. Equation (2.67) now gives

$$g = 2\gamma\rho\int_{z_1}^{z_2}(\pi/2 + \theta)\, dz = 2\gamma\rho\left(\pi t/2 + \int_{z_1}^{z_2}\theta\, dz\right)$$

We now have:

$$\tan\theta = (x + z\tan\beta)/z = (x/z) + \tan\beta$$

$$z = x/(\tan\theta - \tan\beta)$$

$$dz = -x\sec^2\theta\, d\theta/(\tan\theta - \tan\beta)^2$$

$$= -x\cos^2\beta\, d\theta/\sin^2(\theta - \beta)$$

$$= -x\cos^2\beta\, d\psi/\sin^2\psi$$