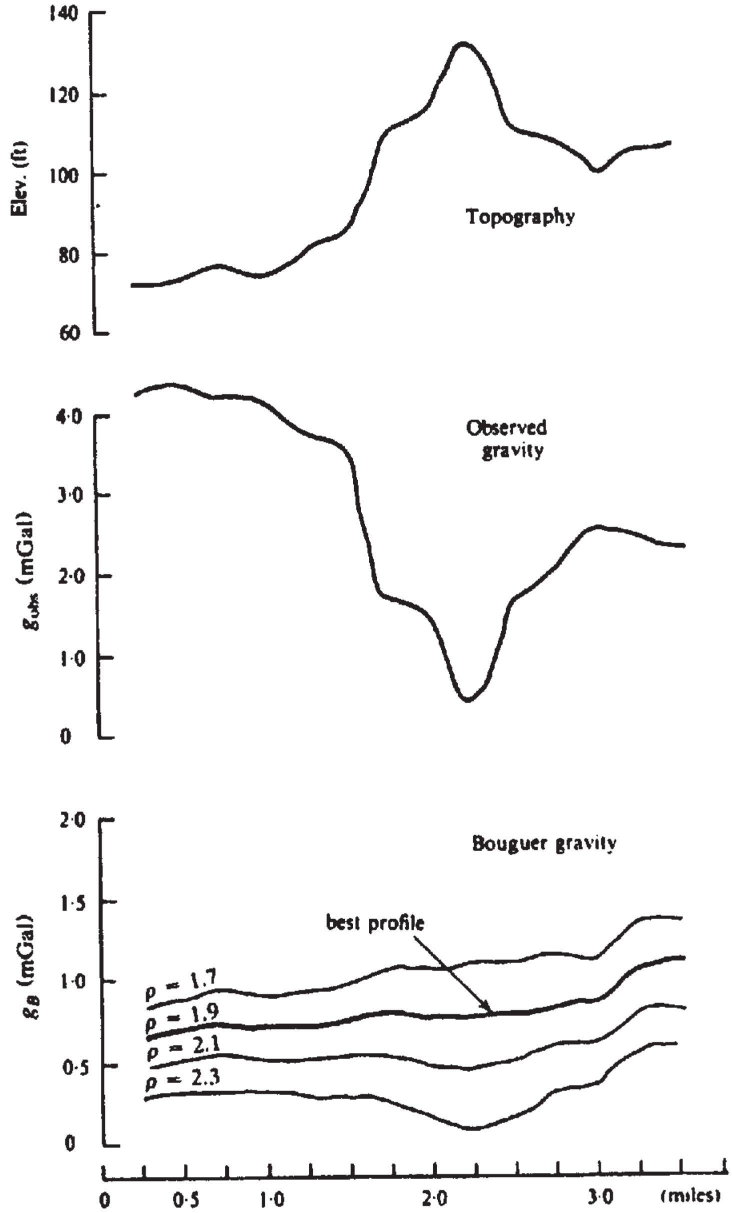

Figure 2.8. Method for estimating surface density.

we obtain

$$(g_{obs} - g_t + \Delta g_L + 0.3086z) - g_B = (0.0419z - \Delta g_T/\rho)\rho \qquad (2.31a)$$

$$(g_{obs} - g_t + \Delta g_L + 0.0941z') - g_B = (0.0128z' - \Delta g_T/\rho)\rho \qquad (2.31b)$$

where z is in meters and z' is in feet. We wish to determine the average bulk density for the data set by considering the Bouguer anomaly g_B to be a random error of mean value zero. If we plot

$$(g_{obs} - g_t + \Delta g_L + 0.3086z)$$

versus $(0.0419z - \Delta g_T/\rho)$ (or the equivalent in terms of z'), the slope of the best-fit straight line through the origin will be ρ.

2.4. GRAVITY INSTRUMENTS

2.4.1. General

The absolute measurement of gravity is usually carried out at a fixed installation by the accurate timing of a swinging pendulum or of a falling weight.

Relative gravity measurements may be made in various ways. Three types of instruments have been used: the torsion balance, the pendulum and the gravimeter (or gravity meter). The latter is the sole instrument now used for prospecting, the others having only historical interest.

2.4.2. Absolute Measurement of Gravity

Although the timing of a freely falling body was the first method of measuring g, the accuracy was poor because of the difficulty in measuring small time intervals. The method has been revived as a result of instrumentation improvements and elaborate free-fall installations are now located at several national laboratories. It is necessary to measure time to about 10^{-8} s and distance to $< \frac{1}{2}$ μm to obtain an accuracy of 1 mGal with a fall of 1 or 2 m.

Until recently, the standard method for measuring g employed a modified form of the *reversible Kater pendulum*. The value of g was obtained by timing a large number of oscillations.

2.4.3. Relative Measurement of Gravity

(a) Portable pendulum. The pendulum has been used for both geodetic and prospecting purposes. Since g varies inversely as the square of the period T, we have

$$T^2 g = \text{constant}$$

Differentiating, we get

$$\begin{aligned} \Delta g &= -2g\Delta T/T \\ &= -2g(T_2 - T_1)/T_1 \end{aligned} \tag{2.32}$$

Thus if we can measure the periods at two stations to about 1 μs, the gravity difference is accurate to 1 mGal. This is not difficult with precise clocks such as quartz crystal, cesium, or rubidium.

The pendulum has been used extensively for geodetic work, both on land and at sea (in submarines). Portable pendulums used in oil exploration during the early 1930s had a sensitivity of about 0.25 mGal. Pendulum apparatus was complex and bulky. Two pendulums, swinging in opposite phase, were used to reduce sway of the mounting; they were enclosed in an evacuated, thermostatically controlled chamber to eliminate pressure and temperature effects. To get the required accuracy, readings took about $\frac{1}{2}$ hr.

(b) Torsion balance. A fairly complete account of the salient features of the torsion balance can be found in Nettleton (1976). Figure 2.9 is a schematic of the torsion balance. Two equal masses m are separated both horizontally and vertically by rigid bars, the assembly being supported by a torsion fiber with an attached mirror to measure rotation by the deflection of a light beam. Two complete beam assemblies were used to reduce the effects of support sway. Readings were taken at three azimuth positions of the beam assemblies, normally 120° apart, to get sufficient data to calculate the required results. Elaborate precautions were required to minimize extraneous effects such as temperature and air convection. Each station had to be occupied for approximately one hour so that daily production was only 8 to 10 stations.

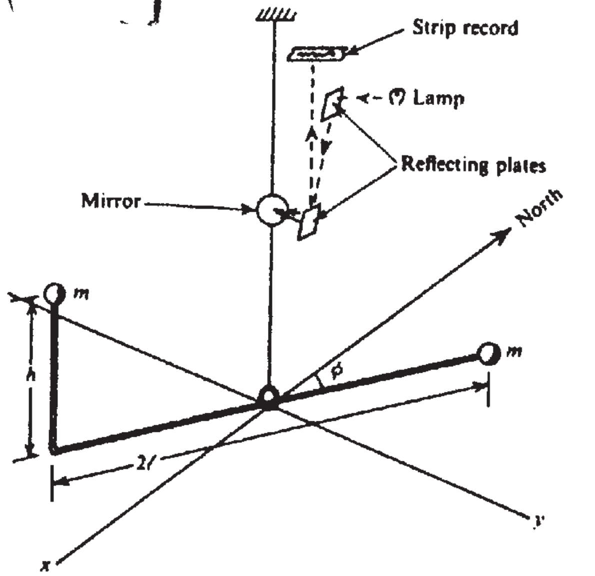

Figure 2.9. Torsion balance (schematic).

The deflection of the torsion balance beam is due to horizontal and vertical changes in the gravity field resulting from curvature of the equipotential surfaces. Torsion-balance measurements permitted calculation of U_{xy}, U_{xz}, U_{yz}, and $|U_{yy} - U_{xx}|$. The plotted values are usually the ***horizontal gradient*** [the vector $(U_{xz}\mathbf{i} + U_{yz}\mathbf{j})$] and the ***differential curvature*** [a vector with magnitude given by Equation (2.19) and direction relative to the x axis of $(1/2)\tan^{-1}(2U_{xy}/|U_{yy} - U_{xx}|)$]. Measurements were usually in Eötvös units (EU) equal to 10^{-6} mGal/cm.

(c) Stable-type gravimeters. The first gravimeters dating from the early 1930s were of the stable type but these have now been superceded by more sensitive unstable meters. Nettleton (1976) describes a number of different gravimeters. All gravimeters are essentially extremely sensitive mechanical balances in which a mass is supported by a spring. Small changes in gravity move the weight against the restoring force of the spring.

The basic elements of a stable gravimeter are shown in Figure 2.10. Whereas the displacement of the spring is small, Hooke's law applies, that is, the change in force is proportional to the change in length; hence,

$$\Delta F = M\,\delta g = k\,\delta s \quad \text{or} \quad \delta g = k\,\delta s/M \tag{2.33}$$

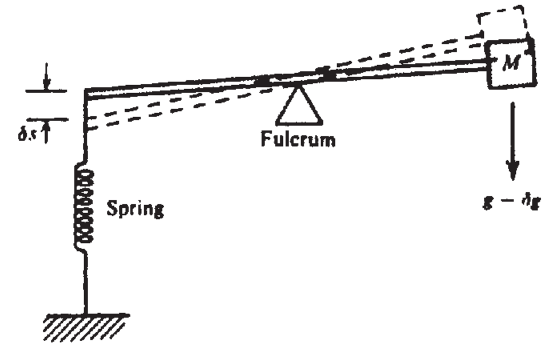

Figure 2.10. Basic principle of the stable gravimeter.

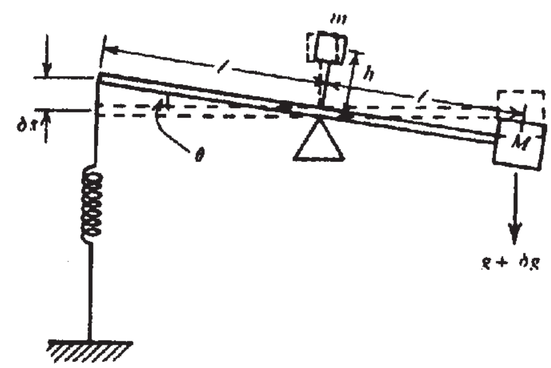

Figure 2.11. Basic principle of the unstable (Thyssen) gravimeter. (After Dobrin, 1960.)

where k is the spring constant in dynes per centimeter. To measure g to 0.1 mGal, we must detect a fractional change in spring length of $1/10^7$ (because $Mg \approx ks$, $\delta g/g \approx \delta s/s$), hence the need for considerable magnification. Mechanically we can make k/M small by using a large mass and a weak spring, but this method of enhancing sensitivity is limited. The period of oscillation of this system is

$$T = 2\pi(M/k)^{1/2}$$

Substituting for M in Equation (2.33), we get

$$\delta g = 4\pi^2 \delta s/T^2 \qquad (2.34)$$

Thus for good sensitivity, the period is very large and measurement of δg requires considerable time. Stable gravimeters are extremely sensitive to other physical effects, such as changes in pressure, temperature, and small magnetic and seismic variations.

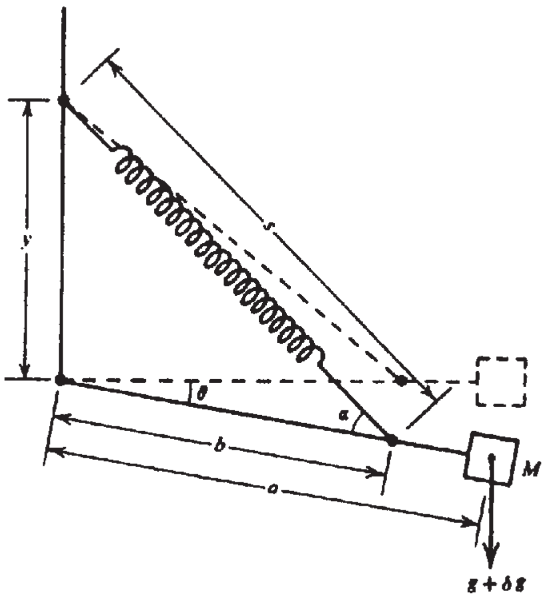

Figure 2.12. Lacoste – Romberg gravimeter.

(d) Unstable-type gravimeters. Also known as *labilized or astatized gravimeters*, these instruments have an additional negative restoring force operating against the restoring spring force, that is, in the same sense as gravity. They essentially are in a state of unstable equilibrium and this gives them greater sensitivity than stable meters. Their linear range is less than for stable gravimeters so they are usually operated as null instruments.

The *Thyssen gravimeter*, although now obsolete, illustrates very clearly the astatic principle (Fig. 2.11). The addition of the mass m above the pivot raises the center of gravity and produces the instability condition. If g increases, the beam tilts to the right and the moment of m enhances the rotation; the converse is true for a decrease in gravity.

At present the Worden and LaCoste–Romberg meters are the only types used for exploration.

(e) LaCoste – Romberg gravimeter. The LaCoste-Romberg gravimeter was the first to employ a zero-length spring, now used by almost all gravimeters (Askania, Frost, Magnolia, and North American). A *zero-length spring* is one in which the tension is proportional to the actual length of the spring, that is, if all external forces were removed the spring would collapse to zero length. The advantage of the zero-length spring is that if it supports the beam and mass M (see Fig. 2.12) in the horizontal position, it will support them in any position (note that $\cos\theta$ in Eq. (2.35) cancels out, and $g = K(1 - c/s)$, which always has a solution since g is finite). Zero-length-springs are built with initial tension so that a threshold force is required before spring extension begins (as with a door spring).

To derive the expression for the sensitivity of the LaCoste-Romberg gravimeter, we write $k(s - c)$ for the tension in the spring when its length is s; thus, c is a small correction for the fact that the spring is not truly zero length. Taking moments about the pivot in Figure 2.12, we get

$$\begin{aligned} Mga\cos\theta &= k(s - c)b\sin\alpha \\ &= k(s - c)b(y\cos\theta)/s \qquad (2.35) \end{aligned}$$

Figure 2.13. Reading a Worden gravimeter.

using the law of sines. Thus

$$g = (k/M)(b/a)(1 - c/s)y$$

When g increases by δg, the spring length increases by δs where

$$\delta g = (k/M)(b/a)(c/s)(y/s)\,\delta s \quad (2.36)$$

For a given change in gravity δg, we can make δs as large as we wish by decreasing one or more of the factors on the right-hand side; moreover, the closer the spring is to the zero-length spring, the smaller c is and the larger δs becomes.

In operation this is a null instrument, a second spring being used, which can be adjusted to restore the beam to the horizontal position. The sensitivity of gravimeters in use in surface exploration is generally 0.01 mGal. The instrument requires a constant-temperature environment, usually achieved by keeping it at a constant temperature that is higher than the surroundings.

(f) Worden gravimeter. The Worden gravimeter (Fig. 2.13) is especially portable and fast to operate. It uses small, very light weight parts of quartz (for example, the mass M weighs only 5 mg) with small inertia so that it is not necessary to clamp the movement between stations. Sensitivity to temperature and pressure changes is reduced by enclosing the system in a vacuum flask. The meter also employs an automatic temperature-compensating arrangement. The Worden meter is small (instrument dimensions are a few centimeters, the outer case is about 25 cm high and 12 cm in diameter) and weighs about 2.5 kg. Its only power requirement is two penlight cells for illuminating the scale.

A simplified schematic is shown in Figure 2.14. The moving system is similar to the LaCoste–Romberg meter. The arm OP' and beam OM are rigidly connected and pivot about O, changing the length of the main spring $P'C$, which is fixed at C. We have the following relations:

$$\angle OCP' = \angle OP'C = \pi/2 - (\alpha + \theta/2)$$

$$RP \perp CP \qquad P'P \perp OP$$

so

$$\angle RPP' = \pi/2 - \alpha$$

$$s = CP \qquad \delta s = CP' - CP \approx b\theta \sin(\pi/2 - \alpha)$$

so

$$\theta \approx \delta s/(b\cos\alpha)$$

The correction factor c that appeared in the treatment of the LaCoste–Romberg meter is negligible for the Worden meter. Taking moments about the pivot for the case where $\theta = 0$, we get

$$Mga = ksb\cos\alpha$$

When g increases to $(g + \delta g)$, P moves along the