Chapter 2

Gravity Methods

2.1. INTRODUCTION

2.1.1. General

Gravity prospecting involves measurements of variations in the gravitational field of the earth. One hopes to locate local masses of greater or lesser density than the surrounding formations and learn something about them from the irregularities in the earth's field. It is not possible, however, to determine a unique source for an observed anomaly. Observations normally are made at the earth's surface, but underground surveys also are carried out occasionally.

Gravity prospecting is used as a reconnaissance tool in oil exploration; although expensive, it is still considerably cheaper than seismic prospecting. Gravity data are also used to provide constraints in seismic interpretation. In mineral exploration, gravity prospecting usually has been employed as a secondary method, although it is used for detailed follow-up of magnetic and electromagnetic anomalies during integrated base-metal surveys. Gravity surveys are sometimes used in engineering (Arzi, 1975) and archaeological studies.

Like magnetics, radioactivity, and some electrical techniques, gravity is a natural-source method. Local variations in the densities of rocks near the surface cause minute changes in the gravity field. Gravity and magnetics techniques often are grouped together as the *potential methods*, but there are basic differences between them. Gravity is an inherent property of mass, whereas the magnetic state of matter depends on other factors, such as the inducing fields and/or the orientations of magnetic domains. Density variations are relatively small, and the gravity effects of local masses are very small compared with the effect of the background field of the Earth as a whole (often of the order of 1 part in 10^6 to 10^7), whereas magnetic anomalies often are large relative to the main field. The time variation of the magnetic field is complex, whereas the gravity field is constant (ignoring "earth tides"). Corrections to gravity readings are more complicated and more important than in magnetic or other geophysical methods. Gravity field operations are more expensive than magnetic operations, and field work is slower and requires more highly skilled personnel.

2.1.2. History of Gravity Exploration

Galileo Galilei, in about 1589, so legend tells us, dropped light and heavy weights from the Leaning Tower of Pisa in an attempt to determine how weight affects the speed at which a given object falls. Johann Kepler worked out the laws of planetary motion, and this enabled Sir Isaac Newton to discover the universal law of gravitation (*Mathematical Principles of Natural Philosophy*, 1685–87).

The expeditions of the French Academy of Sciences to Lapland and Peru (Ecuador) in 1735–45 gave Pierre Bouguer the opportunity to establish many of the basic gravitational relationships, including variations of gravity with elevation and latitude, the horizontal attraction due to mountains, and the density of the Earth.

Captain Henry Kater, in 1817, introduced the compound pendulum, with interchangeable centers of oscillation and suspension, which became the major tool for gravity investigation for over a century. Because the variations in gravitational attraction are so small, Baron Roland von Eötvös set out to measure derivatives rather than total magnitudes. He completed his first torsion balance (a modification of the Coulomb balance) in 1890 and made the first gravity survey on the ice of Lake Balaton in 1901. F. A. Vening Meinesz, in 1923, measured gravity with pendulums on board a Dutch submarine and demonstrated gravity variations over various areas of the oceans, especially the large gravity effects near the Indonesian trench.

In December 1922, a torsion-balance survey of the Spindletop oil field initiated geophysical exploration for oil. In late 1924, a test well on the Nash salt dome in Brazoria County, Texas, verified the gravity interpretation, becoming the first geophysical hydrocarbon discovery, although the first producing oil well did not come in until January 1926.

The last half of the 1920s saw extensive gravity surveys with the torsion balance. In 1929 the portable pendulum began to be used, followed in 1932 by the stable gravimeter (and the unstable gravimeter, which was not publicly described until 1937). By 1940, gravimeters had become so stable and convenient that torsion balances and portable pendulums disappeared from use. LaCoste (1934) described the zero-length spring, but the first workable LaCoste gravimeter did not appear until 1939. In subsequent years, gravimeters have been adapted (LaFehr, 1980) to measurements under water, on moving ships and aircraft, and in boreholes.

The major addition to our knowledge of gravity in recent years has come from observations of satellite paths (Kahn, 1983). These have considerably increased our knowledge of the detailed shape of the Earth, but this has not changed gravity exploration significantly.

In the 1940s, graphic and grid methods of isolating anomalies were developed, and the anomalies that result from simple shapes were calculated. The computing power made available by digital computers since the 1960s has considerably increased our interpretation capabilities, the ultimate goal being solution of the *inverse problem* (§2.7.9).

2.2. PRINCIPLES OF GRAVITY

2.2.1. Newton's Law of Gravitation

The force of gravitation is expressed by *Newton's law*: The force between two particles of masses m_1 and m_2 is directly proportional to the product of the masses and inversely proportional to the square of the distance between the centers of mass:

$$\mathbf{F} = \gamma(m_1 m_2/r^2)\mathbf{r}_1 \qquad (2.1)$$

where $\mathbf{F}$ is the force on m_2, $\mathbf{r}_1$ is a unit vector directed from m_2 toward m_1, r is the distance between m_1 and m_2, and γ is the universal gravitational constant. Note that the force $\mathbf{F}$ is always attractive. In SI units the value of γ is 6.672×10^{-11} N m^2/kg^2 or in cgs units 6.672×10^{-8} dyne cm^2/g^2.

2.2.2. Acceleration of Gravity

The acceleration of m_2 due to the presence of m_1 can be found by dividing $\mathbf{F}$ by m_2 in Equation (2.1), that is,

$$\mathbf{g} = (\gamma m_1/r^2)\mathbf{r}_1 \qquad (2.2a)$$

The acceleration $\mathbf{g}$ is equal to the gravitational force per unit mass due to m_1. If m_1 is the mass of the Earth, M_e, $\mathbf{g}$ becomes the *acceleration of gravity* and is given by

$$\mathbf{g} = (\gamma M_e/R_e^2)\mathbf{r}_1 \qquad (2.2b)$$

R_e being the radius of the Earth and $\mathbf{r}_1$ extending downward toward the center of the Earth. (It is customary to use the same symbol $\mathbf{g}$ whether it is due to the Earth or a mass m.) The acceleration of gravity was first measured by Galileo in his famous experiment at Pisa. The numerical value of $\mathbf{g}$ at the Earth's surface is about 980 cm/s^2. In honor of Galileo, the unit of acceleration of gravity, 1 cm/s^2, is called the galileo or Gal.

Gravimeters used in field measurements have a sensitivity of about 10^{-5} Gal or 0.01 mGal, although the reading accuracy is generally only 0.03 to 0.06 mGal. As a result, they are capable of distinguishing changes in the value of g with a precision of one part in 10^8. Microgravimeters are available with measuring accuracy of about 5 μGal.

2.2.3. Gravitational Potential

(a) Newtonian or three-dimensional potential. Gravitational fields are conservative; that is, the work done in moving a mass in a gravitational field is independent of the path traversed and depends only on the end points (§A.3.4). If the mass is eventually returned to its original position, the net energy expenditure is zero, regardless of the path followed. Another way of expressing this is to say that the sum of kinetic (motion) energy and potential (position) energy is constant within a closed system.

The gravitational force is a vector whose direction is along the line joining the centers of the two masses. The force giving rise to a conservative field may be derived from a scalar potential function $U(x, y, z)$, called the *Newtonian* or *three-dimensional potential*, by finding the gradient [Eqs. (A.17), (A.30), and (A.31)]:

$$\begin{aligned}\nabla U(x, y, z) &= -\mathbf{F}(x, y, z)/m_2 \\ &= -\mathbf{g}(x, y, z) \qquad (2.3a)\end{aligned}$$

In spherical coordinates (Fig. A.4b) this becomes

$$\begin{aligned}\nabla U(r, \theta, \phi) &= -\mathbf{F}(r, \theta, \phi)/m_2 \\ &= -\mathbf{g}(r, \theta, \phi) \qquad (2.3b)\end{aligned}$$

Alternatively, we can solve this equation for the

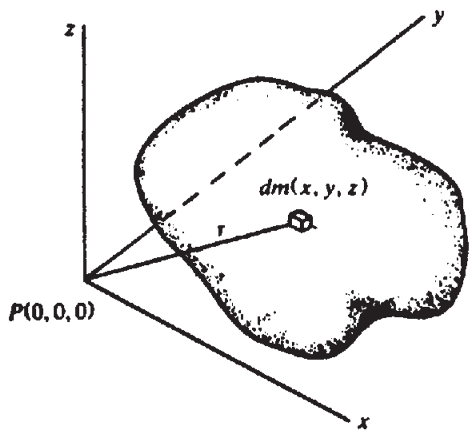

Figure 2.1. Potential of three-dimensional mass.

gravitational potential in the form [Eq. (A.16)]

$$U(r,\theta,\phi) = \int_{\infty}^{r}(\nabla U)\cdot d\mathbf{r}$$
$$= -\int_{\infty}^{r}\mathbf{g}\cdot d\mathbf{r} \qquad (2.4)$$

which is a statement of the work done in moving a unit mass from infinity (that is, a very distant point), by any path, to a point distant r from the point mass producing the gravitational field. Using Equation (2.2a) in scalar form, we get

$$U(r) = -\gamma\int_{\infty}^{r}m(1/r^2)\,dr = \gamma m/r \qquad (2.5)$$

It is often simpler to solve gravity problems by calculating the scalar potential U rather than the vector $\mathbf{g}$ and then to obtain $\mathbf{g}$ from Equation (2.3).

Considering a three-dimensional mass of arbitrary shape as in Figure 2.1, the potential and acceleration of gravity at a point outside the mass can be found by dividing the mass into small elements and integrating to get the total effect. From Equation (2.5), the potential due to an element of mass dm at the point (x, y, z) a distance r from $P(0,0,0)$ is

$$dU = \gamma\,dm/r = \gamma\rho\,dx\,dy\,dz/r$$

where $\rho(x, y, z)$ is the density, and $r^2 = x^2 + y^2 + z^2$. Then the potential of the total mass m is

$$U = \gamma\int_x\int_y\int_z(\rho/r)\,dx\,dy\,dz \qquad (2.6a)$$

Because g is the acceleration of gravity in the z direction (positive vertically downward), and assuming ρ constant,

$$g = -(\partial U/\partial z)$$
$$= \gamma\rho\int_x\int_y\int_z(z/r^3)\,dx\,dy\,dz \qquad (2.7a)$$

Sometimes it is more convenient to use cylindrical coordinates (Figure A.4a). Because $dx\,dy\,dz = r_0\,dr_0\,d\theta\,dz$ and $r^2 = r_0^2 + z^2$, $r_0^2 = x^2 + y^2$, the potential becomes

$$U = \gamma\rho\int_{r_0}\int_\theta\int_z(r_0/r)\,dr_0\,d\theta\,dz \qquad (2.6b)$$

and the acceleration in the z direction is

$$g = \gamma\rho\int_{r_0}\int_\theta\int_z(r_0 z/r^3)\,dr_0\,d\theta\,dz \qquad (2.7b)$$

In spherical coordinates,

$$dx\,dy\,dz = r^2\sin\theta\,dr\,d\theta\,d\phi$$

hence,

$$U = \gamma\rho\int_r\int_\theta\int_\phi r\sin\theta\,dr\,d\theta\,d\phi \qquad (2.6c)$$

Taking the z axis along the polar axis,

$$g = -\partial U/\partial z$$
$$= -\gamma\rho\int_r\int_\theta\int_\phi(z/r)\sin\theta\,dr\,d\theta\,d\phi$$
$$= -\gamma\rho\int_r\int_\theta\int_\phi \sin\theta\cos\theta\,dr\,d\theta\,d\phi \qquad (2.7c)$$

because $z/r = \cos\theta$. (The minus sign indicates that g is directed toward the mass dm at the center of the sphere.)

(b) Logarithmic or two-dimensional potential. If the mass is very long in the y direction and has a uniform cross section of arbitrary shape in the xz plane, the gravity attraction derives from a logarithmic (rather than Newtonian) potential. Then Equation (2.6a) becomes

$$U = \gamma\rho\int_x\int_z dx\,dz\int_{-\infty}^{\infty}(1/r)\,dy$$

With some manipulation (see problem 1), the logarithmic potential becomes

$$U = 2\gamma\rho\int_x\int_z \ln(1/r')\,dx\,dz \qquad (2.8)$$

where $r'^2 = x^2 + z^2$. The gravity effect for the two-dimensional body is

$$g = -\partial U/\partial z = 2\gamma\int_x\int_z\rho(z/r'^2)\,dx\,dz \qquad (2.9)$$

2.2.4. Potential-Field Equations

The divergence theorem [Gauss's theorem; Eq. (A.27)] states that the integral of the divergence of a vector field **g** over a region of space V is equivalent to the integral of the outward normal component of the field **g** over the surface enclosing the region. We have

$$\int_V \nabla \cdot \mathbf{g}\, dv = \int_S g_n\, ds \qquad (2.10)$$

If there is no attracting matter within the volume, the integrals are zero and $\nabla \cdot \mathbf{g} = 0$. But from Equation (2.3a) the gravitational force is the gradient of the scalar potential U, so that

$$-\nabla \cdot \mathbf{g} = \nabla \cdot \nabla U = \nabla^2 U = 0 \qquad (2.11a)$$

that is, the potential in free space satisfies Laplace's equation. In cartesian coordinates, Laplace's equation is

$$\nabla^2 U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0 \qquad (2.11b)$$

[see Eq. (A.37) for Laplace's equation in spherical coordinates]. Also, because $g = -\partial U/\partial z$, and any derivative of a solution of a differential equation is also a solution, we have

$$\nabla^2 g = 0 \qquad (2.11c)$$

If, on the other hand, there is a particle of mass at the center of a sphere of radius r, then

$$\int_S g_n\, ds = -(\gamma m/r^2)(4\pi r^2) = -4\pi\gamma m \qquad (2.12a)$$

the minus meaning that g_n is opposite to **n**, the outward-drawn normal. It can be shown (see problem 2) that this result holds regardless of the shape of the surface and the position and size of the mass within the surface. If the surface encloses several masses of total mass M, we can write

$$\int_V \nabla \cdot \mathbf{g}\, dv = \int_S g_n\, ds = -4\pi\gamma M \qquad (2.12b)$$

If the volume V is very small, enclosing only a point, we can remove the integral sign to give

$$\nabla \cdot \mathbf{g} = -4\pi\gamma\rho \qquad (2.13a)$$

where ρ is the density at the point. Then, from Equation (2.3a),

$$\nabla^2 U = 4\pi\gamma\rho \qquad (2.13b)$$

which is *Poisson's equation*.

Equations (2.11a) and (2.13b) state that the gravity potential satisfies Laplace's equation in free space and Poisson's equation in a region containing mass.

These equations imply that various distributions of mass can produce the same potential field over a surface (Skeels, 1947); this is sometimes called the "inherent ambiguity" of gravity interpretation. Sometimes it is convenient to substitute for masses distributed throughout a volume V a fictitious *surface density* of mass over a surface S enclosing V such that the effect outside S is the same. From Equations (2.12b) and (2.13a) we have

$$\int_V (-4\pi\gamma\rho)\, dv = \int_S g_n\, ds \qquad (2.14)$$

that is, the component of gravity perpendicular to the surface gives the equivalent surface density. For an equipotential surface, this is merely the total gravitational field.

2.2.5. Derivatives of the Potential

Quantitites useful in gravity analysis may be obtained by differentiating the potential in various ways. We have already noted in Equation (2.7a) that vertical gravity $g = -\partial U/\partial z$. This is the quantity measured by gravimeters.

The first vertical derivative of g [from Eq. (2.7a)] is

$$\begin{aligned}\partial g/\partial z &= -\partial^2 U/\partial z^2 \\ &= -U_{zz} \\ &= \gamma\rho \int_x\int_y\int_z (1/r^3 - 3z^2/r^5)\, dx\, dy\, dz \qquad (2.15)\end{aligned}$$

where subscripts indicate derivatives of U. Measurements occasionally are made of the vertical gradient (Falkiewicz, 1976; Jordan, 1978; Ager and Lilard, 1982; Butler, 1984).

The *second vertical derivative* is

$$\begin{aligned}\partial^2 g/\partial z^2 &= -\partial^3 U/\partial z^3 \\ &= -U_{zzz} \\ &= 3\gamma\rho \int_x\int_y\int_z (5z^3/r^7 - 3z/r^5)\, dx\, dy\, dz \qquad (2.16)\end{aligned}$$

This derivative frequently is employed in gravity interpretation for isolating anomalies (§2.6.5) and for upward and downward continuation (§2.6.7).

Derivatives tend to magnify near-surface features by increasing the power of the linear dimension in the denominator. That is, because the gravity effect varies inversely as the distance squared, the first and second derivatives vary as the inverse of the third and fourth powers, respectively (for three-dimensional bodies).

By taking the derivatives of g in Equation (2.7a) along the x and y axes, we obtain the components of the *horizontal gradient of gravity*:

$$U_{xz} = -\partial g/\partial x$$
$$= 3\gamma\rho \int_x \int_y \int_z (xz/r^5)\, dx\, dy\, dz \quad (2.17)$$

and similarly for the y component U_{yz}. The horizontal gradient can be determined from gravity profiles or map contours as the slope or rate of change of g with horizontal displacement. The horizontal gradient is useful in defining the edges and depths of bodies (Stanley, 1977).

The *differential curvature* (or *horizontal directive tendency*, HDT) is a measure of the warped or curved shape of the potential surface. From Equation (2.6a),

$$U_{xx} = \gamma\rho \int_x \int_y \int_z (3x^2/r^5 - 1/r^3)\, dx\, dy\, dz \quad (2.18)$$

Other components are U_{yy} and U_{xy}. The differential curvature (HDT) is given by

$$\mathrm{HDT} = \left\{ (U_{yy} - U_{xx})^2 + (2U_{xy})^2 \right\}^{1/2}$$
$$= 3\gamma\rho \int_x \int_y \int_z \left\{ (x^2 + y^2)/r^5 \right\}\, dx\, dy\, dz \quad (2.19)$$

It is not possible to measure U_{xx}, U_{yy}, U_{xy}, or HDT directly. Differential curvature can be obtained from torsion-balance measurements.

2.3. GRAVITY OF THE EARTH

2.3.1. Figure of the Earth

(a) General. Gravity prospecting evolved from the study of the Earth's gravitational field, a subject of interest to geodesists for determining the shape of the Earth. Because the Earth is not a perfect homogeneous sphere, gravitational acceleration is not constant over the Earth's surface.

The magnitude of gravity depends on five factors: latitude, elevation, topography of the surrounding terrain, earth tides, and density variations in the subsurface. Gravity exploration is concerned with anomalies due to the last factor, and these anomalies generally are much smaller than the changes due to lattitude and elevation, although larger than the anomalies due to tidal and (usually) topographic effects. The change in gravity from equatorial to polar regions amounts to about 5 Gal, or 0.5% of the average value of g (980 Gal), and the effect of elevation can be as large as 0.1 Gal, or 0.01% of g. A gravity anomaly considered large in oil exploration, on the other hand, would be 10 mGal, or 0.001% of g, whereas in mineral exploration a large anomaly would be 1 mGal. Thus, variations in g that are significant in prospecting are small in comparison with the magnitude of g and also in comparison with latitude and elevation effects. Fortunately, we can, with good accuracy, remove most of the effects of factors that are not of interest in prospecting.

(b) The reference spheroid. The shape of the Earth, determined by geodetic measurements and satellite tracking, is nearly spheroidal, bulging at the equator and flattened at the poles. The *polar flattening* is $(R_{eq} - R_p)/R_{eq} = 1/298.25$, where R_{eq} and R_p are the Earth's equatorial and polar radii, respectively.

The *reference spheroid* is an oblate ellipsoid that approximates the mean sea-level surface (*geoid*), with the land above it removed. In 1930 the International Union of Geodesy and Geophysics adopted a formula (Nettleton, 1976, p. 17) for the theoretical value of gravity g_t, but this has been superseded (Woolard, 1979) by the Geodetic Reference System 1967 (GRS67):

$$g_t = 978{,}031.846(1 + 0.005{,}278{,}895 \sin^2\phi$$
$$+ 0.000{,}023{,}462 \sin^4\phi)\ \mathrm{mGal} \quad (2.20)$$

where ϕ is latitude.

(c) The geoid. Mean continental elevations are about 500 m, and maximum land elevations and ocean depressions are of the order of 9,000 m referred to sea level. Sea level is influenced by these variations and other lateral density changes. We define mean sea level (the equipotential for the Earth's gravity plus centrifugal effects), called the *geoid*, as the average sea level over the oceans and over the surface of sea water that would lie in canals if they were cut through the land masses.

The simplified *figure of the Earth* allows for increasing density with depth, but not for lateral variations, which are the objects of gravity exploration. Because of the lateral variations, the geoid and reference spheroid do not coincide. Local mass anomalies warp the geoid as in Figure 2.2a. We might expect